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The Idiosyncratic Volatility Puzzle – Anomaly or Data Mining?

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Abstract

In this study, I investigate the robustness of the idiosyncratic volatility puzzle to the configuration of the research design. Using the regression- as well as the portfolio-based concept, I start with the replication of the idiosyncratic volatility puzzle approving the findings of Ang, Hodrick, Xing, and Zhang (2006). However, when idiosyncratic volatility is estimated from monthly data and a time window spanning 1 or 5 years, the puzzle vanishes, regardless of the research method employed. Similar result hold if only stocks with a market capitalization above the cross-sectional median or those with a price higher than 10$ are used. Independent of the weighting scheme, the puzzle is also absent in the regression-based context when the risk premia are estimated by generalized least squares weighting returns by the inverse of their variance estimates. The same finding is derived in the portfolio-based context by extending the holding period to 12 months or controlling for the past month maximum daily return.

Keywords: Idiosyncratic Volatility; Cross-section of stock returns; Predictability; Risk Premium; Robustness.

1. Introduction

Common asset pricing models such as the CAPM argue that systematic market risk is the only type of risk that should be compensated with a return premium in the equilibrium. Idiosyncratic risk, on the other hand, should not be priced, as it is assumed to be perfectly diversifiable by holding a combination of the market portfolio and a risk-free investment. Nonetheless, some authors claim that perfect diversification might not be possible, if investors cannot attain the market portfolio due to market frictions such as transaction costs or incomplete information (e.g. Malkiel & Xu, 2006; Merton, 1987). Merton (1987) argues that in this case investors no longer only care for the market, but the total risk which includes the idiosyncratic volatility (IVOL hereafter) as special part. Therefore, idiosyncratic risk should be of importance to the investors decision making process and hence be compensated with a positive risk premium in the cross-section of average returns from a theoretical point of view. However, in an influential study, Ang et al. (2006) empirically discovered a negative relation between IVOL and subsequent stock returns instead. This finding is particularly puzzling when considering standard asset pricing theory and has started the debate about the importance of a pricing effect on idiosyncratic volatility. Many papers were published that verified the negative relation but claim to be able to explain it with new theories supported by empirical results, whereas other studies emerged that completely challenged the finding of a negative relationship. One example is the study of Fu (2009). His findings fit to the standard asset pricing theory, as he empirically confirmed a positive link between conditional IVOL and expected returns. This illustrates that not only the empirical findings of a consistent pricing effect induced by IVOL diverge from its theoretical predictions but also that the empirical evidence varies across studies. As there are various studies that appear to analyze the same theoretical concept, the question may arise why their results differ so fundamentally from each other. The research concepts related to the IVOL puzzle can be classified into two major groups which consist of the regression- and portfolio-based approach. Whereas the regression-based concept focuses on the estimation of the IVOL risk premium by means of monthly cross-sectional regressions, the portfolio-based method utilizes the idea of sorting stocks into portfolios based on their exposure to IVOL and searches for a systematic pattern in portfolio returns respectively. Even though both of these concepts are able to discover the IVOL puzzle as shown by Ang et al. (2006); Ang, Hodrick, Xing, and Zhang (2009), not all studies come to this same conclusion. Since Ang et al. (2006) for example use value-weighted portfolios for their analysis, Bali and Cakici (2008) find that using equal-weighted portfolio returns instead, lets the puzzle vanish. Also, in the regression-based

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concept, the IVOL puzzle is sensitive to the research design, as Bali, Cakici, and Whitelaw (2011) claim that the puzzle no longer exists when they integrate a control for the maximum daily return over the past month into their cross-sectional regressions. Hence, the choice of the research design is important for answering the question on the existence of an IVOL puzzle. On this background I aim to answer the question on how robust the IVOL puzzle is to a specific research design. To do so, I analyze in what aspects the studies on idiosyncratic volatility mainly differ and how these differences influence their conclusions. In addition, I incorporate further adjustments that I consider adequate and try to verify the robustness of the findings respectively. My analysis is structured along the regression- and the portfolio-based approach on the basis of which I investigate the differences in results that might emerge from adjustments in the research design. Here I start with implementation of adjustments that are applicable to both research concepts and afterwards I focus on those modifications that are related to the methodological peculiarities of the research concepts. By this methodology, I intend to settle the dispute on the existence and nature of the idiosyncratic volatility risk premium as well as to illustrate the influence a researcher has on the results obtained simply by choosing a specific research design. Using the example of idiosyncratic volatility, I try to highlight the importance to also consider the study's econometric design when interpreting its result.

My study is structured as follows. In section 2 I start with a brief overview on the literature related to the pricing effect of idiosyncratic risk as well as the corresponding research approaches and their findings. Afterwards section 3 sets out the methodology of my study. It begins with the description on how I measure idiosyncratic volatility, then gives the explanation on how the reference results based on the regression- and the portfolio-based research concept are derived and closes with an explanation on the kind of adjustments I consider as well how they are incorporated into the research concepts. Section 4 introduces the data used for this study. The empirical analysis starts in section 5 with some summary statistics on the sample. Afterwards I continue with the replication of the IVOL puzzle on the basis of the regression- and portfolio-based research concept that should later on act as a reference point for the further adjustment analysis. The empirical analysis of the general adjustments, consisting of idiosyncratic volatility and sample-related modifications, is located in section 6. In contrast, section 7 presents the evaluation of the method-specific adjustments. For the regression-based concept these adjustments involve changes in the risk-related control variables as well as the regression estimation procedure, while those adjustments for the portfolio-based method focus on modifications in the way portfolios are characterized and analyzed, as well as the interaction of the IVOL puzzle with other firm-specific effects when these are controlled for by means of bivariate dependent portfolio sorts. Lastly, section 8 concludes.

2. Literature Overview and Theoretical Motivation

2.1. The Pricing Effect of Idiosyncratic Risk

Markowitz (1952) was among the first to formulate the idea of a positive relationship between systematic risk and expected returns. In his study, he allows investors to only construct a portfolio-based on all stocks available in the market and measures risk as the portfolio return variance. His advice is then to select a portfolio laying on the efficient frontier, as these attain the optimal risk-return trade-off, which is why he terms them to be "mean-variance efficient" (see Markowitz, 1952). Mean-variance efficient portfolios contain the part of the variance that remains after diversification and thus determines the extent of risk that is equal for all assets in the portfolio. This type of risk is called "systematic risk" and should be compensated by a positive return premium to make a risky investment attractive for risk averse investors. Risk that can be diversified away is called "idiosyncratic risk" and should not yield any compensation, as no influence on the diversified portfolio's payoff is assumed. Based on the portfolio theory of Markowitz (1952) and his concept of mean-variance efficiency, Sharpe (1964), Lintner (1965) and Black (1972) formulate the well-known Capital Asset Pricing Model (CAPM hereafter). In their model, the investment strategy with the optimal risk-return trade-off in equilibrium involves investing in the mean-variance efficient market portfolio and a risk-free investment such as the risk-free interest rate. The market portfolio acts as a proxy for systematic risk and the asset's co-variation with this portfolio is captured by the market beta. This market beta allows for quantification of the asset's exposure to systematic risk and determines the part of the risk that requires compensation by a return premium. Hence, portfolios with a higher market beta are assumed to contain more systematic risk and should therefore deliver higher expected returns. In the CAPM, the linear relationship between securities' expected returns and their market betas implies that, in the equilibrium, idiosyncratic risk can be perfectly diversified and thus does not carry a risk premium.

However, authors have begun to question the CAPM assumptions including the irrelevance of idiosyncratic risk for pricing purposes and provided theoretical evidence that this kind of risk is indeed priced under plausible assumptions, even though the direction of the pricing effect has remained controversial. Merton (1987), for example, argues that when investors have incomplete information on the characteristics of securities, they are prone to hold assets whose risk-return profile they are familiar with. In consequence, they hold under-diversified portfolios and no longer care only for systematic market risk but instead demand a return premium for the total amount of risk including its idiosyncratic part. Hence, incomplete information leads to a situation in which diversifiable risk would be priced as investors require compensation for their imperfect diversification. Therefore, Merton (1987) predicts a positive relationship between idiosyncratic risk and cross-sectional stock returns. Conversely, Miller (1977) sets the framework for an opposite theoretical prediction. His study is based on the assumption that investors diverge in their perception of expected returns for a specific firm and also constrains them in their short-selling activities. He argues that stocks which are characterized by dispersed opinions on their future payoffs are likely systematically overvalued, as their prices are mainly determined by market participants who are most optimistic about the firms expected returns and hence willing to pay the highest price. When this overvaluation is eventually corrected as relevant information becomes available to all investors over time, disagreements on future returns diminish and prices are corrected which leads to lower subsequent returns (see Miller, 1977). In addition Miller (1977) claims that this effect increases with the level of dispersion in opinions on future returns leading to even lower subsequent returns for stocks with a higher degree of disagreement. Diether, Malloy, and Scherbina (2002) measure the heterogeneity of opinions by the dispersion in analysts' forecasts and claim that it can proxy for idiosyncratic risk. They find that expected returns are negatively related to dispersion in analysts' forecasts and thus conclude also on a negative relationship to idiosyncratic risk which contradicts the notion of Merton (1987).

To support the above mentioned theoretical predictions, several studies have emerged that try to unveil the true pricing implications of idiosyncratic risk empirically. Among the first studies to do so is the one of Fama and MacBeth (1973). In a linear regression-based framework, they measure idiosyncratic risk as standard deviation of the least-squares regression residual that was generated by regressing the firm's stock excess return onto the CAPM. Based on their market beta, they sort all stocks into 20 portfolios and regress them each month onto the portfolio market beta, the squared portfolio market beta and the average idiosyncratic risk of all stocks in the portfolio to compute their regression coefficients that proxy for the variables' risk premia. They use monthly returns of all common stocks trading on the New York Stock Exchange (NYSE hereafter) in the period of January 1926 to June 1968 and find a statistically insignificant risk premium, wherefore they conclude that idiosyncratic risk is not priced in the cross-section of average returns (see Fama & MacBeth, 1973).

In a subsequent replication of the study from Fama and MacBeth (1973), Tinic and West (1986) use data from 14 additional years and conclude that idiosyncratic risk is indeed priced, carrying a positive risk premium which corresponds to the predictions of Merton (1987). Malkiel and Xu (1997) draw the same conclusion. Using portfolios sorted by idiosyncratic volatility they find that portfolios containing stocks with the highest idiosyncratic volatility also exhibit the highest returns. It is of note, however, that they do not report any statistics for the verification of statistical significance. In their follow up study, Malkiel and Xu (2006) utilize a regression-based method comparable to that of Tinic and West (1986) as well as Fama and MacBeth (1973) and show that the positive relationship between expected returns and IVOL holds up in statistical tests. Spiegel and Wang (2005) in addition to Fu (2009) argue that past realized idiosyncratic risk is not able to identify the true relationship between IVOL and expected returns, which is why they suggest to use conditional IVOL instead that is estimated using an Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) model. Both studies define idiosyncratic volatility relative to the Fama and French (1993) three-factor model (FF3 model hereafter) again, finding a positive return premium for IVOL (see Fu (2009); Spiegel and Wang (2005)). The FF3 model uses a market (MKT), size (SMB) and value (HML) factor for systematic risk-correction (see Fama & French, 1993).

Even though a positive risk premium for idiosyncratic risk violates the CAPM assumptions, it still retains the notion of Markowitz (1952) that risk averse investors require a return premium for taking on volatility risk. Ang et al. (2006) are among the first to disagree with previous studies as they propose a negative relationship between idiosyncratic risk and expected returns. This relationship is hardly explained by classic risk-return arguments, wherefore it is also known as the "Idiosyncratic Volatility Puzzle". Ang et al. (2006) sort all stocks traded on the American Stock Exchange (AMEX hereafter), National Association of Securities Dealers Automated Quotations (NASDAQ hereafter) and the NYSE into quintile portfolios based on their past month idiosyncratic volatility and point out that the portfolio in the highest IVOL quintile systematically underperform the one in the lower IVOL quintile by a statistically significant average total return difference of about -1.06%. This result also holds for risk-adjusted returns relative to the CAPM and FF3 model as well as when controlling for various cross-sectional effects in bivariately sorted portfolios (see Ang et al., 2006). Authors such as A. G. Huang (2009), Ang et al. (2009) and Duarte, Kamara, Siegel, and Sun (2014) are able to verify the finding of an IVOL puzzle. A. G. Huang (2009) uses both, a regression-based approach in fashion of Fama and MacBeth (1973) as well as a portfolio sorting approach comparable to that of Ang et al. (2006) and confirm that the underperformance of the highest IVOL portfolio remains statistically significant even if risk adjustment is induced by the Carhart (1997) four-factor (CH4 hereafter) model that augments the FF3 model by a momentum factor (MOM hereafter) (see Ang et al., 2006; Carhart, 1997). Ang et al. (2009) add international evidence on the IVOL puzzle by analyzing a data set consisting of 23 developed countries spanning a sample period from January 1980 to December 2003. In addition, they discover an international co-movement in portfolio returns that goes long the quintile portfolio containing firms with the highest IVOL and short those of the lowest IVOL quintile, hinting at systematic risk that is not captured by standard risk factors (see Ang et al., 2009). This idea is picked up by Duarte et al. (2014) who also confirm the negative relationship between idiosyncratic volatility and expected returns and construct a risk factor based on the return difference of a portfolio with high predicted IVOL as well as the one of low predicted IVOL. Adding this risk factor to the FF3 model, they improve in explaining the cross-section of returns, wherefore they argue that the IVOL puzzle arises due to an omitted systematic risk factor (see Duarte et al., 2014).

2.2. Regression- vs. Portfolio-Based Method

To understand the contradictory findings on the relationship between idiosyncratic risk and the cross-section of expected returns as outlined in the previous section, it is necessary to get an overview on the research concepts and adjustments used in different studies. Two branches have emerged in the literature that investigate the pricing implications of idiosyncratic risk. The first branch follows the ideas of Fama and MacBeth (1973) and implements a regression-based methodology that is based on the risk premia estimation by means of cross-sectional linear regressions. In contrast, the second branch uses a portfolio-based methodology as proposed in Ang et al. (2006) where stocks are sorted into portfolios based on their idiosyncratic volatility and are then analyzed respectively.

The regression-based approach is based on the ideas of Fama and MacBeth (1973). They propose a two step procedure that starts with running a time-series regression of all stocks' monthly excess returns onto a risk-correction model which they choose to be the CAPM and compute the firm-specific risk exposure by the stock-related market beta. Afterwards 20 portfolios are formed based on the ranking of all stocks relative to their estimated market betas. These portfolios are constantly rebalanced and the stock-specific betas are regularly recomputed. Then, the simple average over all stock betas within a portfolio is used to compute the portfolio beta. The standard deviation of the regression residual from the time-series regression in the first step is used as a proxy for the stock-specific IVOL that is also averaged across all firms within a portfolio to generate a portfolio level IVOL measure. In the second step they compute cross-sectional regressions in each period and average the estimated coefficients over their sample time horizon to obtain an estimate on the expected risk premium for the corresponding regressor. Their study investigates the premium on the market beta as a proxy for the price of market risk, the premium on the squared market beta as a measure for non-linearity in market risk and the expected risk premium on their IVOL measure drawing the conclusions already mentioned in section 2.1 (see Fama & MacBeth, 1973). Ang et al. (2009) were among the first to verify the existence of the IVOL puzzle with such an approach but using single stocks instead of portfolios as dependent variable for the cross-sectional regressions.

In subsequent studies authors have begun to adjust the regression-based methodology to improve it by handling well-known econometric problems or simply updating the method based on recent empirical findings and new theoretical concepts such as more sophisticated risk-correction models. Malkiel and Xu (2006) try to handle the well-known "errors-in-variables" problem which arises in the IVOL estimation on the firm level. For this they use the ideas of Fama and French (1992) and apply the portfolio IVOL on all stocks within the portfolio. First they sorted all stocks into 200 portfolios based on its exposure to size and market beta. Thereafter they calculate the portfolio IVOL and assign this value to all stocks located in the specific portfolio. This IVOL is then used in the cross sectional regression with individual stocks (see Malkiel & Xu, 2006). Subsequently, Ang et al. (2009) measure IVOL relative to the FF3 model and pick up the idea of using single stock returns instead of portfolios in the cross-sectional regressions. They are also among the first to correct the t-statistics of the average cross-sectional risk premia estimates for serial correlation and heteroskedasticity using the adjustment from Newey and West (1987). Z. Chen and Petkova (2012) use two different techniques for the estimation of time-series risk factor loadings that focus on different degrees of time variation in these estimates. One method uses the complete sample for this time-series regression, whereas the other implements 60-months rolling estimation windows for that purpose which allows for better characterization of the dynamically changing firm-specific risk exposure. Furthermore, changes in the cross-sectional regression estimation technique have also been considered throughout the literature. Among others Ang et al. (2009) as well as Han and Lesmond (2011) suggest to incorporate value weighting into the regression approach of Fama and MacBeth (1973), as they weight all stock returns with the corresponding firm size at the beginning of the month to avoid small firm effects distorting their risk premia estimates. Moreover, some authors consider adjustments related to the control variables used in cross-sectional regressions. Various studies show that the choice of the risk-related control variables capturing systematic or firm-specific risk can have a notable impact on the IVOL risk premium. Boyer, Mitton, and Vorkink (2010) among others regress portfolio excess returns not only onto the firm's CH4 model risk factor loadings to account for systematic risk but also add those on a liquidity and a coskewness risk factor respectively. Additionally they include variables such as expected idiosyncratic skewness, size, the book-to-market ratio, momentum and turnover into their cross-sectional regressions to account for firm-specific risk which was not considered in the study of Fama and MacBeth (1973) (see Boyer et al., 2010). L. H. Chen, Jiang, Xu, and Yao (2012) propose the idea of a subsample analyses to dissect the finding of an IVOL puzzle from the effect of other firm characteristics. Hence they conduct part of their regression-based analysis in subsamples constructed based on attributes such as size, price and calendar time (see L. H. Chen et al. (2012)).

In contrast, the general idea of the portfolio-based research method is to sort stocks based on their idiosyncratic volatility into a pre-specified number of portfolios and rebalance these portfolios regularly. For all portfolios, the next period's return (total or in excess of the risk-free rate) as well as the Jensen's alphas relative to a select risk-correction model are computed each period. Based on the long-short portfolio, which goes long the portfolio of stocks with the highest IVOL and short the one containing those stocks with the lowest IVOL, the alpha and return difference between high and low IVOL portfolios is computed which should deliver evidence on the pricing implications from idiosyncratic risk (see e.g. Ang et al., 2006; Bali & Cakici, 2008; Rachwalski & Wen, 2016). Ang et al. (2006) are among the first to conduct such an analysis. They measure idiosyncratic volatility as standard deviation of the residual from a time-series regression of the stock-specific excess return onto the FF3 model using daily data over the past month. Based on this IVOL measure, they sort stocks into monthly rebalanced quintile portfolios for which they compute the value-weighted (VW hereafter) monthly total returns as well as the Jensen's alpha relative to the CAPM and the FF3 model that they then average over time for presentation purposes. Lastly, by investigating the return patterns from the quintile and the long-short portfolio, they discover the IVOL puzzle (see Ang et al., 2006).

The portfolio-based research concept has also been adjusted by various studies. Ang et al. (2006) argue that the trading strategy used for portfolio construction can influence the findings, which is why they implemented other strategies for robustness purposes but were not able to identify any influence on the IVOL puzzle in their study. In addition, also the number of univariate portfolios analyzed was subject to change. While Ang et al. (2006) and later Duarte et al. (2014) propose quintile portfolios for the analysis, Spiegel and Wang (2005) use decile portfolios and find a positive relationship between IVOL and expected returns instead. Bali and Cakici (2008) show that the choice of the return weighting scheme, the breakpoints necessary to subdivide stocks into portfolios and the data frequency used for IVOL estimation can lead to different conclusions on the existence of the IVOL puzzle. Their analysis of equal-weighted (EW hereafter), instead of VW portfolio returns, delivers no evidence on an IVOL puzzle. Neither do portfolios formed from IVOL breakpoints that should account for potential size effects and hence are calculated from stocks traded on the NYSE only, or are set such that stocks are subdivided into portfolios that each make up an equal market share. They draw the same conclusion when using monthly data over the past two or five years to estimate IVOL (see Bali & Cakici, 2008). Other studies also differ in the choice of the risk-correction model used for computation of the Jensen's alpha. Ang et al. (2006), Bali and Cakici (2008) and Fu (2009) report alphas that correct for the risk factors from the CAPM or FF3 model, whereas Spiegel and Wang (2005) as well as Rachwalski and Wen (2016) use the CH4 model for that purpose. Cao, Chordia, and Zhan (2021) adds the alpha relative to the Fama and French (2015) five-factor model (FF5 model hereafter) that augments the FF3 model by an investment (CMA) and a profitability (RMW) factor. Furthermore, Ang et al. (2006) and A. G. Huang (2009) use bivariate dependent sorted portfolios to investigate the robustness of the IVOL puzzle to firm characteristic effects. For that, they choose a control variable based on which they sort all stocks into portfolios first. Then all stocks within each of these portfolio are sorted again into portfolios based on their IVOL. Lastly, each of these IVOL portfolios is averaged across the control variable sorts resulting in average portfolios that capture the IVOL puzzle effect which is robust to the effects of the selected control (see Ang et al., 2006; A. G. Huang, 2009).

3. Methodology

This section introduces the methodology used throughout my study. Section 3.1 starts with the definition of my idiosyncratic volatility measure. In section 3.2 and 3.3 I present my benchmark specification for the regression- as well as the portfolio-based research approach, on the basis of which I replicate the IVOL puzzle finding and implement further adjustments. Lastly, section 3.4 sets out my adjustment analysis.

3.1. Estimation of Idiosyncratic Volatility

Usually idiosyncratic volatility is computed as the standard deviation of the residual from the regression of the firm's excess returns onto to a risk-correcting model like the FF3. Regardless of the model used, the following general time-series regression is run for each firm to obtain the corresponding idiosyncratic volatility estimate:

$$r_t^i = \alpha^i + \beta^i X_t + \epsilon_t^i, \tag{1}$$

where r_t^i is the return of stock i at time t in excess of the risk-free rate and α^i stands for the time-series intercept for stock i. $\beta^i X_t$ is a 1 x k vector of risk factor loadings for firm i that are estimated relative to the k x 1 vector of risk factors X_t that should proxy for systematic risk. ϵ_t^i presents the regression residual for firm i at time t, on the basis of which idiosyncratic volatility is generally computed as follows:

$$IVOL_i^X(t;t-s) = \sqrt{var_{t;t-s}(\epsilon_t^i)}. \tag{2}$$

Here $IVOL_i^X(t;t-s)$ presents the IVOL of firm i at time t measured relative to risk-correction model X over the time horizon from $t-s$ to t. The right-hand side of the equation states that IVOL is measured as the standard deviation of the time-series regression residuals over the time horizon from $t-s$ to t. Equation 1 allows for the integration of various risk-correcting models to compute IVOL which becomes important in the later adjustment analysis. If not otherwise stated, I use the FF3 model for risk-correction and compute IVOL from the following time-series regression for each stock separately while using daily data:

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \epsilon_t^i, \tag{3}$$

β_{MKT}^i refers to the time-series regression factor loading on the MKT risk factor of firm i. β_{SMB}^i and β_{HML}^i stand for the factor loadings of firm i on the SMB and HML risk factor respectively. MKT_t, SMB_t and HML_t represent the level of the corresponding risk factor at time t. For estimation of the IVOL measure utilized throughout most of my analysis, I use the daily residuals from equation 3 and compute it as follows:

$$IVOL_{i,t}^{FF3} = \sqrt{var(\epsilon_t^i)}. \tag{4}$$

$IVOL_{i,t}^{FF3}$ is then defined as the idiosyncratic volatility relative to the FF3 model of firm i in month t that is measured as the standard deviation of the daily residuals from regression 3 over the current month. For firms with less than 15 daily return observations, the IVOL measure is considered as missing. If not otherwise stated, this definition is used when referring to idiosyncratic volatility. Modifications to this definition are considered in the later adjustment analysis.

3.2. Regression-Based Approach

I use the study of Ang et al. (2009) as a reference point for implementation of the regression-based research approach, as it is among the first to verify the existence of the IVOL puzzle using a linear regression framework comparable to that of Fama and MacBeth (1973). This procedure is subdivided into two steps: a time-series regression and a cross-sectional regression step.

In the first step, a time-series regression equal to that of equation 1 is run for every firm. Here a pre-defined time window as well as risk-correction model are used to obtain the firm-specific risk exposure measured by the risk factor loadings. If not otherwise stated, I compute the risk factor loadings relative to the FF3 model based on equation 3. Similar to the IVOL estimation, monthly factor loadings of firm i are computed using rolling windows of one month that contain daily data. These factor loadings are then used in the second step of the procedure.

In the second step, I compute cross-sectional regressions at every month t across all firms for which data are available at that month. These have the following form:

$$r_i(t;t+1) = c + \gamma IVOL_i^X(t;t-s) + \lambda_b^{'} b_i(t;t+1) + \lambda_z^{'} z_i(t) + \epsilon_{t+1}^i, \quad (5)$$

where $r_i(t;t+1)$ is the stock's excess return from time t to $t+1$, c is the cross-sectional regression intercept, $b_i(t;t+1)$ is the m x 1 vector of time-series risk factor loadings for firm i computed over the window from time t to $t+1$, $z_i(t)$ is the k x 1 vector of firm characteristics for firm i observable at time t and ϵ_{t+1}^i is the cross-sectional regression residual of stock i at time $t+1$. γ, $\lambda_b^{'}$ and $\lambda_z^{'}$ define the 1 x 1, 1 x m and 1 x k vectors of cross-sectional regression coefficients on idiosyncratic volatility, the time-series risk factor loadings and the firm characteristics respectively. These coefficients should approximate the monthly risk premia for the corresponding regressors. Subsequently, I use the time series of monthly risk premia and the intercept to test whether the average premium on the chosen idiosyncratic volatility measure is statistically different from zero. For that I use a standard t-test where I correct the standard errors of the coefficient estimates for serial correlation by the methodology of Newey and West (1987). Lags are selected based on the maximum value proposed by the quadratic spectral or the Bartlett kernel that is then rounded downwards to the next integer value. In addition, I calculate the average monthly R^2 and number of stocks used in the corresponding regression.

Whereas the previous explanations are given generally to enhance the understanding of the later adjustment analysis, I will now concretize the regression-based research methodology that is used as reference point throughout my empirical analysis. I use monthly data to estimate the cross-sectional regression of equation 5 at each month t by means of the ordinary least squares (OLS hereafter). On the left-hand side of this equation, I use the next month excess return for stock i measured from the end of month t to $t+1$. As IVOL measure, I utilize the one defined in equation 4 that is computed from daily data over the current month. Risk-correction is implemented by usage of the time-series risk factor loadings on the FF3 model from equation 3 that are obtained from daily data over the next month $t+1$. For the calculation of the time-series risk factor loadings, I again require each stock to report at least 15 daily return observations during the month. As I rely on the concept of Ang et al. (2009) for the regression-based methodology, I apply almost the same firm characteristics and start further adjustment analyses from there on. Hence, my vector of firm characteristics at each t consists of the firms' size measured by the natural logarithm of market capitalization, the firm's book-to-market ratios of equity (BM hereafter) calculated based on Fama and French (1992) and the total return measured from the end of the previous month $t-1$ until the end of current month t to control for the one-month return reversal effect discovered by Lehmann (1990) and Jegadeesh (1990). In addition to these firm characteristics I augment the study by sequentially rotating among further firm-related control variables that might be interrelated with the IVOL puzzle effect. To reduce the arbitrariness in selecting these variables, I rely on the findings of L. H. Chen et al. (2012) as well as Hou and Loh (2016) who evaluated the pervasiveness of the IVOL puzzle across different explanations found in the literature. Barberis and Huang (2008) claim that investors have preferences for extreme events when evaluating stocks that they quantify by stock return skewness. Consequently I consider this effect by controlling for the skewness of stock returns over the current month, the expected idiosyncratic skewness measure of Boyer et al. (2010) and the coskewness measure over the recent past 5 years computed based on the procedure of Harvey and Siddique (2000). Lastly, I also account for the liquidity-related IVOL puzzle explanation and include the Amihud (2002) illiquidity measure as a control variable (see Han & Lesmond, 2011). At this step of the analysis I use 7 lags for the standard error correction based on Newey and West (1987). I term the above explained method as equal-weighted Fama-MacBeth regression procedure. However, similar to Ang et al. (2009) I also consider a value-weighted approach. For this I use a weighted least squares (WLS) method to estimate risk premia in the cross-section regression of equation 5, where all returns are weighted by the firm's current month market capitalization (MTCAP hereafter) in U.S. dollars. I call this procedure the value-weighted Fama-MacBeth regression.

3.3. Portfolio-Based Approach

For implementation of the portfolio-based research method I rely on the study of Ang et al. (2006). Here the idea is that all stocks traded on the NYSE/AMEX/NASDAQ are sorted into a pre-defined number of portfolios based on their past idiosyncratic volatility. These portfolios are rebalanced regularly and analyzed, additionally to a long-short portfolio, to identify the relationship between IVOL and expected cross-sectional returns non-parametrically. The procedure starts by defining the trading strategy. Such a strategy consists of an estimation period E, a waiting period W and a holding period H. Using this notation, an $E/W/H$ trading strategy can be computed as follows. At month t the IVOL measure is computed for each firm over the E-month period from $t-E-W$ to month $t-W$ based on equation 2. Afterwards it is annualized by multiplication with $\sqrt{250}$. Based on the calculated IVOL, at each month t breakpoints are determined that subdivide stocks into a pre-defined number of portfolios. These portfolios are then held for the next H-months. As proposed by Ang et al. (2006), for portfolios with $E > 1$ and $H > 1$ I follow Jegadeesh and Titman (1993) and compute portfolios with overlapping holding periods. Hence, for a 6/1/6 strategy at month t the pre-defined number of portfolios are constructed from the IVOL estimate over the past 6 months ending 1 month prior to the formation date t and using daily return data. The same number of portfolios is constructed based on the IVOL estimate over the past 6 month but ending 2 months, 3 months up to 6 months prior to t. Then a simple average across all the sets of portfolios with different ending periods are calculated, such that only the set consisting of the number of pre-defined portfolios constructed from the averaging procedure remains. Consequently, 1/6th of the composition of these portfolios changes each month. Furthermore, each 1/6th of these portfolios consists of the portfolio from 1 month ago and 1/6th of the portfolio from 2 month ago up to the last 1/6th that consists of the portfolio from 6 month ago. In addition, I construct a long-short portfolio that goes long the portfolio of the highest IVOL firms and short the one with the lowest IVOL firms. This portfolio should directly reveal the effect of IVOL on the cross-section of expected returns. After having constructed the monthly time-series of IVOL portfolios introduced above, measures such as the portfolio's monthly average equal- and value-weighted total and excess returns as well as the Jensen's alphas are computed. Lastly, these averages are tested for significance by a standard t-test where standard errors are corrected by the concept of Newey and West (1987). The objective is to find out whether the average difference of the alphas from the long-short portfolio as well as its average equal- and value-weighted total and excess returns are statistically significant different from zero and have a negative sign which could be interpreted as finding of the IVOL puzzle in the data.

After having explained the general portfolio-based approach, I will now clarify the specification used as a reference point for the adjustment analysis. I mostly use a 1/0/1 trading strategy where IVOL is computed over the past month and equal- as well as value-weighted portfolio returns are computed over the subsequent month. Stocks are sorted based on their past month IVOL relative to the FF3 model into quintile portfolios. Then the long-short portfolio is constructed from the quintile portfolio containing stocks with the highest past month IVOL minus the one containing stocks with lowest IVOL. All these portfolios are rebalanced monthly. For all portfolios I compute the equal- and value-weighted total and excess returns as well as their time-series alphas relative to the FF3 and the Fama and French (2018) six-factor model (FF6 model hereafter) based on the equal- and value-weighted portfolio excess returns respectively. The FF6 model augments the FF5 model for better risk-correction with the momentum factor from Carhart (1997) (see Fama & French, 2018). Value-weighted monthly returns are calculated by averaging the returns of all stocks within the portfolio at month t while weighting them with their corresponding MTCAP. Alphas are calculated from equation 1 over the complete sample time span where the dependent variable is now the monthly equal- or value-weighted portfolio excess return. As further portfolio characteristics I calculate the time-series average of the monthly size and BM of stocks within the portfolio. The average monthly size and BM are computed as simple averages over the respective values for all stocks in the portfolio at the specific month. Furthermore, I compute the standard deviation of the monthly excess returns for every portfolio. For the t-tests of the average portfolio alphas and returns, I implement the Newey and West (1987) correction using 7 lags.

3.4. Adjustment Analysis

I proceed with the explanation of the adjustments to both reference research concepts introduced above with the aim of finding out how these affect the IVOL puzzle finding. I categorize them into the following three groups: general adjustments, regression-related adjustments and portfolio-related adjustments. The general adjustments are explained in section 3.4.1 and consist of all modifications that are applicable to both research methods. Sections 3.4.2 and 3.4.3 present the regression- and the portfolio-related adjustments, which contain alterations that are only employable in their respective research frameworks.

3.4.1. General Adjustments

In this section I introduce all adjustments which are applicable independent of the underlying research approach. These allow me not only to identify how research findings of a single approach are changing but also facilitates a cross-concept comparison of the effect from the same adjustment on both approaches respectively. I classify my general adjustments into the following two categories: idiosyncratic volatility-related adjustments and sample-related adjustments. They are applied sequentially and independently from each other to both reference concepts. Alongside these modifications, I investigate how findings on the IVOL puzzle change.

The idiosyncratic volatility-related general adjustments focus on the IVOL estimation procedure. Section 3.1 shows that several decisions have to be made by researchers when defining their IVOL measure. Among one of such decisions is the choice of estimation window and the data frequency used for estimation of equation 2. Ang et al. (2009) use different estimation windows for their IVOL estimate showing that their evidence of an IVOL puzzle is robust to these adjustments in the regression-based research approach. In contrast, Bali and Cakici (2008) claim that when they change the data frequency used for the IVOL computation from daily to monthly data they are not able to find any significant relationship between IVOL and the cross-section of expected returns. Consequently I modify these two dimensions as well, while I keep using the FF3 model for risk-correction purposes. I begin by extending the estimation window using daily data over the past 12 months where I require a minimum of 200 days of stock return data for the estimation. Afterwards I estimate IVOL from monthly data using a rolling window of 1 and 5 years where I require at least 10 and 24 monthly stock returns to exist.

Another IVOL-related adjustment focuses on the choice of risk-correction model that is usually assumed to be the FF3 model. Malkiel and Xu (2006) use the FF3 model and the CAPM for IVOL estimation and find a positive relationship between IVOL and expected returns. So if the FF3 model insufficiently captures the relevant systematic risk, then some of it might remain in the risk that is assumed to be idiosyncratic and consequently could be the cause of the IVOL puzzle. Newer models have emerged that claim to explain returns better than the FF3 model does, which is why I decided to also use them for the IVOL estimation and analyze these estimates respectively (see e.g. Fama & French, 2018; Hou, Mo, Xue, & Zhang, 2020). Again I use daily data over the current month and compute for each stock i equation 1 using the respective risk-correction model to obtain the residuals, which are then used in equation 2 to compute the corresponding IVOL measure. As additional risk-correction models I use the FF6, the four-factor model proposed by Stambaugh and Yuan (2017) (SY4 model hereafter) and the five-factor model of Hou et al. (2020) (HOU5 model hereafter). The SY4 model consists of the MKT and the SMB factor from Fama and French (1993) as well as two mispricing-related factors, which are constructed from bivariate sorts of stocks relative to a total of 11 prominent asset pricing anomalies. One of these factors captures the exposure to anomalies related to quantities such as net stock issues that can be affected by the firms' management, whereas the other is directed at performance-related anomalies like momentum that cannot directly be influenced by the management (see Stambaugh & Yuan, 2017). With this model I try to account for the mispricing-related explanation of the IVOL puzzle put forward by Stambaugh, Yu, and Yuan (2015). They argue that only the overpriced stocks show an IVOL puzzle which is even more pronounced for overpriced stocks that are put under short selling restrictions (see Stambaugh et al., 2015). Hence I try to investigate whether researchers accounting for that fact can resolve the IVOL puzzle finding. On the other hand, the HOU5 model takes an investment-based asset pricing approach. The model consists of factors related to the market excess return, size, investment-to-assets, profitability as well as an expected-growth factor. The first four factors are constructed from triple sorting stocks according to a 2 x 3 x 3 strategy on size, investment-to-assets and return on equity, whereas the last factor is constructed by independent 2 x 3 double sorts related to market equity and the expected 1-year-ahead investment-to-assets changes (see Hou et al., 2020; Hou, Xue, & Zhang, 2015). I include the model here to incorporate the investment-based asset pricing idea and because Hou et al. (2020) argue that it outperforms most of the other asset pricing models including the FF6 model.

Besides the IVOL-related adjustments I investigate adjustments related to the sample used for the analysis. Here I sort all stocks into subsample based on stock-specific criteria and conduct my analysis within each of these subsamples separately. I form two kinds of subsamples based on the following criteria: price and size. Price-related subsamples should account for the fact that the IVOL puzzle might be specific to low price stocks and excluding these stocks from the analysis can diminish or even resolve the puzzle completely (see Bali & Cakici, 2008). To disentangle this relationship I subdivide stocks according to their price into three groups using the cutoffs from L. H. Chen et al. (2012). The first group contains so-called "penny stocks" with a price lower than 5$, whereas the second group contains low price stocks with a price higher than or equal to 5$ but lower than 10$. Lastly, high price stocks have a price higher than or equal to 10$ and hence form the last group. The prices used for subsample formation are adjusted for delisting based on the ideas of Shumway (1997). Bali and Cakici (2008) argue that size-related effects also affect the IVOL puzzle, wherefore I analyze size based subsamples as well. Size groups are constructed in reference to Fama and French (2008) who classify stocks as microcaps, small or big stocks based on the cross-sectional distribution of MTCAP at each month. Hence, each month the breakpoints used to categorize stocks are calculated as the 20th and the 50th percentile of the cross-sectional distribution of the market capitalization for all stocks traded on the NYSE. Consequently, stocks with a MTCAP lower than this 20th percentile are classified as microcaps during the corresponding month. In contrast a MTCAP within the 20th and the 50th percentile classifies stocks as small, whereas one larger than the 50th percentile characterizes them as big stocks.

3.4.2. Regression-Related Adjustments

This section presents all adjustments that are only applicable in the regression-based research concept as they address methodological peculiarities. I summarize them in the following two categories: risk-related control variables and estimation procedure.

The category named "risk-related control variables" refers to modifications in the variables that are used for risk-correction in equation 5. These variables can be grouped into two classes where the first consists of all variables that are related to the risk-correction model responsible to capture the systematic risk and the second class subsumes all the firm characteristic control variables that should characterize the left over firm-specific risk. Systematic risk-correction model-related adjustments consist of modification to the time-series risk factor loadings used in equation 5 that should correct for risk when estimating the risk premia. In the reference concept explained above, the FF3 model was used for that purpose. Now I rotate independently among four further risk-correction models. Nevertheless, the computation of time-series factor loadings and their incorporation into the cross-sectional regression remains similar as explained in section 3.2. I employ the following models as an adjustment here: the FF6, the SY4 and the HOU5 model and a model that augments the FF3 model by a short and long-term reversal factor. These two short and long-term reversal factors are calculated as proposed by French (2021) from double sorting stocks relative to their past returns over the last month or the prior months from $t-60$ to $t-13$ and size respectively. I call the FF3 model augmented with the reversal factors the augmented Fama-French model (FFA hereafter). The second class of risk-related control variable adjustments focuses on the firm characteristic control variables from the vector of firm characteristics used in equation 5. Here I integrate 3 additional control variables. First I include the stock's monthly trading volume to account for possible trading volume-related effects that have also been considered in Ang et al. (2006). The effect of trading volume on expected returns was first discovered by Gervais, Kaniel, and Mingelgrin (2001). As a further control variable related to the liquidity effect, I include the monthly average over the daily bid-ask-spreads calculated from the difference between the ask and bid price, as Han and Lesmond (2011) argue that bid-ask bounces are among the drivers of the IVOL puzzle. If one or both of the bid and ask prices are not available for a specific day, I use the lowest and highest trading prices, or if these do also not exist, I choose the closing bid and ask prices for approximation of the bid-ask spread from that day instead. Lastly, I control for the maximum daily return during the current month computed based on the ideas of Bali et al. (2011) as an additional proxy for the investor's skewness preferences. I again rotate among all additional firm characteristic control variables sequentially such that they are integrated independently from each other to the benchmark setting and shifts in results can be investigated accordingly. In a last step, I integrate all additional control variables jointly and analyze the IVOL premium respectively.

The estimation procedure-related adjustment, on the other hand, refers to all modifications in the estimation of the first step time-series regressions and the second step cross-sectional regressions introduced in section 3.2. Related to the time-series regression step, I adjust the horizon over which the risk factor loadings are estimated. This idea is based on the arguments of Z. Chen and Petkova (2012) who claim that estimates over the full-sample would be most precise if the risk factor loadings are truly constant but as risk exposure of firms is likely not constant over time, the optimal time horizon maximizing the precision of the estimate remains an unsolved question. Therefore, I allow for varying degrees of time-variation in the risk factor loadings by using different window sizes for the estimation of equation 1 instead of the usual one month period. I choose window sizes covering the full-sample, the past 12 months and the past 60 months for which the existence of at least 800, 200 and 800 daily stock return observations are required. The cross-sectional regression step adjustments focus on the technique used for estimation of the risk premia by the cross-sectional regression of equation 5. This idea is motivated by the studies of Bali and Cakici (2008) as well as Hollstein and Prokopczuk (2020) who point out that findings and estimation precision of the risk premia change if returns are weighted differently. Hollstein and Prokopczuk (2020) claim that relying less on highly volatile stocks might improve the estimation precision of the risk premia, which is why they use a weighting matrix based on the empirical variances. I pick up this idea and apply a generalized least squares (GLS) technique where a diagonal weighting matrix consisting of the inverse of the estimated stock return variances is utilized. The estimation of the return variance is conducted over the past 1, 12 and 60 months as well as the complete sample using daily data where I require a stock to report at least 15, 200, 800 and 800 past daily return observations to allow for a valid estimate of the firm return variance at the specific month. These estimates are then annualized by multiplication with 250 and used in the weighting matrix respectively.

3.4.3. Portfolio-Related Adjustments

In the following I present all adjustments unique to the portfolio-based research concept that I consider in my study. I separate them into the following two groups: portfolio characterization and bivariate portfolio sorts.

In the category named "portfolio characterization" I subsume all modifications that are related to the way the univariate IVOL portfolios are computed and analyzed as explained in section 3.3. More precisely, I adjust the breakpoints used for portfolio sorting, the number of portfolios analyzed, the computation of risk-adjusted portfolio returns as well as the trading strategy used to construct portfolios.

Adjustments to the breakpoints used for sorting stocks into portfolios are primarily motivated by mitigation of the size effect that might distort the findings related to the IVOL puzzle as pointed out by Bali and Cakici (2008). These adjusted breakpoints again define the IVOL cutoffs on the basis of which I assign all stocks in my sample to portfolios subsequently. For the calculation of the breakpoints I first use only stocks traded on the NYSE and afterwards focus on breakpoints that sort stocks into equal market share portfolios. For the equal market share breakpoints I follow Bali and Cakici (2008) and rank all stocks in the sample according to their idiosyncratic volatility in the specific month. Based on this ranking, I adjust the breakpoints such that within all IVOL quintile portfolios the stocks in these portfolios together make up the same share of the total market capitalization. This means that, for example, the lowest IVOL quintile portfolio consists of all the stocks with the lowest IVOL that together account for approximately 20% of the total MTCAP.

In addition to the different breakpoints, the number of univariate portfolios is a further aspect that is adjusted here.

The idea is to vary the degree of dispersion in the data and hence also the measures of comparison including returns and alphas by increasing the number of portfolios analyzed. Additionally to the standard quintile portfolios, I sort all stocks into 10 and 15 portfolios respectively and implement the reference research concept accordingly. Ten portfolios have also been selected in the study of Spiegel and Wang (2005) who found a positive IVOL risk premium.

The adjustment in risk-adjusted portfolio returns addresses the choice of the risk-correction model used to calculate the portfolio alphas. These models are responsible for capturing the systematic risk in stock returns and should be able to explain the stock return differences by their exposure to the model specific systematic risk. As various studies use different models for this purpose, the choice might influence findings consequently (see e.g. Bali & Cakici, 2008; Bali, Del Viva, Lambertides, & Trigeorgis, 2020; Boehme, Danielsen, Kumar, & Sorescu, 2009). In addition to the models used in the reference concept, I consider the SY4, the HOU5 and the model proposed by Daniel, Hirshleifer, and Sun (2020). The Daniel et al. (2020) (DANIEL3 hereafter) model consists of three factors where one is the market factor also used in the Fama-French models and the other two are theory-based behavioral factors that should capture short- and long-term mispricing dynamics which are induced by investor behavior. The short-term mispricing factor is related to the post-earnings announcement drift phenomenon, whereas the long-term mispricing factor is derived from investors issuance and repurchase activities as well as related misperceptions (see Daniel et al. (2020)).

Lastly, the trading strategies are also modified and analyzed as done by Ang et al. (2006). In addition to the usual 1/0/1 strategy, I compute the results related to the following trading strategies: 1/1/1, 1/1/12, 1/0/12, and 12/0/12. These are implemented based on the procedure of Jegadeesh (1990) as explained in section 3.3. I include the strategies that include a one month waiting period to find out whether this waiting period would allow researchers to mitigate the IVOL puzzle effect and hence deliver evidence on a short-term reversal effect based explanation as argued by W. Huang, Liu, Rhee, and Zhang (2010). The longer holding period of 12 months should account for the explanation that the IVOL puzzle is driven by a short-term overreaction and can even reverse over a period longer than one month, as found by Rachwalski and Wen (2016). Similar to the study of Ang et al. (2006), the 12 months formation period is included again to mitigate the effect of potential short-term events.

As previously only univariate IVOL portfolios were considered, effects from further firm characteristic-related variables that might influence the IVOL puzzle were not taken into account. Hence, I now construct average portfolios out of bivariate dependent portfolio sorts, as also proposed by Ang et al. (2006) and Boyer et al. (2010), that should account for further firm-specific effects. I control for the following variables: size, BM, the stock's current month return, the Amihud (2002) illiquidity measure calculated over the current month, return skewness over the current month, the expected idiosyncratic skewness measure of Boyer et al. (2010), the Harvey and Siddique (2000) coskewness measure computed over the past 5 years, the stock's monthly trading volume, the current month average daily bid-ask-spreads and the maximum daily return of the current month as proposed by Bali et al. (2011). Based on each of these control variables I form the bivariate portfolios using the usual 1/0/1 strategy and again compute EW and VW portfolio returns. First I sort all stocks each month respective to the control variable currently considered into quintile portfolios. Within these quintile portfolios, the stocks are again sorted into quintile portfolios relative to their IVOL on the basis of which also the corresponding long-short portfolios are formed. To disentangle the IVOL effect from those of the control variables, I calculate the average IVOL quintile portfolios where each has the same dispersion in the control variable. So every month I average each IVOL quintile portfolio, as well as the long-short portfolio, over all the quintiles of the control variable to obtain the monthly time-series of these six IVOL-related average portfolios. Finally the time-series average across the monthly average IVOL portfolios is calculated. With these average portfolios I can investigate the return pattern of the quintiles IVOL portfolios that remains after controlling for the effect of the control variable.

4. Data

My study is primarily based on US-based individual stock data that is derived from the Center for Research in Security Prices database database (CRSP hereafter). From CRSP I obtain the closing stock returns, their closing prices, trading volume and end-of-period market capitalization on a daily and monthly basis. I use all stocks that are traded on the NYSE, AMEX and NASDAQ regardless of their security type captured by the share code. Further information on the stocks including information on the security type, the exchange on which the corresponding stocks are listed, the Standard Industrial Classification (SIC) code that classifies the companies into industry groups as well as the shares outstanding are also derived from the CRSP database. To calculate the bid-ask spreads I derive the daily and monthly bid and ask price as well as the corresponding lowest bid and the highest ask price again from CRSP. The lowest bid (highest ask) price either consist of the lowest (highest) trading price or the closing bid (ask) price during the time interval respectively. Throughout the analysis I generally use delisting adjusted returns and prices following the methods of Shumway (1997) for which I obtain delisting data such as the delisting code, the delisting date, the delisting price and the delisting return from CRSP as well. The accounting data including stockholders equity, deferred taxes, investment tax credit, preferred stock redemption value, preferred stock liquidating value and preferred stock par value are obtained from the CRSP/Compustat Merged (CCM) database. In general my sample period covers the horizon from July 1963 until December 2020. This starting point is chosen, as at that point in time most of the accounting-related and factor data are available for the majority of the models. Data on the risk factors from the FF3 and FF5 as well as the MOM, long-term and short-term reversal factors are obtained on a daily and monthly basis from the Kenneth French's Data Library (see French (2021)). From here also the data on the risk-free rate, that is the one-month Treasury Bill rate, is acquired. Daily and monthly factor data for the SY4 and HOU5 model come from Robert Stambaugh's and the global-q webpage respectively (see Hou, Xue, & Zhang, 2021; Stambaugh, 2021). However, it is of note that the SY4 model data is only available until December 2016 and hence a shorter sample is used when this model is applied. The factor data related to the DANIEL3 model are only available at a monthly frequency up to December 2018 and are obtained from Kent Daniel's webpage (see Daniel, 2021). Lastly, the data related to the expected idiosyncratic skewness measure of Boyer et al. (2010) is obtained from Brian Boyer's webpage until December 2016 and is afterwards calculated manually based on their proposed concept until December 2020 (see Boyer, 2021).

5. Reference Results

In this section I replicate the findings of the IVOL puzzle using the reference research concepts introduced in section 3.2 and 3.3. Prior to that I provide cross-sectional summary statistics on the data used for replication in section 5.1.

5.1. Summary Statistics

The summary statistics for all relevant variables from the reference concepts can be found in table 1. These allow for a preliminary explorative analysis that can reveal first insights on interactions between the IVOL puzzle and related firm characteristic effects. Panel A provides the time-series average of all parameters that characterize the monthly cross-sectional distribution of the corresponding variable, whereas Panel B shows the time-series average of their monthly cross-sectional Pearson product-moment and Spearman rank pairwise correlations.

The mean column in Panel A shows the cross-sectional means of the variables in the average month that can be interpreted as the level of the respective variable for the average stock in the sample. Hence, the average stock has risk factor loadings relative to the MKT, SMB and HML of about 0.806, 0.643 and 0.202, an annualized IVOL of 40.83%, a size measured as the natural logarithm of the MTCAP of 4.601, a book-to-market ratio of equity of 3.214, a monthly total return of 0.966%, a one-month ahead excess return of 0.596%, a monthly return skewness of 0.209, an Amihud (2002) illiquidity measure over the current month of 5.396, an expected idiosyncratic skewness of 1.157 and a coskewness of -1.085. Comparing mean and median it can be seen that the median values are mostly smaller than the cross-sectional means on average except for $CoSkew$ and that they lay rather close together. The only exceptions are BM, R_t, r_{t+1} and $Illiq_{1M}$ that have a median in the average month of 0.711, 0.019%, -0.352% and 0.310 respectively. However, all these values remain within a bandwidth of one standard deviation from their corresponding mean. About half of the variables including IVOL, BM, R_t, r_{t+1}, $Illiq_{1M}$ and $CoSkew$ are positively skewed with an average monthly cross-sectional skewness ranging from 2.085 for $CoSkew$ up to 32.125 for BM, whereas the remaining variables are rather symmetric around the mean. Additionally, almost all variables except for $Size$ are leptokurtic with an average cross-sectional excess kurtosis ranging from 2.634 for $Skew_{1M}$ to 1337.492 for BM when excluding $Size$. The high level of kurtosis and the positive skewness of IVOL, BM, R_t, r_{t+1}, $Illiq_{1M}$ and $CoSkew$ shows that the cross-sectional distribution of these variables is characterized by a small number of extremely positive observations which also cause most of their variability. This can also be seen by the percentiles of their distribution as the right tails are longer than their left tails because the average distance between the 95th percentile and the maximum value is larger than the one between the 5th percentiles and the minimum in all these cases. The extreme observation might also cause the differences between the mean and median values of the BM, R_t, r_{t+1} and $Illiq_{1M}$. Nevertheless, it is of note that the distribution of IVOL, BM, R_t, r_{t+1} and $Illiq_{1M}$ has heavy right tails partly due to the mechanical reason, as these variables have a lower bound and are unbounded from above because IVOL, BM and $Illiq_{1M}$ are not defined for values below zero and R_t as well as r_{t+1} cannot become smaller than -100%. All other variables except for $Size$ are more concentrated around their means as they are leptokurtic but barely skewed. However, they have a heavy tailed distribution as the average distance between the minimum and 5th percentile as well as 95th percentile and maximum exceeds, except for $Skew_{1M}$, a distance of three standard deviations. The last column of Panel A depicts the average number of stocks for which the corresponding variable is available. Excluding BM these values range from 4646 for $EIdioSkew$ to 5730 for r_{t+1}. BM has on average 3727 valid observations.

Panel B of table 1 presents the average monthly cross-sectional pairwise correlations for all variables, where the upper-diagonal matrix present the Pearson product–moment and the lower-diagonal one the average Spearman rank correlations. The aim is to understand the cross-sectional relations between the variables with a focus on the relationship to IVOL. The Pearson product-moment correlations reveal linear relationships, whereas the Spearman rank correlations detect monotonic but not necessarily linear dependencies. I begin with the analysis of the pairwise correlations involving IVOL to derive insights on potential drivers of the IVOL puzzle. Then I briefly repeat the analysis for r_{t+1} to investigate the variables return predictability properties. Finally I comment on some of the remaining correlations.

The Pearson product-moment (Spearman rank) correlation between IVOL and the MKT, SMB and HML risk factor loadings are low with values of 0.064 (0.138), 0.096 (0.148) and 0.000 (0.010) indicating that the information content captured by these variables and IVOL is likely different. Nevertheless, stocks with high IVOL are also more likely to have a high loading on the MKT and SMB. The correlations to the

Table 1: Cross-Sectional Summary Statistics

This table presents summary statistics for all variables of my sample that are related to the replication of the reference research concepts. The sample covers the period from July 1963 until December 2020 and includes all stocks traded on the NYSE/AMEX/NASDAQ. $\beta(MKT)$, $\beta(SMB)$ and $\beta(HML)$ refer to the time-series loadings onto the MKT, SMB and the HML risk factors respectively that were calculated for each stock based on daily data over the current month using equation 3. IVOL is the annualized idiosyncratic volatility (in percentage points) calculated via equation 4 by benchmarking the daily excess returns to the FF3 model over the current month. $Size$ refers to the natural logarithm of the firm's market capitalization. BM gives the firm's book-to-market ratio of equity for the specific month. R_t is the stock return at the current month. r_{t+1} is the excess stock return over the next month. $Skew_{1M}$ reports the total return skewness calculated over the current month. $Illiq_{1M}$ is the Amihud (2002) illiquidity measure computed over the current month. $EIdioSkew$ reports the expected idiosyncratic skewness of a stock calculated by the method of Boyer et al. (2010). $CoSkew$ stands for the firm's coskewness measure computed as proposed by Harvey and Siddique (2000). Panel A presents the time-series means of the monthly mean (*Mean*), standard deviation (*SD*), skewness (*Skew*), excess kurtosis (*Kurt*), minimum (*Min*), fifth percentile (*5%*), 25th percentile (*25%*), median (*Median*), 75th percentile (*75%*), 95th percentile (*95%*), and maximum (*Max*) values of the cross-sectional distribution for each variable. The column labeled n indicates the average number of stocks for which the corresponding variable is available. Panel B reports the time-series averages of the monthly cross-sectional Pearson product-moment and Spearman rank pairwise correlations between each of the variables. Here the above-diagonal entries present the average Pearson product–moment correlations, whereas the below-diagonal entries present the average Spearman rank correlations.

Panel A: Cross-Sectional Distribution

	Mean	*SD*	*Skew*	*Kurt*	*Min*	*5%*	*25%*	*Median*	*75%*	*95%*	*Max*	n
$\beta(MKT)$	0.806	1.996	-0.071	67.008	-26.265	-1.804	-0.060	0.695	1.625	3.736	26.523	5716
$\beta(SMB)$	0.643	2.906	0.379	56.398	-37.052	-3.115	-0.560	0.432	1.748	4.966	38.887	5716
$\beta(HML)$	0.202	3.437	-0.017	77.171	-47.098	-4.561	-1.095	0.156	1.511	4.989	46.163	5716
$IVOL$	40.833	37.413	5.567	110.012	0.785	9.259	19.455	31.354	50.661	101.802	820.888	5715
$Size$	4.601	1.935	0.297	-0.015	-2.294	1.644	3.229	4.473	5.858	8.002	11.771	5734
BM	3.214	41.670	32.125	1337.492	0.005	0.146	0.405	0.711	1.167	3.168	1893.320	3727
R_t	0.966	15.165	3.555	84.860	-83.581	-18.509	-5.848	0.019	6.249	22.654	292.276	5725
r_{t+1}	0.596	15.164	3.527	83.627	-83.822	-18.888	-6.222	-0.352	5.883	22.297	291.173	5730
$Skew_{1M}$	0.209	0.990	0.061	2.634	-4.000	-1.354	-0.276	0.182	0.688	1.851	4.155	5662
$Illiq_{1M}$	5.396	40.307	22.698	902.555	0.000	0.007	0.056	0.310	1.809	19.288	1883.481	4885
$EIdioSkew$	1.157	0.664	0.616	16.003	-2.659	0.258	0.684	1.089	1.590	2.219	7.409	4646
$CoSkew$	-1.085	9.442	2.085	86.961	-77.130	-14.323	-4.909	-0.952	2.609	11.543	151.906	4744

Panel B: Correlations

	$\beta(MKT)$	$\beta(SMB)$	$\beta(HML)$	$IVOL$	$Size$	BM	R_t	r_{t+1}	$Skew_{1M}$	$Illiq_{1M}$	$EIdioSkew$	$CoSkew$
$\beta(MKT)$		0.213	0.296	0.064	0.108	-0.007	0.020	-0.011	0.030	-0.039	-0.100	0.011
$\beta(SMB)$	0.237		0.211	0.096	-0.035	-0.012	0.021	-0.010	0.028	0.001	0.011	-0.003
$\beta(HML)$	0.280	0.197		0.000	-0.010	0.004	0.005	-0.003	0.010	0.009	-0.001	0.005
$IVOL$	0.138	0.148	0.010		-0.382	-0.004	0.150	-0.026	0.195	0.366	0.386	-0.039
$Size$	0.168	-0.018	-0.002	-0.412		-0.091	0.058	-0.005	-0.032	-0.284	-0.674	0.077
BM	-0.069	-0.029	0.078	-0.014	-0.252		0.001	0.001	-0.005	0.084	0.128	0.007
R_t	0.023	0.011	0.010	-0.011	0.109	0.020		-0.037	0.350	-0.009	-0.019	0.005
r_{t+1}	-0.007	-0.014	-0.002	-0.078	0.051	0.021	-0.030		-0.010	0.009	-0.007	-0.002
$Skew_{1M}$	0.038	0.030	0.011	0.179	-0.042	0.010	0.351	-0.024		0.025	0.076	-0.005
$Illiq_{1M}$	-0.155	0.047	0.021	0.478	-0.915	0.269	-0.067	-0.044	0.059		0.201	-0.017
$EIdioSkew$	-0.132	0.021	-0.001	0.441	-0.742	0.121	-0.081	-0.057	0.077	0.696		-0.069
$CoSkew$	0.020	-0.007	0.005	-0.069	0.117	0.022	0.010	0.005	-0.013	-0.116	-0.122	

BM and $CoSkew$ amount to -0.004 (-0.014) and -0.039 (-0.069), showing that these variables likely capture different information than IVOL does, wherefore they are improbable to explain the IVOL puzzle. Conversely, $Skew_{1M}$, $Illiq_{1M}$ and $EIdioSkew$ show correlations to IVOL of 0.195 (0.179), 0.366 (0.478) and 0.386 (0.441) respectively. Hence, stocks with higher IVOL likely exhibit high average return skewness, expected idiosyncratic skewness and are highly illiquid. To phrase it differently, it might be stocks with high skewness, high expected idiosyncratic skewness or those which are highly illiquid that drive the IVOL puzzle. Following the arguments of Barberis and Huang (2008) as well as Boyer et al. (2010), the low returns from high IVOL stocks might indeed be explained by their strong exposure to $Skew_{1M}$ and $EIdioSkew$, as stocks with high levels of these characteristics are assumed to have low returns respectively. The liquidity-related explanation of Han and Lesmond (2011) could also explain the IVOL puzzle, as high IVOL stocks also appear to be illiquid and for these stocks the bias in the IVOL estimate from microstructure effects is often high. An illiquidity premium as found by Amihud (2002), however, is unlikely explaining the puzzle, as then the high IVOL stocks would have to be liquid in order to posses low returns. $Size$ is the only variable with a non-negligible negative correlation to IVOL of -0.382 (-0.412), indicating that mainly small stocks exhibit high IVOL and consequently drive the IVOL puzzle. Ultimately, IVOL is positively correlated to the current month's return and negatively to the one-month ahead excess return with correlations of 0.150 (-0.011) and -0.026 (-0.078). Despite that the correlations to the next month excess returns are low, they still hint at the existence of the IVOL puzzle. It is of note that, even though the Pearson product-moment and the Spearman rank correlations to R_t have a different signs, they are low in magnitude wherefore an impact on the results is unlikely. For all variables the Pearson product–moment and Spearman rank measures are similar enough such that no non-linearity concerns in the relations arise that might distort the results.

Considering the Pearson product-moment correlations between r_{t+1} and all other variables as evidence on the direction of return predictability, I find that $\beta(MKT)$, $\beta(SMB)$, $\beta(HML)$, IVOL, $Size$, R_t, $Skew_{1M}$, $EIdioSkew$ and $CoSkew$ negatively predict the cross-section of next-month returns, whereas BM and $Illiq_{1M}$ do so in a positive direction. These relations are all in line with their theoretical predictions from the literature (see e.g. Amihud, 2002; Ang et al., 2006; Barberis & Huang, 2008; Boyer et al., 2010; Fama & French, 1993; Harvey & Siddique, 2000; W. Huang et al., 2010). It has to be noted that all correlations involving r_{t+1} are low ranging from -0.057 in case of the Spearman rank correlation with $EIdioSkew$ up to 0.009 for the Pearson product-moment correlation with $Illiq_{1M}$ and hence can only provide a rough evidence on the corresponding return predictability, meaning that there might be economically relevant relationships that are not found here.

The negative correlations of $Size$ to BM, $Illiq_{1M}$ and $EIdioSkew$ show that small stocks often have a high BM, high $EIdioSkew$ and are less liquid than big stocks. Especially the relationship to $Illiq_{1M}$ and $EIdioSkew$ with correlations of -0.284 (-0.915) and -0.674 (-0.742) has to be highlighted. The Spearman rank correlation between $Size$ and $Illiq_{1M}$ exceeds the Pearson product-moment correlation significantly and therefore indicates a non-linear relationship between the variables. BM is positively related to $Illiq_{1M}$ and $EIdioSkew$, wherefore value stocks are likely less liquid and have a high expected idiosyncratic skewness. R_t is only non-negligibly positively related to $Skew_{1M}$ implying that stocks with high returns in the current month also have on average positively skewed returns during that month. Finally illiquid stocks seem to exhibit on average a high expected idiosyncratic skewness and low coskewness.

5.2. Regression-Based Analysis

In this paragraph I empirically implement the regression-based reference research concept introduced in section 3.2 with the objective to replicate the IVOL puzzle that implies a negative cross-sectional relationship between stocks' IVOL and their expected returns. The corresponding results can be found in table 2. Here I display the average coefficients from the monthly cross-sectional regressions of equation 5, where I regress the next-month stock excess returns onto the stock-specific risk factor loadings relative to the FF3 model as well as further firm characteristics. In Panel A I use an OLS regression-based approach in the cross-sectional regression step to estimate the risk premia where all stock returns are weighted equally. Conversely, in Panel B I repeat the cross-sectional regression step with a WLS method where the stock excess returns are weighted by their contemporaneous MTCAP.

Panel A of table 2 shows that the average coefficient on IVOL is negative and significant in all regression settings except for the simple regression in column (1) that includes IVOL as unique independent variable. No matter which control variable is added to the benchmark regression in column (2), the IVOL puzzle from Ang et al. (2006) can successfully be replicated. I discover that the effect is strongest, with a coefficient of -1.248 and a Newey-West robust t-statistic of -4.843 indicating statistical significance at the 1% level, when the Amihud (2002) illiquidity measure is added as control in column (6). In contrast, excluding the simple model of column (1), the effect is least significant in column (5) where I control for the coskewness measure of Harvey and Siddique (2000). There the average coefficient is -0.456 and has a robust t-statistic of -1.907 making it statistically significant at the 10% significance level. Considering all IVOL coefficients in Panel A, their magnitude ranges from -0.389 in the simple regression of column (1) to -1.248 when controlling for illiquidity in column (6). To interpret the economic magnitude of the IVOL puzzle, I compute the excess return effect related to a one standard deviation change in IVOL while all other characteristics are held constant. For that I multiply the average IVOL coefficients with the average cross-sectional standard deviation of IVOL computed in Panel A of table 1. Hence, a one standard deviation increase in IVOL is associated with

Table 2: Regression-Based Results - Reference Results

This table presents the average coefficients from monthly Fama and MacBeth (1973) cross-sectional regressions for individual stocks where each column refers to a different cross-sectional regression specification. Using equation 5, each month I regress the next-month excess firm returns (in percentage points) on a constant, the annualized idiosyncratic volatility measure (in decimals) calculated from equation 4 using daily data over the current month, the stock-specific risk factor loadings relative to the FF3 model over the next month, the end of month firm size defined as the natural logarithm of the market capitalization, the end of month firm book-to-market ratio of equity, the monthly return (in percentage points) computed from the end of the previous month to the current month, as well as additional control variables related to further firm characteristics that are incorporated into the regression equation successively. Among these firm characteristic variables are the following: $Skew_{1M}$ which is the monthly return skewness measured over daily return data from the current month, $EIdioSkew$ that is the expected idiosyncratic skewness measure from Boyer et al. (2010), $CoSkew$ which is the coskewness measure from Harvey and Siddique (2000) and $Illiq_{1M}$ being the Amihud (2002) illiquidity measure computed over the current month. In Panel A I use simple stock excess returns to run the OLS regression wherefore this procedure is named to be the equal-weighted Fama-MacBeth regression as all stocks are weighted equally. On the other hand, in Panel B I estimate the Fama-MacBeth regression with a weighted-least squares approach, where all individual monthly stock returns are weighted by their current month market capitalization, which is why this method is named value-weighted Fama-MacBeth regression. I report the t-statistics testing the null hypothesis that the average coefficient is equal to zero in parenthesis below each coefficient where corresponding standard errors are corrected for autocorrelation and heteroskedasticity by implementation of the Newey and West (1987) method using 7 lags. *, ** , and *** indicate significance at the 10%, 5%, and 1% level, respectively. The row "Adjusted R^2" reports the time-series average of the cross-sectional adjusted R^2's. The last row reports the average number of stocks used for the monthly cross-sectional regressions. The sample period covers July 1963 to December 2020.

Panel A: Equal-Weighted Fama-MacBeth Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.706***	1.328***	1.326***	1.779***	1.318***	1.389***	1.845***
	(3.151)	(3.594)	(3.579)	(4.140)	(3.546)	(3.828)	(4.579)
IVOL	- 0.389	-0.616**	-0.627**	-0.431**	-0.456*	-1.248***	-0.975***
	(-1.369)	(-2.520)	(-2.567)	(-2.010)	(-1.907)	(-4.843)	(-4.133)
$\beta(MKT)$		0.253***	0.249***	0.197*	0.237**	0.268***	0.193*
		(2.645)	(2.615)	(1.709)	(2.494)	(2.738)	(1.649)
$\beta(SMB)$		0.077**	0.079**	0.019	0.079**	0.067*	0.005
		(2.009)	(2.072)	(0.382)	(2.198)	(1.703)	(0.104)
$\beta(HML)$		-0.038	-0.041	-0.073	-0.036	-0.039	-0.066
		(-0.871)	(-0.965)	(-1.352)	(-0.775)	(-0.910)	(-1.113)
Size		-0.160***	-0.160***	-0.186***	-0.153***	-0.134***	-0.159***
		(-4.176)	(-4.199)	(-4.752)	(-3.990)	(-3.664)	(-4.349)
BM		-0.001	-0.001	0.222***	0.002	0.007	0.140**
		(-1.282)	(-1.308)	(3.302)	(0.628)	(1.063)	(2.222)
R_t		-0.047***	-0.050***	-0.041***	-0.049***	-0.048***	-0.043***
		(-11.273)	(-11.421)	(-10.217)	(-11.323)	(-11.103)	(-10.694)
$Skew_{1M}$			0.068***				0.102***
			(2.808)				(4.129)
EIdioSkew				-0.353***			-0.302**
				(-2.778)			(-2.561)
CoSkew					-0.004		-0.011**
					(-0.953)		(-2.307)
$Illiq_{1M}$						0.046***	0.035***
						(4.880)	(3.492)
Adjusted R^2	0.017	0.061	0.061	0.060	0.064	0.070	0.067
n	5648	3676	3647	3520	3366	3234	3056

Panel B: Value-Weighted Fama-MacBeth Regressions

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.546***	1.096***	1.104***	1.523***	1.096***	1.062***	1.531***
	(2.913)	(3.529)	(3.574)	(4.062)	(3.522)	(3.365)	(4.104)
IVOL	-0.660	-0.927***	-0.967***	-0.747**	-0.799**	-0.963***	-0.795**
	(-1.422)	(-2.665)	(-2.738)	(-2.241)	(-2.295)	(-2.720)	(-2.287)
$\beta(MKT)$		0.191	0.190	0.105	0.178	0.202	0.096
		(1.442)	(1.433)	(0.649)	(1.348)	(1.501)	(0.593)
$\beta(SMB)$		-0.009	-0.009	-0.081	-0.016	-0.011	-0.087*
		(-0.215)	(-0.216)	(-1.599)	(-0.361)	(-0.252)	(-1.748)
$\beta(HML)$		-0.011	-0.009	-0.031	-0.009	-0.013	-0.027
		(0.150)	(-0.131)	(-0.325)	(-0.126)	(-0.179)	(-0.279)
Size		-0.072***	-0.073***	-0.082**	-0.070***	-0.070**	-0.081**
		(-2.688)	(-2.720)	(-2.497)	(-2.608)	(-2.566)	(-2.443)
BM		0.002	0.002	-0.016	-0.002	0.013	-0.026
		(0.596)	(0.620)	(-0.172)	(-0.312)	(0.785)	(-0.284)
R_t		-0.026***	-0.029***	-0.022***	-0.027***	-0.026***	-0.025***
		(-6.174)	(-6.687)	(-4.896)	(-6.173)	(-6.250)	(-5.350)
$Skew_{1M}$			0.092***				0.105***
			(3.074)				(3.308)
EIdioSkew				-0.249			-0.237
				(-1.621)			(-1.572)
CoSkew					-0.006		-0.016*
					(-0.786)		(-1.959)
$Illiq_{1M}$						0.042	0.001
						(1.601)	(0.096)
Adjusted R^2	0.027	0.124	0.127	0.142	0.132	0.128	0.153
n	5644	3676	3647	3520	3366	3234	3056

a decrease in expected excess returns ranging from 0.146% (0.389 x 0.374) to 0.467% (1.248 x 0.374) per month. Investigating the same one standard deviation IVOL increase in context of the model from column (7) that incorporates all controls simultaneously, the expected returns decrease by about 0.365% per month. This effect remains economically relevant and proves the negative relationship between IVOL and expected excess returns implied by the IVOL puzzle.

The average coefficients for the remaining variables are mostly in line with their theoretical predictions. Here the average risk premia for all FF3 model risk factor loadings, except for the on $\beta(HML)$, carry a positive sign as predicted by Fama and French (1993). However, it is of note that only $\beta(MKT)$ achieves a positive risk premium that is statistically significant in all regressions with Newey-West robust t-statistics between 1.649 and 2.738. The $\beta(SMB)$ premium is not statistically significant when the models of column (4) or (7) are considered, whereas the one of $\beta(HML)$ is not significant in any of the models. A negative size premium ranging from -0.134 in column (6) to -0.186 in column (4) is verified in all regression models and is furthermore always highly significant at the 1% level where the robust t-statistics lay between -3.664 when controlling for illiquidity and -4.752 in the regression that controls for $EIdioSkew$. In contrast to that, the BM risk premium ranges from 0.222 to -0.001 and is on average only significant at the 1% level when controlling for $EIdioSkew$ or at the 5% level in the complete regression model of column (7) with t-statistics of 3.302 and 2.222 respectively. As predicted by Jegadeesh (1990), I find a negative risk premium for R_t that is significant at the 1% level in all regression models and varies between -0.050 in column (3) and -0.041 when controlling for $EIdioSkew$. In addition, I confirm the negative return premium of expected idiosyncratic skewness found by Boyer et al. (2010) that is statistically significant at a level not less than 5% and the positive illiquidity premium found by Amihud (2002) which is significant at the 1% level in all regressions. $CoSkew$ carries a negative premium consistent with Harvey and Siddique (2000) that is, however, only significant at the 5% level in the complete regression model. Only the estimates on $Skew_{1M}$ do not match the predictions of Barberis and Huang (2008). Here I find a positive premium that varies between 0.068 and 0.102 and is always significant at the 1% level.

Panel B confirms most of the findings from Panel A, however, the IVOL risk premia are higher in magnitude for most of the regression models even though the overall maximum has reduced slightly. Now these premia range from -0.660 in the simple model to -0.967 when controlling for $Skew_{1M}$ and are statistically significant at a level not less than 5%, except for the simple model where it is not significant at any reasonable level. The economic magnitude computed by a one standard deviation increase in IVOL, when holding all other characteristics constant, amounts to a decrease of expected returns ranging from 0.247% to 0.362% per month respectively. In consequence I discover the IVOL puzzle found by Ang et al. (2006) again.

The remaining coefficient estimates are mostly in line with those from Panel A. However, all the FF3 model factor loadings risk premia are no longer significant at any reasonable level except for the one on the $\beta(SMB)$ that is significant at the 10% level in column (7). In addition, the $\beta(SMB)$ premium has switched signs for all models and is, therefore, no longer in line with the predictions of Fama and French (1993). The size-related risk premium decreased in magnitude to values ranging from -0.070 for regressions (5) and (6) to -0.082 in regression (4) with robust t-statistics of -2.566 or -2.608 and -2.497 respectively. It is of note that when controlling for illiquidity, expected idiosyncratic skewness or incorporating all controls, the premium is now only significant at the 5% level. On the other hand, BM is no longer significant in any model and the R_t premium remains negative and highly significant at the 1% level in all settings, however, it has decreased to values between -0.025 and -0.029. Lastly, the negative premium on $EIdioSkew$ and the positive premium on $Illiq_{1M}$ are no longer significant at any reasonable level, no matter which model is considered, whereas the negative premium of $CoSkew$ is now only significant at the 10% level in the complete regression model with an estimated risk premium of -0.016.

When comparing the average fit of the cross-sectional models by means of the adjusted R^2, I find a better fit for all models in Panel B as the adjusted R^2 are largely almost twice as large as the corresponding values from Panel A. This indicates that value-weighting enhances the model fit on average and therefore the results from the VW Fama-MacBeth regressions might be slightly more reliable than those of the EW Fama-MacBeth regression. From the last row of table 2 I conclude that, on average, approximately the same number of stocks is used for the monthly cross-sectional regression estimation, regardless of the weighting scheme employed. This number ranges from 5648 stocks in the simple model to 3056 stocks in the complete regression model, hence allowing, on average, for a valid risk premia estimation.

To sum up the results from Panel A and B, the negative relationship between IVOL and the cross-section of expected returns, implied by the IVOL puzzle, can be found in the risk premium estimates of the EW as well as the VW context. The effect is robust when controlling for skewness preferences and illiquidity measured by $Skew_{1M}$, $EIdioSkew$, $CoSkew$ and $Illiq_{1M}$ respectively and it is always statistically significant except when analyzing the simple regression model. Hence, in the regression-based reference setting, the findings of Ang et al. (2006, 2009) can be verified.

5.3. Portfolio-Based Analysis

This section deals with the replication of the IVOL puzzle using the portfolio-based concept introduced in section 3.3. If the puzzle exists, the portfolio of highest IVOL stocks has to underperform compared to the one with lowest IVOL stocks. Table 3 presents the average portfolio returns, Jensen's time-series alphas relative to the FF3 and the FF6 model, the standard deviation of portfolio excess returns as well as the average size and BM of all stocks within the corresponding portfolio. All stocks traded on the NYSE/AMEX/NASDAQ are

Table 3: Portfolio-Based Results - Reference Results

This table presents the average results from the univariate portfolio sorting procedure as explained in section 3.3 covering all stocks traded on the NYSE/AMEX/NASDAQ. Every month stocks are sorted into quintile portfolios based on their idiosyncratic volatility relative to the FF3 model over the past month that has been calculated by the usage of daily data for the respective month. These portfolios are rebalanced monthly according to the 1/0/1 trading strategy. Portfolio 1 (5) contains all stocks with lowest (highest) IVOL over the past month. The column labeled "5-1" refers to the long-short portfolio that goes long the portfolio of stocks with the highest IVOL and short the respective portfolio of stocks with the lowest IVOL, which is then also rebalanced monthly according to the 1/0/1 trading strategy. In the first four rows I report the average monthly total and excess returns for the portfolios (in percentage terms) and below each return in parenthesis the corresponding Newey and West (1987) adjusted t-statistics where I use 7 lags for the adjustment. Here it is of note that return differences for the long-short portfolio are the same for total and excess returns, wherefore I report them only once in the row for the total returns. SD illustrates the monthly standard deviation of the portfolio excess returns also denoted in percentage terms. The $Size$ and BM report the average natural logarithm of the market capitalization for firms within the portfolio as well as their average book-to-market ratio respectively. The α^{FF3} and the α^{FF6} denote the monthly average of Jensen's time-series alphas relative to the FF3 and the FF6 model that were calculated by equation 1 using the monthly portfolio excess returns over the complete sample horizon. Below each of these alphas I provide the corresponding t-statistics that were again corrected by the procedure of Newey and West (1987) using 7 lags. In Panel A I use the equal-weighted portfolio returns for the analysis, where all firms are getting the same weight. On the other hand, Panel B uses value-weighted portfolio returns that were calculated by weighting the stock returns within the portfolio by their market capitalization which is observable at beginning of the month in order to give higher weights to bigger stocks respectively and therefore diminish the effects that might be explicitly related to small stocks. The complete sample period covers July 1963 to December 2020.

Panel A: Equal-Weighted Portfolio Sorts by Idiosyncratic Volatility

	1	2	3	4	5	5 - 1
Total Return	0.868	1.094	1.158	1.046	0.886	0.018
	(3.893)	(4.510)	(4.169)	(3.167)	(2.216)	(0.064)
Excess Return	0.495	0.721	0.785	0.674	0.514	
	(2.227)	(2.977)	(2.823)	(2.028)	(1.273)	
SD	5.128	6.038	6.877	8.001	9.680	6.786
Size	7.465	7.099	6.320	5.488	4.413	
BM	3.220	4.268	3.518	2.695	2.479	
α^{FF3}	0.018	0.056	0.010	-0.200	-0.455	-0.474
	(0.093)	(0.393)	(0.087)	(-1.614)	(-2.314)	(-2.712)
α^{FF6}	-0.070	0.018	0.079	0.082	0.129	0.199
	(-0.343)	(0.135)	(0.665)	(0.579)	(0.523)	(0.965)

Panel B: Value-Weighted Portfolio Sorts by Idiosyncratic Volatility

	1	2	3	4	5	5 - 1
Total Return	0.804	0.838	0.905	0.648	0.126	-0.678
	(4.146)	(3.911)	(3.492)	(2.067)	(0.350)	(-2.425)
Excess Return	0.432	0.466	0.532	0.275	-0.246	
	(2.219)	(2.178)	(2.053)	(0.872)	(-0.672)	
SD	5.342	5.970	6.880	8.035	9.288	6.659
Size	7.465	7.099	6.320	5.488	4.413	
BM	3.220	4.268	3.518	2.695	2.479	
α^{FF3}	-0.024	-0.108	-0.160	-0.510	-1.145	-1.122
	(-0.192)	(-0.515)	(-1.052)	(-3.244)	(-6.143)	(-6.574)
α^{FF6}	-0.149	-0.165	-0.036	-0.182	-0.599	-0.450
	(-1.714)	(-0.412)	(-0.166)	(-0.807)	(-2.556)	(-2.924)

sorted each month into univariate quintile portfolios based on their past month IVOL calculated from daily data. The column named "5-1" reports the long-short portfolio results. It is of note that I report the long-short monthly average returns just in the row of total returns as the return difference between portfolio 5 and 1 is the same regardless of the return type used for its calculation. Panel A analyzes the EW and Panel B the VW portfolio returns respectively, where in both cases the 1/0/1 trading strategy is used for portfolio formation.

Panel A shows that the EW total (excess) portfolio returns increase monotonically from 0.868% (0.495%) per month for portfolio 1, which contains the lowest IVOL stocks, until portfolio 3 with a average total (excess) returns of 1.158% (0.758%) per month and declines afterwards again to 0.886% (0.514%) per month for portfolio 5 that contains the stocks with the highest IVOL. All total returns are statistically significant at the 1% level, except for the one of portfolio 5 which is only significant at the 5% level. On the other hand, the excess returns for portfolio 2 and 3 are statistically significant at the 1% level with t-statistics of 2.977 and 2.823 respectively, whereas for portfolio 1 and 4 they are only significant at the 5% level and for portfolio 5 they are not significant at any level. Comparing total and excess returns shows that significant results are more likely when analyzing the total returns which might be one reason why researchers tend to report these in their studies respectively (see e.g. Ang et al., 2006; Bali & Cakici, 2008). Nevertheless, the long-short portfolio return shows that the return difference between portfolio 5 and 1 is only 0.018% per month and not significant at any reasonable level. In line with Bali and Cakici (2008), the return pattern from the IVOL puzzle cannot be detected in my equal-weighted setting regardless of analyzing total or excess returns.

Nonetheless, the previous conclusions change when investigating the alphas instead. The α^{FF3} decrease almost monotonically from portfolio 1 to 5 with values ranging from 0.018% to -0.455% per month respectively, but only the one of portfolio 5 is also statistically significant at the 5% level. Nonetheless, the alpha difference from the long-short portfolio amounts to -0.474% per month, which is not only economically relevant but also statistically significant at the 1% level with a t-statistic of -2.712. So in terms of the alphas relative to the FF3 model, I can verify the existence of an IVOL puzzle as found by Ang et al. (2006), especially when considering the long-short portfolio. Therefore, my results contradict those of Bali and Cakici (2008) who negate its existence in the equal-weighted portfolio context. Their conclusion is only supported by the analysis of the α^{FF6}. Here alphas increase monotonically from -0.070% for portfolio 1 to 0.129 for portfolio 5, albeit none of them are statistically significant at any level. Also the long-short portfolio alpha is no longer significant at any level and is even positive. In conclusion, the FF6 model seems to most appropriately capture the risk dynamics behind the IVOL puzzle as it produces insignificant alphas only. Hence, researchers that decide to compute the α^{FF6} instead of the α^{FF3} can modify their results in a way such that they find no evidence of the puzzle.

The portfolio excess return standard deviations show that portfolio return volatility is monotonically increasing with portfolio IVOL, such that the portfolio with highest IVOL stocks is also the most volatile one. Also the average size of the portfolios exhibits a distinct pattern. Like Ang et al. (2006), I find that stocks with higher IVOL are mostly smaller than low IVOL stocks as the average size decreases across portfolios from 7.465 in portfolio 1 to 4.413 for portfolio 5. In this case, the size effect predicts higher returns for small stocks, which would correspond to higher returns for portfolio 5 compared to portfolio 1 (see Fama & French, 1993). This is indeed observed when looking at the total or excess portfolio returns, but at the same time it contradicts the return pattern implied by the IVOL puzzle. Nevertheless, this size effect is not found in the α^{FF3} as they show lower average returns for portfolio 5 compared to portfolio 1. The average BM increases from 3.220 in portfolio 1 to 4.268 in portfolio 2 and declines afterwards monotonically to 2.479 contradicting the increasing pattern found by Ang et al. (2006). When comparing the BM of portfolio 5 and 1, the value effect predicts lower returns for portfolio 5 as it shows a lower BM relative to portfolio 1. While this effect does not match the EW total or excess portfolio return pattern, it indeed fits to the α^{FF3} pattern. Therefore, the IVOL puzzle found when considering the FF3 alphas might be related to a underlying value effect in the stocks.

Panel B delivers clearer evidence on the existence of the IVOL puzzle, as now all different alphas and the return on the long-short portfolio confirm the underperformance of the highest IVOL portfolio compared to the lowest IVOL one. The VW returns show a similar pattern as the EW ones from Panel A. Here the total (excess) returns increase from 0.804% (0.432%) per month for portfolio 1 to 0.905% (0.532%) for portfolio 3 and afterwards they decline again to 0.126% (-0.246%) per month for portfolio 5. However, the average total return on portfolio 5, as well as the excess returns on portfolios 4 and 5, are not statistically significant at any plausible level, whereas the remaining portfolios show significant returns at a level not less than 5%. The portfolio return difference between portfolio 5 and 1, captured by the long-short portfolio return, illustrates that stocks with high IVOL underperform those of low IVOL by about -0.678% per month which is significant at the 5% level with a Newey-West robust t-statistic of -2.425. In line with Ang et al. (2006), the VW total and excess returns computed here verify the existence of the IVOL puzzle.

Investigating the portfolio alphas delivers the same results. Moving from portfolio 1 to portfolio 5, alphas to all risk-correction models decrease monotonically indicating that the average risk-adjusted performance of the portfolios decreases in IVOL. The only exception is the α^{FF6} that increases from portfolio 2 to 3 but decreases again afterwards. Regardless of the risk-correction model used, all portfolios seem to underperform relative the corresponding risk-correction model, as all alphas in Panel B carry a negative sign. The α^{FF3} ranges from -0.024% for portfolio 1 to -1.145% per month for portfolio 5, whereas the α^{FF6} spans the interval from -0.036% per month for portfolio 3 to -0.599% for portfolio 5. However, only the α^{FF3} of portfolio 4 and 5 are statistically significant at the 1% level with t-statistics of -3.244 and -6.143. In contrast, only portfolio 1 and 5 carry a statistically significant α^{FF6} with robust t-statistics of -1.714 and -2.559. The long-short portfolio alpha is also statistically significant at a level of 1% now, regardless of the risk-correction model employed. It ranges from -0.450% for the α^{FF6} to -1.122% per month for the α^{FF3} with t-statistics of -2.924 and -6.574 respectively. Therefore, I can economically and statistically verify the existence of the IVOL puzzle from these risk-adjusted returns as well. Comparing the alphas of different risk-correction models also delivers evidence on their attractiveness for researchers to integrate them into their study. The fact that long-short portfolio alphas are largest in magnitude when correcting for the FF3 model, no matter whether EW or VW returns are used, might incentivise researchers to use this model for risk-correction in their studies if they seek to discover the IVOL puzzle. This tendency can indeed be observed throughout the literature (see e.g. Ang et al., 2006, 2009; Bali & Cakici, 2008; Boyer et al., 2010).

From the SD the same positive relationship between IVOL and portfolio excess return volatility as in Panel A appears. As portfolios are still sorted similarly to Panel A and only the return-weighting has been modified, the average portfolio size and BM remain the same. However, as the high IVOL portfolio underperforms the low IVOL one when VW returns are analyzed, this pattern is in line with the predictions of the value effect because high IVOL portfolios have a lower BM on average and should hence have lower returns. As portfolio 5 contains the smallest stocks on average, the size effect would argue for higher returns in the high IVOL portfolio relative to the low IVOL one instead. Nevertheless, this effect seems to be dominated by the value effect, as the size effects might be accounted for by the value-weighting.

In summary, the IVOL puzzle is more apparent in Panel B as here all long-short portfolio performance measures approve its existence. For the EW portfolios of Panel A only the α^{FF3} gives evidence on the existence of the IVOL puzzle. However, the EW long-short portfolio return and its α^{FF6}, albeit not statistically significant, predict higher returns for portfolios with higher IVOL respectively. These results point out that, in contrast to the previous regression-based results, the IVOL puzzle finding in the portfolio-based analysis is sensitive to the choice of the weighting scheme.

6. General Adjustments

After having empirically analyzed the IVOL puzzle with the reference research concepts, I start with adjusting them in a way that is independent of the concrete approach implemented as explained in section 3.4.1. In section 6.1 I modify the way the IVOL is estimated. Section 6.2 then presents all changes related to the sample used for the analysis respectively. All adjustments implemented here are explained in detail in section 3.4.1. Due to space constraints I present only a subset of the regression- and portfolio-based results depicted in tables 2 and 3. However, this selective reporting behavior preserves all information that is necessary for investigation of the IVOL puzzle finding and how it evolves across adjustments.

6.1. Idiosyncratic Volatility-Related

I start with the modification of the way IVOL is estimated by changing the time-window and the data frequency that is used for its computation as well as the underlying risk-correction model towards which it is measured. These adjusted IVOL estimates are then analyzed using the regression- and the portfolio-based approach respectively. For the regression-based method I report the complete model results of column (7) from table 2 that includes all control variables simultaneously.

Table 4 presents the regression-based results where each column corresponds to the findings from a specific IVOL estimate used in the analysis accordingly. The columns labeled $IVOL_{12m}$, $IVOL_{1y}$, $IVOL_{5y}$ depict the results for the IVOLs that are computed relative to the FF3 model but using rolling windows over the past 12 month of daily data as well as the past 1 and 5 years of monthly data respectively. In the remaining columns, $IVOL^{FF6}$, $IVOL^{SY4}$ and $IVOL^{HOU5}$ show the results when IVOL is computed from daily data over the current month but using the FF6, SY4 or the HOU5 model for risk-correction. Panel A displays the EW and Panel B the VW regression results. Panel A shows negative average IVOL risk premia that are statistically significant at the 1% level regardless of the IVOL estimate considered. Since I use the complete regression model for this analysis, these results are robust when controlling for the effects from the control variables introduced previously. The IVOL risk premia estimates fluctuate around -0.238 when using $IVOL_{1y}$ in the regression and -1.189 for $IVOL_{12m}$ with corresponding robust t-statistics of -3.529 and -3.119. Considering the economic significance of the risk premia estimates, a one standard deviation increase in the respective IVOL measure, holding all other variables constant, translates into a reduction of expected returns between 0.281% per month for $IVOL_{1y}$ to 0.391% per month for $IVOL_{12m}$. So no matter which measure is used, the IVOL risk premium remains economically and statistically relevant in the EW Fama-MacBeth regressions and hence verifies the existence of the IVOL puzzle. The standard deviations used for the previous calculation and further summary statistics of the adjusted IVOL estimates can be found in table A.1 of Appendix A (vgl. Anhang, Tabelle A.1). However, it is of note that while the risk premia estimates for all risk-correction models are largely similar and vary from -1.019 for $IVOL^{HOU5}$ to -1.038 for $IVOL^{SY4}$, this is not true for the IVOLs that use different estimation windows and data frequencies. Comparing the IVOL estimates over the same estimation window of one year but with different data frequencies, I find that the risk premium from $IVOL_{12m}$, which uses daily data, is -1.189, whereas the one of $IVOL_{1y}$ that uses monthly data is only -0.238. Nonetheless, when considering the economic significance of these estimates by a one standard deviation increase in the respective IVOL measure, both predict a decrease in expected returns that is close together with values of 0.391% and 0.281% per month. This is most likely explained by the fact that $IVOL_{1y}$ is more volatile than $IVOL_{12m}$ with values of 118.230% and 32.883% as shown in table A.1 of Appendix A (vgl.Anhang, Tabelle A.1). When keeping the data frequency fixed and adjusting the estimation window only, the risk premia estimates hardly change as can be seen by the results from $IVOL_{1y}$ and $IVOL_{5y}$. Hence, in contrast to Bali and Cakici (2008) as well as Spiegel and Wang (2005), the IVOL estimate from monthly data still possess a negative risk premium here. So in line with the findings of Ang et al. (2009), the IVOL puzzle exists in this EW regression setting regardless of the IVOL estimate analyzed. The patterns, as well as the magnitudes, of all other risk premia are almost similar to the reference results of table 2, no matter which IVOL estimate is considered.

Panel B shows that all estimated IVOL risk premia remain negative in sign, but the t-statistics decrease notably when value-weights are used. Only the risk premia from modified risk-correction model IVOL estimates remain significant but only at a level of 5% and with t-statistics around -2.176 for $IVOL^{FF6}$ and -2.570 for $IVOL^{SY4}$. In absolute terms, they are slightly weaker then in the EW context and range from -0.780

Table 4: Regression-Based Results - Idiosyncratic Volatility-Related Adjustments

This table presents the average coefficients from monthly Fama and MacBeth (1973) cross-sectional regressions for individual stocks. Using equation 5, each month I regress the next-month excess firm returns (in percentage points) on a constant, an annualized idiosyncratic volatility measure (in decimals), the stock-specific risk factor loadings relative to the FF3 model over the next month, the end of month firm size defined as the natural logarithm of the market capitalization, the end of month firm book-to-market ratio of equity, the monthly return (in percentage points) computed from the end of the previous month to the current month as well as additional control variables related to further firm characteristics. These firm characteristic variables are the following: $Skew_{1M}$ which is the monthly return skewness measured over daily return data from the current month, $EIdioSkew$ that is the expected idiosyncratic skewness measure from Boyer et al. (2010), $CoSkew$ which is the coskewness measure from Harvey and Siddique (2000) and $Illiq_{1M}$ being the Amihud (2002) illiquidity measure computed over the current month. Instead of the usual IVOL measure, the regression models in each column integrate an idiosyncratic volatility estimate that was computed either over a different time window, with a different data frequency or against an other risk-correction model. More specifically $IVOL_{12m}$, $IVOL_{1y}$, $IVOL_{5y}$ are the idiosyncratic volatilities computed relative to the FF3 model but using a window of the past 12 month of daily data as well as of the past 1 and 5 years of monthly data respectively. $IVOL^{FF6}$, $IVOL^{SY4}$ and $IVOL^{HOU5}$ refer to the IVOL computed based on daily data over the current month but using the FF6, SY4 and the HOU5 model for risk-correction instead. In Panel A I use equal-weighted stock excess returns to run the OLS regression of equation 5. On the other hand, in Panel B I estimate the Fama-MacBeth regression with a weighted-least squares approach, where all individual monthly stock returns are weighted by their current month market capitalization. I report the t-statistics testing the null hypothesis that the average coefficient is equal to zero in parenthesis below each coefficient, where corresponding standard errors are corrected for autocorrelation and heteroskedasticity by implementation of the Newey and West (1987) method using 7 lags. *, ** , and *** indicate significance at the 10%, 5%, and 1% level, respectively. The row "Adjusted R^2" reports the time-series average of the cross-sectional adjusted R^2's. The last row reports the average number of stocks used for the monthly cross-sectional regressions. The sample covers the period from July 1963 until December 2020 and includes all stocks traded on the NYSE/AMEX/NASDAQ. It is shorter when the SY4 model is used in the analysis, which is due a lack of data availability wherefore it then covers the horizon until November 2016 only.

Panel A: Equal-Weighted Fama-MacBeth Regressions

	$IVOL_{12m}$	$IVOL_{1y}$	$IVOL_{5y}$	$IVOL^{FF6}$	$IVOL^{SY4}$	$IVOL^{HOU5}$
$Constant$	1.971***	1.786***	1.884***	1.816***	2.091***	1.835***
	(5.186)	(4.642)	(5.235)	(4.534)	(5.546)	(4.553)
$IVOL$	-1.189***	-0.238***	-0.254***	-1.022***	-1.038***	-1.019***
	(-3.119)	(-3.529)	(-3.188)	(-4.121)	(-4.039)	(-4.169)
$\beta(MKT)$	0.212*	0.198*	0.209*	0.189	0.318***	0.191
	(1.847)	(1.725)	(1.826)	(1.619)	(3.352)	(1.634)
$\beta(SMB)$	0.010	0.010	0.014	0.005	0.021	0.005
	(0.214)	(0.223)	(0.301)	(0.112)	(0.435)	(0.113)
$\beta(HML)$	-0.073	-0.066	-0.071	-0.064	-0.040	-0.065
	(-1.292)	(-1.158)	(-1.237)	(-1.090)	(-0.853)	(-1.102)
$Size$	-0.166***	-0.152***	-0.160***	-0.156***	-0.178***	-0.158***
	(-4.770)	(-4.215)	(-4.503)	(-4.305)	(-4.652)	(-4.321)
BM	0.143**	0.140**	0.112*	0.141**	0.156**	0.140**
	(2.278)	(2.179)	(1.868)	(2.238)	(2.251)	(2.214)
R_t	-0.045***	-0.044***	-0.045***	-0.043***	-0.044***	-0.043***
	(-11.675)	(-11.208)	(-11.691)	(-10.705)	(-11.017)	(-10.644)
$Skew_{1M}$	0.095***	0.092***	0.097***	0.098***	0.103***	0.099***
	(3.563)	(3.182)	(3.436)	(3.917)	(3.790)	(3.950)
$EIdioSkew$	-0.281***	-0.277**	-0.216**	-0.301**	-0.329**	-0.301**
	(-2.631)	(-2.378)	(-2.018)	(-2.544)	(-0.329)	(-2.543)
$CoSkew$	-0.013***	-0.012**	-0.013***	-0.011**	-0.010*	-0.011**
	(-2.683)	(-2.385)	(-2.670)	(-2.303)	(-1.820)	(-2.296)
$Illiq_{1M}$	0.036***	0.029***	0.029***	0.035***	0.033***	0.035***
	(3.539)	(2.947)	(2.953)	(3.487)	(3.194)	(3.492)
$Adjusted\ R^2$	0.069	0.067	0.067	0.067	0.065	0.067
n	3056	3056	3056	3056	3022	3056

Panel B: Value-Weighted Fama-MacBeth Regressions

	$IVOL_{12m}$	$IVOL_{1y}$	$IVOL_{5y}$	$IVOL^{FF6}$	$IVOL^{SY4}$	$IVOL^{HOU5}$
$Constant$	1.414***	1.161***	1.049**	1.508***	1.843***	1.543***
	(3.066)	(2.963)	(2.293)	(4.056)	(5.066)	(4.121)
$IVOL$	-0.689	-0.025	-0.004	-0.780**	-0.959**	-0.831**
	(-1.124)	(-0.262)	(-0.033)	(-2.176)	(-2.570)	(-2.342)
$\beta(MKT)$	0.109	0.070	0.083	0.088	0.160	0.092
	(0.702)	(0.438)	(0.531)	(0.544)	(0.942)	(0.569)
$\beta(SMB)$	-0.085*	-0.084	-0.080	-0.084	-0.076	-0.087*
	(-1.678)	(-1.637)	(-1.603)	(-1.662)	(-1.446)	(-1.710)
$\beta(HML)$	-0.039	-0.017	-0.012	-0.024	0.051	-0.025
	(-0.427)	(-0.183)	(-0.135)	(-0.250)	(0.641)	(-0.256)
$Size$	-0.077**	-0.054	-0.049	-0.079**	-0.097***	-0.081**
	(-2.119)	(-1.636)	(-1.367)	(-2.407)	(-2.782)	(-2.474)
BM	-0.013	-0.009	-0.005	-0.026	-0.026	-0.028
	(-0.151)	(-0.100)	(-0.063)	(-0.285)	(-0.257)	(-0.310)
R_t	-0.027***	-0.027***	-0.027***	-0.026***	-0.026***	-0.025***
	(-5.842)	(-5.811)	(-5.972)	(-5.431)	(-5.205)	(-5.336)
$Skew_{1M}$	0.100***	0.095***	0.099***	0.101***	0.117***	0.104***
	(3.108)	(3.044)	(3.221)	(3.169)	(3.515)	(3.277)
$EIdioSkew$	-0.203	-0.225	-0.207	-0.238	-0.222	-0.239
	(-1.368)	(-1.526)	(-1.442)	(-1.571)	(-1.345)	(-1.585)
$CoSkew$	-0.021**	-0.019**	-0.022**	-0.016*	-0.013	-0.017**
	(-2.407)	(-2.138)	(-2.511)	(-1.924)	(-1.514)	(-1.960)
$Illiq_{1M}$	-0.001	-0.001	0.002	0.000	0.004	0.001
	(-0.096)	(-0.051)	(0.108)	(0.025)	(0.267)	(0.071)
$Adjusted\ R^2$	0.157	0.153	0.156	0.152	0.151	0.152
n	3056	3056	3056	3056	3022	3056

to -0.959 respectively. Matching the results from Bali and Cakici (2008) as well as Rachwalski and Wen (2016), the risk premia estimates decrease remarkably and are no longer statistically significant when IVOL is estimated with a modified data frequency or estimation window. Comparing $IVOL_{12m}$, $IVOL_{1y}$ and $IVOL_{5y}$, the IVOL risk premium increases monotonically from -0.689 for $IVOL_{12m}$ to -0.004 for $IVOL_{5y}$. Using monthly data for IVOL estimation also seems to lead to a risk premium that is not only statistically insignificant but is also very close to zero in absolute terms. This pattern is likewise found when analyzing the economic significance of the IVOL risk premium estimates by a one standard deviation increase in the corresponding measure, holding all other variables constant. The average decrease in expected returns by such an effect amounts to 0.005%, 0.030% and 0.223% per month for $IVOL_{5y}$, $IVOL_{5y}$ and $IVOL_{1y}$ correspondingly up to 0.263%, 0.296% and 0.341% per month for the $IVOL^{FF6}$, $IVOL^{HOU5}$ and $IVOL^{SY4}$. Thus only the IVOL from daily data and these of the different risk-correction models have economically significant risk premia estimates in the VW regression context.

The risk premia pattern of the remaining variables has changed slightly compared to the results from table 2. The $\beta(SMB)$ risk premia remain all negative and similar in magnitude but are now only statistically significant for $IVOL_{12m}$ and $IVOL^{HOU5}$. $\beta(HML)$ carries a positive risk premium when using $IVOL^{SY4}$ that is still not statistically significant. For size, the risk premium remains negative in all cases and similar in magnitude to the reference setting but is no longer significant for $IVOL_{1y}$ and $IVOL_{5y}$ and in contrast significant the 1% level for $IVOL^{SY4}$. While the negative coskewness risk premium increased in significance to a level of 5% for $IVOL_{12m}$, $IVOL_{1y}$, $IVOL_{5y}$ and $IVOL^{HOU5}$, it is no longer significant when using $IVOL^{SY4}$. Also interesting is that when using the $IVOL_{12m}$ and $IVOL_{1y}$, the illiquidity premium is now negative albeit not significant.

The above findings show that, using the EW Fama-MacBeth regressions, a researcher is more likely to detect an IVOL puzzle, no matter which data frequency, estimation window or risk-correction model is used and therefore makes its implementation more attractive to researchers that seek to find the IVOL puzzle in their study. Conversely, when using the adjusted IVOL estimates in the VW Fama-MacBeth regressions, the results less frequently deliver evidence on the existence of the IVOL puzzle, which is why this method is more attractive to researchers that try to negate the presence of the IVOL puzzle. Ang et al. (2009) use also different estimation windows to estimate the IVOL measure and conclude that the existence of the IVOL puzzle is robust to all these estimates. However, it is of note that they implemented an EW Fama-MacBeth regression approach which is, as revealed earlier, more likely to approve this finding and leave the VW results unreported. This again highlights that the IVOL puzzle finding can be influenced by the choice in research design.

Table 5 presents the average total portfolio returns as well as the time-series alphas relative to the FF3 and the FF6 model from the portfolio-based approach for portfolios 1, 5 and the long-short portfolio respectively. Each column corresponds to the portfolio-based results when using one of the modified IVOL measures introduced above for the construction of the corresponding monthly quintile portfolios. Panel A depicts the results of the EW and Panel B those of the VW portfolio returns respectively. In both cases the 1/0/1 trading strategy is used for portfolio formation.

Panel A shows almost identical results to those from the reference analysis in table 3 which are independent of the IVOL measure analyzed. The long-short portfolio shows positive, albeit not statistically significant, total returns around 0.049% to 0.296% per month for all IVOL measures except for $IVOL^{SY4}$ and $IVOL^{HOU5}$ that display negative total returns instead. But these negative returns are economically small with values of -0.001% and -0.011% per month and are not statistically significant at any reasonable level. The portfolios 1 and 5 total returns are both positive and statistically significant at a level not less than 5%, no matter which IVOL measure is employed. Consequently, from the EW total portfolio returns only, a researcher finds no IVOL puzzle irrespective of the data frequency, estimation window or risk-correction model he uses for IVOL estimation. The α^{FF3} results, on the other hand, do mostly not match those from table 3 showing that the choice of the IVOL estimate influences the alpha pattern of the EW portfolio-based analysis and hence potentially also the conclusions drawn from it. Even though the α^{FF3} of the long-short portfolios are all negative, they are not always statically significant at the 1% level. Only the long-short portfolio α^{FF3} of the $IVOL^{SY4}$ sort that has a return of -0.465% per month with a robust t-statistic of -2.612. Conversely, the $IVOL_{1y}$, $IVOL^{FF6}$ and $IVOL^{HOU5}$ sorts show a negative average long-short α^{FF3} around -0.362% to -0.465% that are significant at the 5% level, whereas those from the $IVOL_{12m}$ and $IVOL_{5y}$ sorts are not significant at any reasonable level. In contrast to the α^{FF3} from the reference setting of section 5.3, here it are mostly the comparatively high and statistically significant α^{FF3} from portfolio 1 that drive the long-short portfolio α^{FF3} and not the statistically significant low alphas from portfolio 5. The positive α^{FF6} from the long-short portfolio in table 3 is robust to all the modified IVOL estimates, as this alpha remains positive, no matter which IVOL estimate is considered. Nevertheless, it is not statistically significant, except for the portfolios that are constructed from the $IVOL_{5y}$ which possesses a long-short α^{FF6} of 0.414% per month with a t-statistic of 2.047 that implies statistical significance at the 5% level. Here portfolio 1 and portfolio 5 seem to drive the long-short α^{FF6} as both are positive and statically significant at a level not less than 5% for all sorts, except for those on $IVOL^{HOU5}$ and $IVOL^{FF6}$ that show a significant alpha for portfolio 5 only. Hence, the positive, albeit statically insignificant, α^{FF6} of the long-short portfolios imply an overperformance of the high IVOL portfolio relative to the low IVOL one independent of the IVOL measured considered which is directly opposite to

Table 5: Portfolio-Based Results - Idiosyncratic Volatility-Related Adjustments

This table presents the average results from the univariate portfolio sorting procedure as explained in section 3.3 covering all stocks traded on the NYSE/AMEX/NASDAQ. Every month stocks are sorted into quintile portfolios based on the idiosyncratic volatility measure depicted in the respective column, where $IVOL_{12m}$, $IVOL_{1y}$, $IVOL_{5y}$ are the idiosyncratic volatilities that are computed relative to the FF3 model but using a window of the past 12 month of daily data as well as of the past 1 and 5 years of monthly data respectively. $IVOL^{FF6}$, $IVOL^{SY4}$ and $IVOL^{HOU5}$, on the other hand, refer to the IVOL computed based on daily data over the current month but using the FF6, SY4 and the HOU5 model for risk-correction instead. These portfolios are rebalanced monthly according to the 1/0/1 trading strategy. In the first row I report the average monthly total returns for the portfolios 1, 5 as well as the long-short portfolio called "5-1" (in percentage points) and in parenthesis below each of these returns I depict the corresponding Newey and West (1987) adjusted t-statistics where I use 7 lags for the adjustment. Portfolio 1 (5) contains all stocks with lowest (highest) IVOL, whereas the long-short portfolio is the one that goes long the portfolio of stocks with the highest IVOL and short the respective portfolio of stocks with the lowest IVOL. This long-short portfolio is also rebalanced monthly according to the 1/0/1 trading strategy. The α^{FF3} and the α^{FF6} in the second and third row denote the monthly average of Jensen's time-series alphas relative to the FF3 and the FF6 model that were calculated by equation 1 using the monthly portfolio excess returns over the complete sample horizon and are again reported for portfolio 1, 5 and the long-short portfolio respectively. Below each of the alphas I provide the corresponding t-statistics that were again corrected by the procedure of Newey and West (1987) using 7 lags. In Panel A I use the equal-weighted portfolio returns for the analysis, where all firms are getting the same weight. On the other hand, Panel B uses value-weighted portfolio returns that were calculated by weighting the stock returns within the portfolio by their market capitalization which is observable at beginning of the month in order to give higher weights to bigger stocks respectively and therefore diminish the effects that might be explicitly related to small stocks. The complete sample period covers July 1963 to December 2020. It is shorter when the SY4 model is used in the analysis, which is due a lack of data availability wherefore it then covers the horizon until November 2016 only.

Panel A: Equal-Weighted Portfolio Sorts

		$IVOL_{12m}$	$IVOL_{1y}$	$IVOL_{5y}$	$IVOL^{FF6}$	$IVOL^{SY4}$	$IVOL^{HOU5}$
Total Return	1	0.871	0.939	0.876	0.863	1.033	0.862
		(4.309)	(4.454)	(4.374)	(3.874)	(5.977)	(3.650)
	5	1.167	1.039	1.212	0.912	1.023	0.861
		(2.718)	(2.586)	(2.975)	(2.284)	(2.530)	(2.078)
	5-1	0.296	0.101	0.337	0.049	-0.011	-0.001
		(0.870)	(0.337)	(1.058)	(0.170)	(-0.035)	(-0.003)
α^{FF3}	1	0.416	0.449	0.410	0.384	0.532	0.396
		(2.462)	(2.413)	(2.481)	(1.663)	(7.094)	(1.639)
	5	0.173	0.086	0.270	-0.051	0.067	-0.062
		(0.796)	(0.436)	(1.294)	(-0.263)	(0.440)	(-0.297)
	5-1	-1.187	-0.362	-0.140	-0.435	-0.465	-0.458
		(0.438)	(-2.038)	(-0.749)	(-2.510)	(-2.612)	(-2.518)
α^{FF6}	1	0.339	0.394	0.340	0.296	0.455	0.303
		(2.054)	(1.963)	(2.008)	(1.097)	(6.434)	(1.113)
	5	0.777	0.580	0.755	0.523	0.715	0.538
		(2.905)	(2.480)	(3.056)	(2.116)	(3.383)	(2.050)
	5-1	0.438	0.186	0.414	0.227	0.260	0.236
		(1.839)	(1.015)	(2.047)	(1.098)	(1.190)	(1.099)

Panel B: Value-Weighted Portfolio Sorts

		$IVOL_{12m}$	$IVOL_{1y}$	$IVOL_{5y}$	$IVOL^{FF6}$	$IVOL^{SY4}$	$IVOL^{HOU5}$
Total Return	1	0.787	0.810	0.756	0.794	0.915	0.826
		(4.167)	(4.292)	(3.975)	(4.124)	(5.852)	(4.107)
	5	0.210	0.639	0.514	0.176	0.214	0.087
		(0.509)	(1.803)	(1.396)	(0.486)	(0.592)	(0.087)
	5-1	-0.577	-0.171	-0.242	-0.618	-0.700	-0.739
		(-1.706)	(-0.616)	(-0.838)	(-2.213)	(-2.374)	(-2.582)
α^{FF3}	1	0.324	0.350	0.285	0.337	0.473	0.375
		(2.571)	(2.720)	(2.301)	(3.089)	(10.401)	(2.648)
	5	-0.788	-0.206	-0.370	-0.724	-0.628	-0.814
		(-3.764)	(-1.146)	(-1.927)	(-4.225)	(-4.398)	(-4.215)
	5-1	-1.112	-0.557	-0.654	-1.061	-1.101	-1.189
		(-5.596)	(-3.304)	(-3.941)	(-6.647)	(-6.447)	(-6.919)
α^{FF6}	1	0.187	0.232	0.140	0.216	0.361	0.241
		(2.098)	(2.467)	(1.415)	(2.721)	(6.931)	(2.431)
	5	-0.120	0.106	-0.040	-0.170	0.014	-0.217
		(-0.579)	(0.644)	(-0.211)	(-0.714)	(0.096)	(-0.832)
	5-1	-0.307	-0.126	-0.180	-0.386	-0.348	-0.458
		(-1.778)	(-0.899)	(-1.344)	(-2.373)	(-2.042)	(-2.868)

the pattern implied by the IVOL puzzle.

The VW portfolio sorts from Panel B show a total return pattern that is almost identical to the one from section 5.3, irrespective of the IVOL measure used. Here the long-short portfolio returns are all negative but only statistically significant for the risk-correction model adjusted IVOL estimates at a level not less than 5% with corresponding returns around -0.618% and -0.739% per month. Among the estimation windows and data frequencies adjusted IVOLs, only the $IVOL_{12m}$ long-short portfolio return is statistically significant at the 10% level and amounts to -0.577% per month. So when using monthly data for the IVOL estimation, the VW total return differences, measured by the long-short portfolio, are not statistically significant at any level. In contrast to table 3, for all IVOL estimates the underperformance of portfolio 5 relative to portfolio 1 appears to be driven by the positive total returns from the low IVOL portfolio that are statistically significant at the 1% level, as those from portfolio 5 are barely significant at any level. When investigating the α^{FF3}, all modified IVOL based sorts confirm the existence of the IVOL puzzle similarly to the results from Panel B of table 3. The long-short portfolios show negative α^{FF3} that are statistically significant at the 1% level for all IVOL measures and fluctuate between -0.557% and -1.189% per month respectively. Also the IVOL puzzle implied negative relationship between IVOL and expected returns becomes apparent from the alpha pattern across portfolios throughout all IVOL sorts, as on average portfolio 5 has negative and portfolio 1 positive α^{FF3} that are statistically significant at a level not less than 5% when excluding the sorts on $IVOL_{1y}$ and $IVOL_{5y}$. Similar to section 5.3, the VW long-short α^{FF6} verify the existence of the IVOL puzzle for all IVOL estimates, except for $IVOL_{1y}$ and $IVOL_{5y}$, as these long-short α^{FF6} are not statistically significant at any plausible level. All other long-short α^{FF6} range from -0.307% for the $IVOL_{12m}$ sort to -0.458% per month for the sort on $IVOL^{HOU5}$ and are always statistically significant at the 5% level, except for the $IVOL_{12m}$ sort where it is only significant at the 10% level. Also, when comparing the α^{FF6} from portfolio 5 and 1, the negative relationship between risk-adjusted expected returns and IVOL appears regardless of the IVOL estimate analyzed. In contrast to the reference results of section 5.3, this pattern is now driven by the positive alphas from portfolio 1 that are statistically significant at a level not less than 5% for all IVOL estimates except for $IVOL_{5y}$.

In summary, the EW reference results from Panel A of table 3 are mostly robust to adjustments in the way IVOL is estimated. As in the reference setting and consistent with the study of Bali and Cakici (2008), the EW total returns and the α^{FF6} show no evidence of an IVOL puzzle, regardless of the IVOL adjustment implemented. Solely the α^{FF3} still confirm the existence of the puzzle which holds for all IVOL sorts, except for those on $IVOL_{12m}$ and $IVOL_{1y}$. The VW portfolio sorts show that the IVOL puzzle finding is sensitive to the choice of the data frequency used for IVOL estimation as all sorts based on an IVOL measure from monthly data are not able to detect the puzzling return pattern in the total returns and the α^{FF6}. This result is in line with Bali and Cakici (2008) as well as Rachwalski and Wen (2016) who also have difficulties detecting an IVOL puzzle for a monthly data based IVOL measure. Only for the VW α^{FF3} the pattern of section 5.3 remains unchanged, showing negative long-short alphas that are statistically significant at the 1% level regardless of the IVOL measured analyzed. When comparing the results from the IVOL estimation adjustments among the portfolio- and the regression-based method conclusions differ notably. In the EW setting, the IVOL puzzle finding is more robust to the usage of different IVOL estimates in the regression-based method as then a statistically significant IVOL risk premium is found regardless of the chosen IVOL estimate. With the portfolio-based analysis it is only found when analyzing the α^{FF3} and excluding the sorts on $IVOL_{12m}$ and $IVOL_{5y}$. However, both approaches deliver similar conclusions in the VW context, as here the IVOL puzzle can only be confirmed statistically for all risk-correction model adjusted IVOLs but not for the IVOLs from different data frequencies or estimation windows.

6.2. Sample-Related

In this section I recompute the regression- and portfolio-based reference analyses from chapter 5, only that I now focus on the corresponding results from specific subsamples. These subsamples are constructed by subdividing the complete data according to the stock price or its size as explained in section 3.4.1.

Table 6 presents the average regression-based results for the different subsamples. In the first column labeled "Price" the complete sample data is subdivided into low, medium high price stocks with cutoffs of 5$ and 10$ and analyzed respectively. The column named "Size" refers to the subsamples that split the data each month into micro, small and big stocks based on the 20th and the 50th percentile of the cross-sectional distribution of market capitalization from all stocks traded on the NYSE. Within each of the subsamples I re-estimate the risk premia using the monthly cross-sectional regressions of equation 5. Then, these risk premia estimates are averaged over time again. The corresponding EW Fama-MacBeth regression results are located in Panel A, whereas the VW ones can be found in Panel B. Panel A shows that the IVOL puzzle finding is robust to all size and price subsamples in the EW regression approach as the estimated IVOL risk premium remains negative and statistically significant at the 1% level in all cases, except for the big stocks where it is only significant at the 10% level. The average IVOL risk premium ranges from -0.796 for low price to -2.402 for medium price stocks in the price subsamples and from -0.657 for big stocks to -1.852 for all small stocks in the size subsamples. As I use the complete regression model for the analysis, none of the corresponding control variables can explain these premia. The economic magnitude of the IVOL risk premia estimates for the price subsamples, measured by the return difference from a one standard deviation change in IVOL while holding all other variables constant, ranges from -0.157% for high

Table 6: Regression-Based Results - Sample-Related Adjustments

This table presents the average coefficients from monthly Fama and MacBeth (1973) cross-sectional regressions for individual stocks. Using equation 5, each month I regress the next-month excess firm returns (in percentage points) on a constant, the annualized idiosyncratic volatility measure of equation 4 calculated from daily data over the current month (in decimals), the stock-specific risk factor loadings relative to the FF3 model over the next month, the end of month firm size defined as the natural logarithm of the market capitalization, the end of month firm book-to-market ratio of equity, the monthly return (in percentage points) computed from the end of the previous month to the current month as well as additional control variables related to further firm characteristics. These firm characteristic variables are the following: $Skew_{1M}$ which is the monthly return skewness measured over daily return data from the current month, $EIdioSkew$ that is the expected idiosyncratic skewness measure from Boyer et al. (2010), $CoSkew$ which is the coskewness measure from Harvey and Siddique (2000) and $Illiq_{1M}$ being the Amihud (2002) illiquidity measure computed over the current month. Instead of the complete sample, the cross-sectional regression of equation 5 are now estimated over subsamples of the data set that matches certain criteria. These criteria can be found in each column accordingly. In the first column named "Price" the complete sample data are subdivided into low, medium high price stocks with cutoffs of 5$ and 10$. The subsample criteria called "Size" refers to the subdivision of the data according to the stock-specific market capitalization. Here the complete data are split each month into micro, small and big stocks based on the 20th and the 50th percentile of the cross-sectional distribution of market capitalization when using all stocks traded on the NYSE. Panel A uses equal-weighted stock excess returns to run the OLS regression of equation 5. On the other hand, in Panel B I estimate the Fama-MacBeth regression with a weighted-least squares approach, where all individual monthly stock returns are weighted by their current month market capitalization. I report the t-statistics testing the null hypothesis that the average coefficient is equal to zero in parenthesis below each coefficient, where corresponding standard errors are corrected for autocorrelation and heteroskedasticity by implementation of the Newey and West (1987) method using 7 lags. *, ** , and *** indicate significance at the 10%, 5%, and 1% level, respectively. The row "Adjusted R^2" reports the time-series average of the cross-sectional adjusted R^2's. n reports the average number of stocks used for the monthly cross-sectional regressions. The last row shows the sample specific time-series mean of the monthly cross-sectional standard deviation of the IVOL estimate (in percentage terms) that was calculated relative to the FF3 model based on daily data over the current month. The sample covers the period from July 1963 until December 2020 and includes all stocks traded on the NYSE/AMEX/NASDAQ.

Panel A: Equal-Weighted Fama-MacBeth Regressions

	Price			Size		
	Low	Medium	High	Microcap	Small	Big
$Constant$	4.857***	2.826***	1.658***	4.165***	2.099***	1.595***
	(7.419)	(5.554)	(4.556)	(7.815)	(3.047)	(3.753)
$IVOL$	-0.796***	-2.402***	-0.834***	-1.172***	-1.852***	-0.657*
	(-3.389)	(-8.743)	(-3.278)	(-5.008)	(-5.797)	(-1.913)
$\beta(MKT)$	0.354***	0.327**	0.094	0.353***	0.160	0.017
	(3.145)	(2.516)	(0.681)	(3.055)	(1.217)	(0.106)
$\beta(SMB)$	0.082	0.041	-0.100**	0.045	-0.023	-0.112**
	(1.327)	(1.016)	(-2.502)	(0.793)	(-0.498)	(-2.456)
$\beta(HML)$	-0.098*	-0.111	-0.047	-0.083	-0.120	0.013
	(-1.726)	(-1.379)	(-0.614)	(-1.593)	(-0.120)	(0.147)
$Size$	-0.939***	-0.297***	-0.111***	-0.696***	-0.148	-0.089**
	(-10.572)	(-4.687)	(-3.544)	(-7.661)	(-1.539)	(-2.512)
BM	0.033	0.174**	0.054	0.114	0.097	0.000
	(0.475)	(2.186)	(0.668)	(1.840)	(0.975)	(-0.003)
R_t	-0.065***	-0.025***	-0.025***	-0.049***	-0.021***	-0.025***
	(-9.049)	(-5.884)	(-6.737)	(-10.398)	(-4.830)	(-5.186)
$Skew_{1M}$	0.076	0.101***	0.089***	0.113***	0.077**	0.085***
	(1.251)	(2.606)	(4.327)	(3.208)	(2.300)	(3.596)
$EIdioSkew$	-0.913***	-0.543***	-0.163	-0.665***	-0.285**	-0.141
	(-4.295)	(-3.915)	(-1.543)	(-4.637)	(-2.005)	(-0.914)
$CoSkew$	-0.016**	0.003	-0.010*	-0.012**	-0.007	-0.017**
	(-2.287)	(0.485)	(-1.787)	(-2.429)	(-1.236)	(-2.139)
$Illiq_{1M}$	0.026**	-0.017	-0.108**	0.030***	0.014	5.394
	(2.515)	(-0.748)	(-2.117)	(2.919)	(0.021)	(0.660)
$Adjusted\ R^2$	0.073	0.069	0.078	0.065	0.076	0.114
n	730	512	1814	1344	758	953
$\sigma(IVOL)$	52.374	28.775	18.822	45.431	21.246	14.135

Panel B: Value-Weighted Fama-MacBeth Regressions

	Price			Size		
	Low	Medium	High	Microcap	Small	Big
$Constant$	3.821***	3.301***	1.537***	3.387***	2.433***	1.509***
	(4.651)	(4.711)	(4.016)	(6.671)	(3.498)	(3.713)
$IVOL$	-1.836***	-2.722***	-0.583	-2.005***	-1.711***	-0.563
	(-6.764)	(-7.627)	(-1.588)	(-7.793)	(-5.334)	(-1.367)
$\beta(MKT)$	0.405***	0.164	0.092	0.371***	0.115	0.102
	(3.413)	(1.134)	(0.554)	(3.250)	(0.854)	(0.597)
$\beta(SMB)$	0.085	0.088	-0.107**	0.042	-0.039	-0.101*
	(1.451)	(1.551)	(-2.149)	(0.892)	(-0.843)	(-1.839)
$\beta(HML)$	-0.088	-0.100	-0.013	-0.087	-0.116	-0.010
	(-1.270)	(-1.016)	(-0.130)	(-1.577)	(-1.423)	(-0.100)
$Size$	-0.518***	-0.341***	-0.082**	-0.432***	-0.199**	-0.079**
	(-5.329)	(-3.943)	(-2.388)	(-5.503)	(-2.072)	(-2.180)
BM	0.115	0.063	-0.042	0.134*	0.068	-0.064
	(1.044)	(0.522)	(-0.415)	(1.939)	(0.656)	(-0.638)
R_t	-0.048***	-0.019***	-0.026***	-0.029***	-0.022***	-0.026***
	(-5.894)	(-3.459)	(-5.002)	(-6.781)	(-4.998)	(-4.783)
$Skew_{1M}$	0.060	0.057	0.110***	0.109***	0.065*	0.110***
	(0.709)	(0.875)	(3.513)	(3.635)	(1.866)	(3.251)
$EIdioSkew$	-0.786***	-0.448**	-0.275*	-0.485***	-0.302**	-0.286
	(-2.950)	(-2.279)	(-1.702)	(-3.299)	(-2.133)	(-1.629)
$CoSkew$	-0.011	0.011	-0.019**	-0.008**	-0.007	-0.018*
	(-1.352)	(1.293)	(-2.092)	(-2.061)	(-1.145)	(-1.814)
$Illiq_{1M}$	0.020**	-0.065	-0.044	0.021**	0.069	4.621
	(2.398)	(-1.514)	(-0.391)	(2.340)	(0.089)	(0.496)
$Adjusted\ R^2$	0.126	0.139	0.163	0.059	0.079	0.174
n	730	512	1814	1345	758	953
$\sigma(IVOL)$	52.374	28.775	18.822	45.431	21.246	14.135

price to -0.691% per month for medium price stocks. In the size subsamples the same effect amounts to -0.093% for the big stocks to -0.532% per month for small stocks. Thus, the negative relationship between IVOL and expected returns remains significant also in economic terms. These results are in line with L. H. Chen et al. (2012) who show that the IVOL puzzle is unlikely caused by market microstructure effects like size, price, liquidity or the security type.

The remaining risk premia estimates show almost the same pattern as the reference results of table 2 with few exceptions that I want to highlight. For $\beta(SMB)$ the premium turns negative in the high price and big stocks subsamples with values of -0.100 and -0.112 respectively that are statistically significant at a level not less than 5% and hence contradict the predictions of Fama and French (1993). Furthermore, the estimated size risk premium is now no longer statistically significant at any reasonable level for the small stock subsample, whereas it is particularly high for low price and microcap stocks with coefficients of -0.939 and -0.695 respectively. The BM premium is now only significant for the medium price subsample, though it generally remains positive. Additionally, in the low price subsample the skewness effect is absent, as the corresponding average risk premium is not significant at any plausible level. It is also interesting, that the effect documented by Boyer et al. (2010) seems to be caused by microstructure effects, as the $EIdioSkew$ premium is highest in magnitude for low price and microcap stocks with values of -0.913 and -0.665 that are also statistically significant at the 1% level and lowest for high price and big stocks with estimates of -0.163 and -0.141 that are not significant at any reasonable level. In addition to that, the coskewness premium turned positive for medium price stocks, although it is not significant at any reasonable level. Lastly, the illiquidity premium turns negative for the medium and high price stocks ranging from -0.017 to -0.108 and is even significant in the high price stock subsample at the 5% level.

The VW regression results of Panel B from table 6 still show negative IVOL risk premia for all subsamples, however, they are not statistically significant at any reasonable level when considering high price or big stock subsamples only. While the low and mid price, as well as the micro and small stocks, show IVOL risk premia around -1.711 and -2.722 that are all significant at the 1% level, they are remarkably lower and insignificant for high price and big stocks with values of -0.583 and -0.563 respectively. Hence, at least in the VW setting small and low price stocks indeed seem to drive the IVOL puzzle, wherefore the results appear to be related to microstructure effects that contradict the findings of L. H. Chen et al. (2012). This pattern is also found when analyzing the economic significance of the risk premia estimates by a one standard deviation increase in IVOL, holding all other variables constant. For the price subsamples the economic effect is the strongest for low price stocks causing a decrease in returns of on average 0.918% per month and the weakest for high price stocks with a associated return decrease of 0.110% per month. Similarly for the size subsamples, the effect ranges from -0.911% per month for the microcap stocks to -0.080% per month for big stocks. Consequently, the economic significance of the results also decreases as higher price or bigger stocks are considered. Hence, focusing on low to medium price or microcap to small stocks might enable researchers to tilt their results towards the discovery of the IVOL puzzle.

Related to the remaining risk premia estimates only a few changes have to be highlighted when compared to the results from Panel B of table 2. In accordance with Fama and French (1993), the $\beta(SMB)$ premium turns positive in the low and medium, as well as microcap stock subsamples, though it is not statistically significant at any plausible level. Also the BM premium has become positive for low to medium price, as well as microcap to small stocks, even though it is only significant for the microcap stock subsample at a level of 10%. Lastly, the illiquidity premium turns negative for medium to high price subsamples contradicting the finding of Amihud (2002), but it is of note that they are not statistically significant at any reasonable level.

In conclusion, the EW and the VW regression-based approach verify the existence of an IVOL puzzle regardless of the price or size subsample analyzed. The VW regressions, however, illustrate that the IVOL puzzle might be caused by microstructure effects, as also argued by Han and Lesmond (2011) as well as W. Huang et al. (2010). This likely holds, as the IVOL risk premium becomes insignificant for high price and big stocks in statistical, as well as economic terms, whereas it is significant in both interpretations for low price and microcap stocks. Hence, researchers that use only low price and microcap stocks in their regression-based analysis are more likely to find the IVOL puzzle than those using stocks with higher price or bigger size.

Table 7 presents the average portfolio total returns as well as the time-series alphas relative to the FF3 and FF6 model of the portfolio-based research concept for portfolios 1, 5 and the long-short portfolio in the previously introduced price and size subsamples. Each column corresponds to a subsample of stocks on the basis of which the monthly quintile portfolios have been constructed and returns as well as alphas have been calculated. Panel A depicts the portfolio-based analysis of the EW and Panel B for the VW portfolio returns respectively. The equal-weighted portfolio sorts from the price subsamples show that the IVOL puzzle finding is robust when looking at the subset of medium and high price stocks but less for the low price stock subsample. Analyzing the average EW long-short portfolio total returns of the price subsamples in Panel A, the positive reference returns of section 5.3 are only found for low price stocks. For the medium and high price stocks, the corresponding average return becomes negative and ranges from -0.210% for high price to -1.144% per month for low price stocks with t-statistics of -5.551 and -1.030. These negative total return differences found by the long-short portfolio are driven by the comparatively high performance of the low IVOL portfolios that is statistically significant at the 1% level in both subsamples. The long-short portfolio α^{FF3} and α^{FF6} show a mainly simi-

Table 7: Portfolio-Based Results - Sample-Related Adjustments

This table presents the average results from the univariate portfolio sorting procedure as explained in section 3.3 covering all stocks traded on the NYSE/AMEX/NASDAQ. Here the portfolio analysis is conducted by subdividing the complete data set into subsamples that are formed based on stock price and stock size as denoted in the respective column and applying the analysis to the subsamples accordingly. In the first column named "Price" the complete sample data are subdivided into low, medium and high price stocks with cutoffs of 5$ and 10$. The subsample criteria called "Size" refers to the subdivision of the data according to the stock-specific market capitalization. Here the complete data is split each month into micro, small and big stocks based on the 20th and the 50th percentile of the cross-sectional distribution of market capitalization when using all stocks traded on the NYSE. Every month stocks from these subsamples are then sorted again into quintile portfolios based on their idiosyncratic volatility relative to the FF3 model over the past month that has been calculated by the usage of daily data for the respective month. These portfolios are rebalanced monthly according to the 1/0/1 trading strategy and the corresponding results for alphas and total returns are calculated. In the first row I report the average monthly total returns for the portfolios 1, 5 as well as the long-short portfolio called "5-1" (in percentage terms) and in parenthesis below each of these returns I show the corresponding Newey and West (1987) adjusted t-statistics where I use 7 lags for the adjustment. Portfolio 1 (5) contains all stocks with lowest (highest) IVOL, whereas the long-short portfolio is the one that goes long the portfolio of stocks with the highest IVOL and short the respective portfolio of stocks with the lowest IVOL. This long-short portfolio is also rebalanced monthly according to the 1/0/1 trading strategy. The α^{FF3} and the α^{FF6} in the second and third row denote the monthly average of Jensen's time-series alphas relative to the FF3 and the FF6 model that were calculated by equation 1 using the monthly portfolio excess returns over the complete sample horizon and are again reported for portfolio 1, 5 and the long-short portfolio respectively. Below each of the alphas I provide the corresponding t-statistics that were again corrected by the procedure of Newey and West (1987) using 7 lags. In Panel A I use the equal-weighted portfolio returns for the analysis, where all firms are getting the same weight. On the other hand, Panel B uses value-weighted portfolio returns that were calculated by weighting the stock returns within the portfolio by their market capitalization which is observable at beginning of the month in order to give higher weights to bigger stocks respectively and therefore diminish the effects that might be explicitly related to small stocks. The complete sample period covers July 1963 to December 2020.

Panel A: Equal-Weighted Portfolio Sorts

		Price			Size		
		Low	Medium	High	Microcap	Small	Big
$TotalReturn$	1	1.316	1.052	0.813	1.020	0.884	0.813
		(3.546)	(3.768)	(3.794)	(3.841)	(3.777)	(4.117)
	5	1.490	-0.092	0.604	0.981	0.381	0.673
		(3.067)	(-0.255)	(1.938)	(2.274)	(1.079)	(2.202)
	5-1	0.175	-1.144	-0.210	-0.039	-0.503	-0.140
		(0.706)	(-5.551)	(-1.030)	(-0.133)	(-1.956)	(-0.641)
α^{FF3}	1	0.217	0.153	-0.002	0.191	0.025	-0.064
		(0.934)	(0.681)	(-0.008)	(0.843)	(0.105)	(-0.498)
	5	0.113	-1.379	-0.555	-0.373	-0.960	-0.506
		(0.355)	(-8.540)	(-4.372)	(-1.540)	(-5.774)	(-3.608)
	5-1	-0.104	-1.533	-0.554	-0.563	-0.986	-0.442
		(-0.443)	(-10.358)	(-4.780)	(-2.631)	(-5.795)	(-3.073)
α^{FF6}	1	0.443	0.132	-0.112	0.149	-0.063	-0.182
		(1.813)	(0.458)	(-0.626)	(0.539)	(-0.222)	(-1.548)
	5	0.870	-1.028	-0.378	0.282	-0.505	-0.131
		(2.374)	(-6.102)	(-3.124)	(0.915)	(-2.731)	(-0.931)
	5-1	0.427	-1.160	-0.267	0.133	-0.442	0.051
		(1.607)	(-8.897)	(-2.749)	(0.513)	(-2.875)	(0.427)

Panel B: Value-Weighted Portfolio Sorts

		Price			Size		
		Low	Medium	High	Microcap	Small	Big
$TotalReturn$	1	1.024	0.901	0.776	1.009	0.890	0.802
		(2.821)	(3.487)	(4.054)	(4.007)	(3.808)	(4.198)
	5	-0.343	-0.297	0.481	-0.256	0.414	0.648
		(-0.708)	(-0.746)	(1.546)	(-0.634)	(1.171)	(2.143)
	5-1	-1.367	-1.198	-0.295	-1.265	-0.475	-0.154
		(-4.388)	(-4.241)	(-1.323)	(-4.431)	(-1.837)	(-0.724)
α^{FF3}	1	-0.108	-0.031	-0.033	0.175	0.016	-0.004
		(-0.409)	(-0.159)	(-0.262)	(0.774)	(0.065)	(-0.038)
	5	-1.810	-1.675	-0.619	-1.648	-0.919	-0.441
		(-5.582)	(-8.243)	(-4.301)	(-8.311)	(-5.415)	(-3.106)
	5-1	-1.702	-1.645	-0.586	-1.824	-0.935	-0.437
		(-6.172)	(-7.983)	(-4.149)	(-9.323)	(-5.460)	(-2.909)
α^{FF6}	1	0.246	0.020	-0.186	0.120	-0.077	-0.132
		0.831	(0.092)	(-1.818)	(0.409)	(-0.243)	(-1.655)
	5	-0.978	-1.145	-0.363	-1.029	-0.452	-0.101
		(-2.866)	(-5.347)	(-2.280)	(-4.575)	(-2.342)	(-0.648)
	5-1	-1.224	-1.165	-0.177	-1.149	-0.375	0.031
		(-4.388)	(-6.297)	(-1.460)	(-5.877)	(-2.449)	(0.237)

lar pattern. Both are negative and statistically significant at the 1% level for medium and high price stocks, whereas low price stocks have a positive α^{FF3} and a negative α^{FF6} that are both not significant at any reasonable level. The alphas for the medium and high price stocks range from -1.533% to -0.267% per month and consequently are also economically relevant while matching the reference results. For both alpha types only the low portfolio 5 alpha is statistically significant at the 1% level and hence drives the negative long-short portfolio alphas for medium and high price stocks. Thus, the price subsamples show that researchers using medium to high price stocks in a EW portfolio-based analysis are more likely to discover a statistically significant IVOL puzzle than those using only low price stocks.

On the other hand, the IVOL puzzle appears more sensitive in the EW portfolio-based analysis of the size subsamples. Even though all size subsamples show negative long-short portfolio total returns now, they are only statistically significant at the 10% level for the small stock subsample. This effect is primarily driven by the high total returns of portfolio 1 as these are all statistically significant at the 1% level in all size subsamples. However, similar to the reference results, the long-short portfolio α^{FF3} is negative and statistically significant for all size subsamples ranging from -0.442% to -0.986% per month with Newey-West robust t-statistics of -3.073 and -5.795. Again the low α^{FF3} of portfolio 5 causes this effect, as it is the only one that is statistically significant at the 1% level for all size subsamples, except for the microcap stocks where it is not significant at any reasonable level. In terms of the α^{FF6}, only the long-short portfolio alpha of the small stocks is negative and statistically significant at the 1% level with a value of -0.442% per month that is also economically viable. Those of the big and microcap stocks match the reference results of section 5.3 and are positive albeit not statistically significant. In addition, no pattern can be found when comparing the α^{FF6} of portfolios 5 and 1 as only the small stock subsample portfolio 5 alpha is statistically significant. Thus, researchers are most likely finding an IVOL puzzle in all return measures using small stocks only or when just computing the α^{FF3} for their EW portfolio-based analysis.

The VW sorts from Panel B find the IVOL puzzle more often than the EW sorts do, even though the finding is still sensitive to the price or size subsample analyzed. In the price subsamples the low and medium price stocks are more likely to show the return pattern implied by the IVOL puzzle than when analyzing the high price stocks. The long-short portfolio total returns are negative in all price subsamples but only statistically significant for low and medium price stocks. They range from -1.198% for medium price to -1.367% per month for low price stocks with t-statistics of -4.241 and -4.388. Thus they are not only statistical significant at the 1% level but are also economically meaningful as in the reference results. Similar to the reference result, the comparatively high total return of portfolio 1 mainly drives this effect, as it is the only one that is statistically significant at a level of 1%, irrespective of the price subsample investigated. Regardless of the price subsample considered, the long-short portfolio α^{FF3} is negative and statistically significant at the 1% level with risk-adjusted returns between -0.589% per month for high to -1.702% per month for low price stocks that are also economically relevant. As in the reference result, the low α^{FF3} of portfolio 5 relative to portfolio 1 seems to cause the negative long-short alpha in all price subsamples as it is the only one that is statistically significant at a level of 1%. The α^{FF6} show the same pattern as the total returns implying a negative long-short portfolio alpha in all price subsamples, although they are only statistically significant for low and medium price stocks. Compared to the reference results, these statistically significant alphas are almost twice as high, amounting to values between -1.165% for medium price to -1.224% per month for low price stocks which are hence also economically significant. However, it is still the low alpha of portfolio 5 that is statistically significant at a level not less than 5% and consequently drives the IVOL effect in the long-short α^{FF6}. Overall the price subsamples reveal a negative relationship between the stock price and the statistical relevance of the IVOL puzzle meaning that researchers who use low or medium price stocks are more likely to prove the existence of the IVOL puzzle than those analyzing high price stocks.

The size subsamples of Panel B illustrate that the IVOL puzzle is also sensitive to the choice of the stock sample when filtering for MTCAP. In general, these subsamples show a negative relationship between stocks' MTCAP and the statistical significance of the IVOL puzzle. The long-short portfolio total returns is negative in all size subsamples but decreases in significance as bigger stocks are used for the analysis. Average long-short portfolio returns increase from -1.265% per month for microcap stocks to -0.154% per month for big stocks with t-statistics of -4.431 and -0.724 which are thus only statistically and economically significant for microcap and small stocks. Therefore, the big stock long-short portfolio total returns match the reference results only in sign but not in statistical significance. Across all size subsamples the high return of portfolio 1 causes this effect again, as it is the only one that is statistically significant at a level not less than 1%. A similar pattern is found for the long-short portfolio α^{FF3} that increases from -1.824% for microcap to -0.473% per month for big stocks and is economically meaningful and statistically significant at the 1% level in all size subsamples. The long-short portfolio α^{FF6} is negative for the microcap, as well as small stocks, matching the reference results but is positive, albeit not significant, for big stocks. It ranges from -1.149% per month for microcap to 0.031% per month for big stocks with t-statistics of -5.877 to 0.237 indicating that only the microcap and small stock α^{FF6} are statistically and economically relevant. In the microcap and small stock subsamples the low portfolio 5 alpha is still statistically significant at a level not less than 5% and thus drives the IVOL puzzle alpha pattern. In summary, the same pattern as in the price subsamples appears implying that researchers using microcap or small stocks are more prone to find an IVOL puzzle than those focusing on stocks with a price higher than 10$.

The previous results indicate that a researcher can affect the conclusions drawn from his study by a smart choice of the subsample used for his portfolio-based analysis. Especially for VW portfolio sorts, a monotonic pattern shows up entailing that analyzing bigger stocks or those with a higher price makes it harder to discover a statistically significant negative relationship between IVOL and expected stock returns as implied by the IVOL puzzle. Assuming that small and low price stocks are more likely subject to microstructure effects, these results contradict the findings of L. H. Chen et al. (2012) who argue that the IVOL puzzle is robust to such effects. However, regardless of the weighting-scheme and subsample used in the analysis, an IVOL puzzle is almost always found when analyzing the α^{FF3} only. The cross-concept comparison again shows that the IVOL puzzle finding from the EW analysis is more sensitive to the research concept used in the subsample analysis than the VW one, as it mostly delivers the same conclusions across approaches. In the EW regression results the IVOL puzzle exists regardless of the price or size subsample considered, whereas the portfolio-based results draw a slightly different conclusion as there the puzzle is absent in low price and microcap stocks. Consequently the regression-based approach is more likely to find a subsample robust IVOL puzzle than the portfolio-based method. Conversely, the VW results show that the IVOL puzzle exist in all subsamples but becomes less significant when higher prices or larger MTCAP stocks are used for the analysis, regardless of the research concept employed. Hence, for both VW methods the highest price, as well as biggest stock subsamples, show a IVOL puzzle effect that is not statistically significant at any reasonable level except when investigating the α^{FF3} in the portfolio-based approach.

7. Method-Specific Adjustments

This section covers all adjustments that are method-specific and thus only applied to the corresponding research concept. Section 7.1 presents the regression-based and section 7.2 the portfolio-based research concept adjustments respectively.

7.1. Regression-Related

The regression specific adjustments are structured according to the two categories introduced in section 3.4.2 consisting of risk-related control variables and estimation procedure-related adjustments. Section 7.1.1 starts with the adjustments in the risk-related control variables, whereas section 7.1.2 continues with all changes in the way the time-series risk factor loadings as well as the risk premia are estimated.

7.1.1. Risk-Related Control Variables

This section analysis the results from modification of the variables used for systematic or firm-specific risk-correction in the IVOL risk premia estimation of equation 5. For this I use the complete regression model of column (7) from table 2 and exchange the FF3 model risk factor loadings for those relative to the FF6, SY4, HOU5 and the FFA model respectively. Then I integrate further variables into the vector of firm characteristics including the stock's monthly trading volume ($Volume$), the average daily bid-ask spread over the current month ($Spread$) and the maximum daily stock return of the current month ($Maxret$) as proposed by Bali et al. (2011). The corresponding results are depicted in table 8, where Panel A uses the EW and Panel B the VW Fama-MacBeth regression for risk premia estimation.

The results of Panel A show that the IVOL puzzle is robust to the usage of different risk-correction models as well as the inclusion of $Volume$, $Spread$ and $Maxret$ as further firm characteristic control variables in the cross-sectional regression step. Across all the adjustments considered here, the IVOL risk premium estimate remains negative and statistically significant at the 1% level with coefficients ranging from -0.964 when controlling for the HOU5 model to -1.073 when using the FF3 model for risk-correction and incorporating all further firm characteristic control variables. Investigating a one-standard deviation increase in IVOL while holding all other variables constant, these IVOL risk premia translate into a decrease of expected returns between 0.361% to 0.401% per month that are also economically significant. Compared to the complete regression model in Panel A from table 2, the IVOL risk premium is now higher except when systematic risk is corrected by the HOU5 model or $Volume$ is used as firm-specific control. Even though the FF6, the SY4 and the HOU5 model claim to outperform the FF3 model, none of them can explain the IVOL puzzle (see Fama & French, 2018; Hou et al., 2020; Stambaugh & Yuan, 2017). The FFA model results also show that the short-term reversal IVOL puzzle explanation of W. Huang et al. (2010) is unlikely, as the puzzle persists despite the fact that the model explicitly contains a short-term reversal factor that should control for this effect. In addition, I cannot verify the arguments of Bali et al. (2011) and Han and Lesmond (2011) who claim that researchers who account for $Maxret$ or the bid-ask spread in their study could resolve the IVOL puzzle. Instead I find them to have almost no effect on the IVOL risk premia estimates.

The remaining risk premia estimates are nearly identical to the reference results from table 2. The only difference is that, albeit in almost all regressions the $EIdioSkew$ risk premium remains in the same range as in the reference result, it is statistically significant at the 1% level except for the case when the SY4 or $Maxret$ is used for risk-correction. In addition, the $CoSkew$ risk premium is only statistical significant at the 10% level when the SY4 model is used for risk-correction. Among the further firm characteristic control variables only the $Spread$ risk premium is statistically significant at the 10% level with coefficients between between 0.099 when it is added separately to the regression model and 0.106 when all further controls are integrated simultaneously. In contrast, the $Volume$ risk premium is almost zero and not statistically significant at any level, while $Maxret$ carries a positive risk premium that contradicts the findings of Bali et al. (2011) who predict a negative premium instead.

Table 8: Regression-Based Results - Risk-Related Control Variables

This table presents the average coefficients from monthly Fama and MacBeth (1973) cross-sectional regressions for individual stocks. Using equation 5, each month I regress the next-month excess firm returns (in percentage points) on a constant, the annualized idiosyncratic volatility measure of equation 4 calculated from daily data over the current month (in decimals), the stock-specific risk factor loadings relative to a selected risk-correction model over the next month, the end of month firm size defined as the natural logarithm of market capitalization, the end of month firm book-to-market ratio of equity, the monthly return (in percentage points) computed from the end of the previous month to the current month as well as additional control variables related to further firm characteristics. These firm characteristic variables are the following: $Skew_{1M}$ which is the monthly return skewness measured over daily return data from the current month, $EIdioSkew$ that is the expected idiosyncratic skewness measure from Boyer et al. (2010), $CoSkew$ which is the coskewness measure from Harvey and Siddique (2000) and $Illiq_{1M}$ being the Amihud (2002) illiquidity measure computed over the current month. The first four columns show the results from adjustments in the risk-correction model used in the risk premia estimation of equation 5. Here each of these columns refers to a different model towards which the risk factor loadings were computed using daily data over the next month as usual. These models consist of the FF6, the SY4, the HOU5 and the FFA respectively. I do not present the risk premia of the risk-correction models in this table for the matter of brevity. In columns C1 to CA I use the FF3 model for risk-correction again but incorporate additional firm characteristic variables instead which are the following: $Volume$ which is the stock's monthly trading volume, $Spread$ which is the average daily bid-ask spread over the current month and $Maxret$ that refers to the maximum daily stock return of the current month as proposed by Bali et al. (2011). The last column named "CA" then shows the risk premia estimates when all the additional firm characteristic variables are used in the Fama-MacBeth regression simultaneously. Panel A uses equal-weighted stock excess returns to run the OLS regression of equation 5. On the other hand, in Panel B I estimate the Fama-MacBeth regression with a weighted-least squares approach, where all individual monthly stock returns are weighted by their current month market capitalization. I report the t-statistics testing the null hypothesis that the average coefficient is equal to zero in parenthesis below each coefficient, where corresponding standard errors are corrected for autocorrelation and heteroskedasticity by implementation of the Newey and West (1987) method using 7 lags. *, ** , and *** indicate significance at the 10%, 5%, and 1% level, respectively. The row "Adjusted R^2" reports the time-series average of the cross-sectional adjusted R^2's. The sample covers the period from July 1963 until December 2020 and includes all stocks traded on the NYSE/AMEX/NASDAQ. It is shorter when the SY4 model is used in the analysis, which is due a lack of data availability wherefore it then covers the horizon until November 2016 only.

Panel A: Equal-Weighted Fama-MacBeth Regressions

	FF6	SY4	HOU5	FFA	C1	C2	C3	CA
Constant	1.833***	2.147***	1.813***	1.873***	1.842***	1.886***	1.805***	1.859***
	(4.585)	(5.603)	(4.537)	(4.755)	(4.439)	(4.694)	(4.760)	(4.805)
IVOL	-1.012***	-0.984***	-0.964***	-0.989***	-0.959***	-1.005***	-1.060***	-1.073***
	(-4.457)	(-3.956)	(-4.180)	(-4.299)	(-4.223)	(-4.245)	(-2.597)	(-2.710)
Size	-0.154***	-0.182***	-0.152***	-0.158***	-0.159***	-0.174***	-0.155***	-0.172***
	(-4.262)	(-4.693)	(-4.199)	(-4.370)	(-3.921)	(-4.733)	(-4.579)	(-4.654)
BM	0.138**	0.148**	0.138**	0.138**	0.143**	0.144**	0.139**	0.146**
	(2.230)	(2.137)	(2.191)	(2.211)	(2.284)	(2.299)	(2.218)	(2.350)
R_t	-0.041***	-0.044***	-0.042***	-0.043***	-0.043***	-0.043***	-0.043***	-0.044***
	(-10.900)	(-11.463)	(-10.808)	(-11.098)	(-10.766)	(-10.684)	(-11.076)	(-11.109)
$Skew_{1M}$	0.100***	0.111***	0.102***	0.106***	0.105***	0.103***	0.133***	0.137***
	(4.199)	(4.195)	(4.247)	(4.396)	(4.211)	(4.177)	(3.618)	(3.720)
EIdioSkew	-0.327***	-0.340**	-0.314***	-0.323***	-0.305***	-0.311***	-0.295**	-0.311***
	(-2.752)	(-2.606)	(-2.645)	(-2.769)	(-2.597)	(-2.659)	(-2.529)	(-2.687)
CoSkew	-0.010**	-0.010*	-0.011**	-0.012**	-0.011**	-0.011**	-0.011**	-0.011**
	(-2.283)	(-1.898)	(-2.401)	(-2.353)	(-2.317)	(-2.195)	(-2.286)	(-2.188)
$Illiq_{1M}$	0.033***	0.032***	0.035***	0.035***	0.036***	0.036***	0.037***	0.037***
	(3.520)	(3.324)	(3.577)	(3.556)	(3.489)	(3.502)	(3.539)	(3.543)
Volume					0.000			0.000
					(-0.994)			(-1.083)
Spread						0.099*		0.106*
						(1.699)		(1.789)
Maxret							0.002	0.001
							(0.110)	(0.094)
Adjusted R^2	0.084	0.069	0.079	0.080	0.068	0.067	0.068	0.069
n	3056	3024	3056	3056	3056	3056	3056	3056

Panel B: Value-Weighted Fama-MacBeth Regressions

	FF6	SY4	HOU5	FFA	C1	C2	C3	CA
Constant	1.577***	1.820***	1.531***	1.481***	1.594***	1.556***	1.563***	1.687***
	(4.318)	(5.055)	(4.058)	(4.061)	(3.816)	(4.179)	(4.234)	(4.045)
IVOL	-0.724**	-0.868**	-0.767**	-0.624*	-0.864**	-0.826**	-1.685***	-1.946***
	(-2.271)	(-2.481)	(-2.355)	(-1.882)	(-2.542)	(-2.359)	(-3.133)	(-3.947)
Size	-0.088***	-0.105***	-0.084**	-0.081**	-0.088**	-0.087***	-0.085***	-0.101***
	(-2.657)	(-3.025)	(-2.551)	(-2.478)	(-2.350)	(-2.602)	(-2.607)	(-2.711)
BM	-0.001	0.012	0.022	-0.013	-0.004	-0.018	-0.019	0.012
	(-0.015)	(0.120)	(0.235)	(-0.147)	(-0.040)	(-0.201)	(-0.205)	(0.130)
R_t	-0.030***	-0.029***	-0.031***	-0.037***	-0.026***	-0.025***	-0.029***	-0.030***
	(-6.659)	(-5.632)	(-6.651)	(-8.584)	(-5.602)	(-5.300)	(-5.766)	(-6.057)
$Skew_{1M}$	0.123***	0.131***	0.130***	0.129***	0.106***	0.102***	0.050	0.034
	(4.022)	(3.959)	(4.455)	(4.476)	(3.395)	(3.219)	(1.027)	(0.721)
EIdioSkew	-0.226	-0.225	-0.227	-0.223	-0.218	-0.250*	-0.224	-0.218
	(-1.613)	(-1.438)	(-1.594)	(-1.584)	(-1.460)	(-1.666)	(-1.492)	(-1.478)
CoSkew	-0.016**	-0.014	-0.017**	-0.017**	-0.015*	-0.016*	-0.016*	-0.014*
	(-2.068)	(-1.605)	(-2.142)	(-2.149)	(-1.805)	(-1.901)	(-1.919)	(-1.733)
$Illiq_{1M}$	0.001	0.005	-0.001	-0.005	0.005	0.001	-0.001	0.002
	(0.076)	(0.346)	(-0.091)	(-0.322)	(0.332)	(0.099)	(-0.097)	(0.144)
Volume					0.000			0.000
					(-0.648)			(-0.657)
Spread						0.072**		0.071**
						(2.309)		(2.381)
Maxret							0.052**	0.060***
							(2.088)	(2.660)
Adjusted R^2	0.182	0.164	0.174	0.180	0.162	0.156	0.156	0.169
n	3056	3024	3056	3056	3056	3056	3056	3056

However, the $Maxret$ risk premium found here is not statistically significant.

Moving to the VW regression results of Panel B, the negative IVOL risk premium remains in all regression specifications ranging from -0.624, when controlling risk by the FFA model, to -1.946 when simultaneously controlling for all additional firm-specific control variables. Compared to the reference results, the IVOL risk premium has more than doubled when $Maxret$ is added to the vector of firm characteristics and is still statistically significant at the 1% level, which is at odds with Bali et al. (2011) who claim that the incorporation of $Maxret$ should resolve the IVOL puzzle. However, it appears to have the opposite effect and increases its magnitude instead. On the other hand, when controlling systematic risk by the FFA model, the IVOL risk premium is only significant at the 10% level and decreases in magnitude. The decrease in statistical significance might be related to the inclusion of the short-term reversal factor that accounts for the short-term-related IVOL puzzle explanation of W. Huang et al. (2010). When computing the effect of a one standard deviation increase in IVOL, holding all other variables constant, these risk premia estimates predict a decrease in expected returns between 0.233% and 0.728% per month that appears highly dispersed but remains inside a bandwidth that is economically meaningful.

For the remaining risk premia estimates a few changes need to be highlighted. First, the BM risk premium turns positive when systematic risk is corrected for by the SY4 or the HOU5 model, albeit not statistically significant at any plausible level in both cases. Interestingly the $Skew_{1M}$ risk premium becomes insignificant when $Maxret$ is added to the cross-sectional regression specification. Furthermore, the $Maxret$ risk premium is now statistically significant at the 5% level when added as the only further control variable and even at the 1% level if all additional firm characteristic controls are added to the regression simultaneously. Hence, as both of these coefficients are related to the skewness preferences of investors, $Maxret$ seems to have more explanatory power than $Skew_{1M}$. In addition, the positive $Spread$ risk premium is now statistically significant at the 5% level showing that it has some explanatory power in the VW Fama-Macbeth regression context. Lastly, it is of note that the $Illiq_{1M}$ risk premium turns negative when the HOU5 model, the FFA model or $Maxret$ is used for risk-correction, although it is never statistically significant.

Table 8 shows that the IVOL puzzle finding in the regression-based approach is robust to the inclusion of other systematic risk-correction models or further firm characteristic controls. Whereas for the EW estimation method the results remain similar to those of the reference setting, some changes occur in the VW regressions. When researchers employ the FFA model for risk-correction in the VW context they find only a weakly significant IVOL puzzle, whereas the inclusion of $Maxret$ as further control lets the puzzle appear stronger in absolute terms than in the reference setting.

7.1.2. Estimation Procedure

In this section I analyze the modifications related to the estimation of the first step time-series regressions and the second step cross-sectional regressions as introduced in section 3.4.2. Specifically I address changes in the way the time-series regression of equation 3 is computed to obtain risk factor loadings and the manner in which the risk premia estimates are derived with the cross-sectional regression of equation 5. The corresponding results can be found in table 9. Panel A shows the equal- and value-weighted regression results from the time-series estimation adjustments where the FF3 model risk factor loadings are estimated over rolling windows of 12 and 60 months starting at month $t+1$ as well as over the complete sample horizon as depicted in the respective column. In Panel B I present the results from the cross-sectional regression adjustments where the GLS estimation technique is implemented. Here the stock returns are weighted with a diagonal weighting matrix consisting of the inverse of the estimated stock return variances that are computed over rolling windows from the past 1, 12 and 60 months as well as over the complete sample horizon as indicated in the specific column.

The results of Panel A show that the IVOL risk premium is negative and statistically significant at the 1% level for all risk factor loading estimation windows, no matter whether the equal- or the value-weighted Fama-MacBeth regression procedure is applied. For the EW regressions the IVOL premium ranges from -0.847 for the 60 month window factor loadings to -1.008 when computing them over the complete sample horizon, whereas it is slightly higher in the VW context with values of -1.070 for the 60 months to -1.149 for the 12 months window factor loadings. Using these estimates for the computation of economic significance by a one standard deviation increase in IVOL, holding all other variables constant, the corresponding decrease in expected returns amounts to values between 0.317% and 0.377% per month for the EW as well as 0.400% and 0.430% per month for the VW regression context. So the IVOL puzzle remains economically, as well as statistically, meaningful and is, moreover, comparable in magnitude to the reference results of column (7) in table 2, regardless of the weighting-scheme applied. However, it is of note that 60 months factor loadings lead to the lowest IVOL premium among all estimation windows, irrespective of the weighting-scheme used. Nevertheless, most of the regression specifications show more negative IVOL risk premia when compared to the reference results with exception of the 12 and 60 months factor loading regressions in the EW context that lead to higher IVOL premia instead. Additionally, the adjusted R^2 shows for both weighting-schemes that the model performance decreases as the window used for the factor loading estimation increases. Interestingly the adjusted R^2 for the 12 months factor loadings are higher for both weighting-schemes when compared to the corresponding reference results. This shows that 12 months factor loadings might more precisely capture the underlying risk than the usual 1 months loadings do and hence

Table 9: Regression-Based Results - Estimation Procedure Adjustments

This table presents the average coefficients from monthly Fama and MacBeth (1973) cross-sectional regressions for individual stocks. Using equation 5, each month I regress the next-month excess firm returns (in percentage points) on a constant, the annualized idiosyncratic volatility measure of equation 4 calculated from daily data over the current month (in decimals), the stock-specific risk factor loadings relative to the FF3 model, the end of month firm size defined as the natural logarithm of the market capitalization, the end of month firm book-to-market ratio of equity, the monthly return (in percentage points) computed from the end of the previous month to the current month as well as additional control variables related to further firm characteristics. These firm characteristic variables are the following: $Skew_{1M}$ which is the monthly return skewness measured over daily return data from the current month, $EIdioSkew$ that is the expected idiosyncratic skewness measure from Boyer et al. (2010), $CoSkew$ which is the coskewness measure from Harvey and Siddique (2000) and $Illiq_{1M}$ being the Amihud (2002) illiquidity measure computed over the current month. Panel A shows the results of the Fama-MacBeth regression procedure from adjustments to the way the factor loadings relative to the FF3 model were computed. Instead of computing them only over the next month I now use the next month including the previous 11 or 59 months as well as estimate them over complete sample described as 12M, 60M and FULL in the respective column. These three estimation windows for risk factor loading estimation are implemented in the EW and the VW Fama-MacBeth regression context as shown in the respective column. Panel B reports the results associated with the adjustments in the way the cross-sectional regression of equation 5 is estimated. Here I use a GLS approach where observations are weighted according to a diagonal weighting matrix that consists of the inverse of the estimated stock return variances. The estimation of the return variance is conducted by calculation over rolling windows from the past 1, 12 and 60 months as well as the complete sample while using daily data as shown in the corresponding column. I report the t-statistics testing the null hypothesis that the average coefficient is equal to zero in parenthesis below each coefficient, where corresponding standard errors are corrected for autocorrelation and heteroskedasticity by implementation of the Newey and West (1987) method using 7 lags. *, ** , and *** indicate significance at the 10%, 5%, and 1% level, respectively. The row "Adjusted R^2" reports the time-series average of the cross-sectional adjusted R^2's. The last row reports the average number of stocks used for the monthly cross-sectional regressions. The sample covers the period from July 1963 until December 2020 and includes all stocks traded on the NYSE/AMEX/NASDAQ.

Panel A: Time-Series Regression Step - Different Estimation Windows

	EW			VW		
	12M	60M	FULL	12M	60M	FULL
$Constant$	1.940***	1.847***	2.366***	1.321***	1.254***	2.771***
	(5.214)	(4.964)	(6.121)	(3.281)	(2.796)	(5.507)
$IVOL$	-0.923****	-0.847***	-1.008***	-1.149***	-1.070***	-1.075***
	(-4.844)	(-4.751)	(-5.357)	(-4.050)	(-3.851)	(-3.595)
$\beta(MKT)$	0.102	0.114	1.612***	0.249	0.116	0.577*
	(0.538)	(0.508)	(5.391)	(1.035)	(0.461)	(1.833)
$\beta(SMB)$	0.031	0.000	-0.842***	0.014	0.071	-0.816***
	(0.295)	(0.002)	(-5.047)	(0.126)	(0.548)	(-5.879)
$\beta(HML)$	-0.033	0.002	-0.589**	-0.131	-0.160	-0.642***
	(-0.268)	(0.012)	(-2.552)	(-0.907)	(-0.999)	(-3.800)
$Size$	-0.156***	-0.146***	-0.344***	-0.072**	-0.063*	-0.259***
	(-4.304)	(-3.581)	(-7.213)	(-2.537)	(-1.743)	(-6.968)
BM	0.117**	0.077	0.131***	0.055	0.088	0.093
	(2.285)	(1.561)	(2.717)	(0.710)	(1.007)	(1.147)
R_t	-0.043***	-0.046***	-0.046***	-0.024***	-0.025***	-0.025***
	(-10.456)	(-11.185)	(-11.079)	(-5.351)	(-5.134)	(-5.083)
$Skew_{1M}$	0.110***	0.116***	0.115***	0.126***	0.131***	0.133***
	(4.470)	(4.765)	(4.736)	(4.119)	(3.934)	(4.163)
$EIdioSkew$	-0.336***	-0.275**	-0.383***	-0.193	-0.158	-0.187
	(-3.077)	(-2.534)	(-3.437)	(-1.400)	(-1.099)	(-1.272)
$CoSkew$	-0.011**	-0.008	-0.014***	-0.015*	-0.019**	-0.024***
	(-2.382)	(-1.593)	(-3.115)	(-1.902)	(-2.424)	(-3.130)
$Illiq_{1M}$	0.039***	0.042***	0.043***	0.027	0.034	0.009
	(3.953)	(4.193)	(4.519)	(1.416)	(1.558)	(0.514)
$Adjusted\ R^2$	0.075	0.063	0.058	0.175	0.159	0.145
n	3056	2818	3038	3056	2818	3038

Panel B: Cross-Sectional Regression Step - GLS

	1M	12M	60M	FULL
$Constant$	0.553	1.162***	1.166***	1.339***
	(1.600)	(3.696)	(3.836)	(4.442)
$IVOL$	0.567	-0.818***	-0.842***	-0.549**
	(1.378)	(-2.929)	(-3.325)	(-2.300)
$\beta(MKT)$	0.214*	0.153	0.125	0.149
	(1.796)	(1.265)	(1.017)	(1.201)
$\beta(SMB)$	-0.004	-0.028	-0.015	-0.022
	(-0.105)	(-0.774)	(-0.415)	(-0.629)
$\beta(HML)$	-0.045	-0.045	-0.027	-0.025
	(-0.752)	(-0.704)	(-0.426)	(-0.397)
$Size$	-0.029	-0.061**	-0.059**	-0.092***
	(-1.169)	(-2.540)	(-2.286)	(-3.515)
BM	0.064	0.065	0.041	0.077
	(1.192)	(1.144)	(0.707)	(1.454)
R_t	-0.036***	-0.035***	-0.040***	-0.042***
	(-8.399)	(-9.157)	(-10.745)	(-11.174)
$Skew_{1M}$	0.002	0.084***	0.088***	0.090***
	(0.095)	(5.112)	(5.379)	(4.966)
$EIdioSkew$	-0.128	-0.099	-0.050	-0.027
	(-1.227)	(-0.898)	(-0.444)	(-0.235)
$CoSkew$	-0.011**	-0.014**	-0.016**	-0.014**
	(-1.982)	(-2.600)	(-2.375)	(-2.468)
$Illiq_{1M}$	0.022**	0.021***	0.019**	0.021**
	(2.459)	(2.884)	(2.436)	(2.612)
$Adjusted\ R^2$	0.085	0.071	0.073	0.071
n	3054	3056	2802	3038

researchers should reconsider to use these estimates to improve their model performance. In conclusion I can verify the findings of Z. Chen and Petkova (2012) that the adjustments in the estimation window used for risk factor loading estimation alone, do not influence the IVOL puzzle finding remarkably, as the IVOL risk premia remain negative and statistically significant.

With a few exceptions, the remaining risk premia estimates have hardly changed in comparison to the reference results. The risk premia of the FF3 model risk factor loadings turn statistically significant when the full sample is used for their computation, regardless of the weighting-scheme employed. It is of note, however, that for both weighting-schemes the average risk premia of the full sample $\beta(SMB)$ and $\beta(HML)$ are negative, which contradicts the predictions of Fama and French (1993) who argue for positive premia instead. Furthermore, the negative BM risk premium of the VW regressions from the reference result turns positive here and thus is more in line with the value effect also mentioned by Fama and French (1993). Nevertheless, this premium stays insignificant for all window sizes in the VW context and for the 60 months factor loadings in the EW regression analysis. Panel B shows that weighting stock returns by their estimated variance can have a non-trivial influence on the IVOL risk premium estimate. When the return variance is estimated from rolling windows over the current month, the IVOL risk premium estimate turns positive but also statistically insignificant. Hence, the IVOL puzzle seems to be related to the general stock return variance, as already found in section 5.3 as well as by Barinov and Chabakauri (2020) and accounting for this relationship by, for example, weighting observations as done here, would enable researchers to resolve the IVOL puzzle consequently. However, the remaining columns show that when the weighting is induced by return variance estimates from rolling window estimates covering 12 and 60 months or the complete sample horizon, the negative and statistically significant IVOL premium prevails. In consequence, when the return variance is measured over a wider window than 1 month, the IVOL puzzle persists almost unchanged hinting at a relationship only between IVOL and return variance measured over the same window size. Nevertheless, when using the variance estimates from the complete sample horizon in the weighting matrix, the IVOL risk premium is slightly less significant at a level of 5% and decreased in magnitude to -0.549. The effect of a one standard deviation increase in IVOL, holding all other variables constant, shows that the statistically significant risk premia are also economically meaningful as they predict a decrease in expected returns ranging from 0.205% to 0.315% per month.

Also for the remaining risk premia estimates some differences relative to the reference results have to be highlighted. The $Size$ premium is positive throughout all adjustments here. But interestingly, the level, at which it is statistically significant, increases with the window size used for return variance estimation. When the current month return variances are used in the weighting matrix, the $Size$ premium is statistically insignificant, but if I use the complete sample horizon variance estimates instead, it becomes significant at the 1% level. This indicates that the same relationship between current month return variance and IVOL might also exists for current month return variance and size. A comparable pattern exists for the $Skew_{1M}$ risk premium that also turns statistically insignificant when returns are weighted by their current month variances but is significant at the 1% level in the rest of the cases. In addition, the BM premium is now always positive as in the reference EW complete regression result, albeit not statistically significant. Lastly, the average adjusted R^2 show that the model, which uses the current month return variances in the weighting matrix, has the best fit compared to the other models.

In conclusion the IVOL puzzle is robust to adjustments in the window size used for risk factor loading estimation and mainly also when the GLS estimation technique is employed for estimation of the cross-sectional regressions. Panel A illustrates that, regardless of the choice of the estimation window utilized for computation of the FF3 model risk factor loadings, researchers can only verify the existence of the IVOL puzzle. In contrast, Panel B shows that the estimation of the cross-sectional regression using GLS with a diagonal weighting matrix consisting of the inverse of the current month return variances, enables researchers to argue on the absence of the puzzle respectively. However, if the variances are estimated from rolling windows over 12, 60 months or the complete sample horizon, the IVOL puzzle prevails.

7.2. Portfolio-Related

Finally I analyze the portfolio method-specific adjustments that are introduced in section 3.4. I begin with the modification related to the way the univariate IVOL portfolios are computed and analyzed in section 7.2.1 and afterwards continue with the investigation of the bivariate portfolio sorts in section 7.2.2.

7.2.1. Portfolio Characterization

This section has the objective of finding out whether researchers are able to influence their findings by the way they compute and analyze the univariate portfolios. To do so, as explained in section 3.4, I implement further trading strategies, use different breakpoints for assigning stocks to portfolios, adjust the number of portfolios constructed and also use different benchmarks towards which I compute the time-series alphas. Table 10 depicts the corresponding result of the long-short IVOL portfolio that goes long the portfolio of stocks with the highest IVOL and short the one containing stocks with the lowest IVOL. Here Panel A shows the results of the EW and Panel B those of the VW portfolio sorts.

The average EW total long-short portfolio return of Panel A is always positive and statistically insignificant, regardless of the trading strategy that is used for portfolio formation. Except for an increase in the magnitude of the average total returns, with the highest value of 0.472% per month for the 12/0/12 strategy, the results remain robust to modifications in the trading strategy, wherefore the conclusions drawn here

Table 10: Portfolio-Based Results - Portfolio Characterization

This table presents the average long-short portfolio results from the reference univariate portfolio sorting procedure introduced in section 3.3 that was adjusted as explained in section 3.4.3. Still all stocks traded on the NYSE/AMEX/NASDAQ are covered here. The first column called "Trading strategies" builds the univariate quintile IVOL portfolios based on other trading strategies than the reference 1/0/1 strategy including the 1/1/1, 1/1/12, 1/0/12 and the 12/0/12 strategy respectively. In the column named "Breakpoints" I cover different breakpoints used for portfolio formation other than the usual NYSE/AMEX/NASDAQ ones. NYSE uses only stocks traded on the NYSE to calculate the IVOL breakpoints, whereas "EQUAL MKT" assigns IVOL ranked stocks based on breakpoints such that the stocks in each quintile portfolio make up the same share of the overall market capitalization. The third column is related to the adjustments in the number of univariate portfolios that are constructed. Instead of 5 monthly univariate portfolios I also compute 10 or 15 portfolios and calculate their time-series averages respectively. The last column named "Risk-Correction" refers to the way the risk-adjusted returns in form of the time-series alphas are computed. For this adjustment I compute the alphas for the following additional risk-correction benchmark models: the SY4, the HOU5 and the DANIEL3 model. As this modification is only extending the reference setting for additional alpha estimates, the results from the total returns as well as the α^{FF3} and the α^{FF6} remain unchanged and are therefore left blank. For each of these adjustments, except for those in the column named "Risk-Correction", I compute the average total return, α^{FF3} and α^{FF6} of the long-short portfolio that goes long portfolio 5 that consists of the stocks with the highest IVOL and short portfolio 1 which contains stocks with the lowest IVOL. The α^{FF3} and the α^{FF6} denote the monthly average of Jensen's time-series alphas relative to the FF3 and the FF6 model that were calculated by equation 1 using the monthly portfolio excess returns over the complete sample horizon. Below each of these metrics I depict the corresponding Newey and West (1987) adjusted t-statistic using 7 lags. The last row named "α^{X}" reports the results of the risk-correction benchmark adjustments for the time-series alphas where the respective models used can be found in the last column. In Panel A I use the equal-weighted portfolio returns for the analysis, where all firms are getting the same weight. On the other hand, Panel B uses value-weighted portfolio returns that were calculated by weighting the stock returns within the portfolio by their MTCAP observable at beginning of the month in order to give higher weights to bigger stocks respectively and therefore diminish the effects that might be explicitly related to small stocks. The complete sample period covers July 1963 to December 2020. It is shorter when the SY4 or the DANIEL3 models are used which is due a lack of data availability and therefore the corresponding analysis cover only the horizon until November 2016 and November 2018 respectively.

Panel A: Equal-Weighted Portfolio Sorts											
	Trading Strategies				Breakpoints		# of Portfolios		Risk-Correction		
	1/1/1	*1/1/12*	*1/0/12*	*12/0/12*	*NYSE*	*EQUAL MKT*	*10*	*15*	*SY4*	*HOU5*	*DANIEL3*
Total Return	0.007	0.260	0.226	0.472	0.165	0.238	0.112	0.178			
	(0.025)	(0.906)	(0.784)	(1.417)	(0.685)	(1.088)	(0.338)	(0.510)			
α^{FF3}	-0.482	-0.203	-0.237	-0.028	-0.327	-0.226	-0.480	-0.464			
	(-2.751)	(-1.182)	(-1.376)	(-0.139)	(-2.456)	(-1.925)	(-2.259)	(-2.002)			
α^{FF6}	0.147	0.350	0.333	0.556	0.159	0.205	0.295	0.350			
	(0.718)	(1.838)	(1.727)	(2.412)	(1.132)	(1.610)	(1.182)	(1.329)			
α^{X}									0.230	0.595	0.383
									(0.896)	(1.824)	(1.410)

Panel B: Value-Weighted Portfolio Sorts											
	Trading Strategies				Breakpoints		# of Portfolios		Risk-Correction		
	1/1/1	*1/1/12*	*1/0/12*	*12/0/12*	*NYSE*	*EQUAL MKT*	*10*	*15*	*SY4*	*HOU5*	*DANIEL3*
Total Return	-0.493	-0.201	-0.260	-0.287	-0.173	-0.018	-1.002	-1.216			
	(-1.801)	(-0.771)	(-0.991)	(-0.915)	(-0.778)	(-0.092)	(-3.035)	(-3.512)			
α^{FF3}	-0.945	-0.609	-0.674	-0.780	-0.562	-0.334	-1.509	-1.761			
	(-5.323)	(-4.192)	(-4.599)	(-4.257)	(-4.246)	(-2.761)	(-6.944)	(-7.876)			
α^{FF6}	-0.308	-0.089	-0.134	-0.134	-0.106	0.033	-0.700	-0.913			
	(-2.170)	(-0.783)	(-1.142)	(-0.881)	(-0.837)	(0.310)	(-2.923)	(-4.260)			
α^{X}									-0.314	0.033	-0.155
									(-1.509)	(0.139)	(-0.808)

match those of the EW reference results from section 5.3. The findings from the α^{FF3} and α^{FF6}, however, seem to be sensitive to the choice of the trading strategy when analyzing the equal-weighted portfolio sorts. Even though the α^{FF3} are negative for all trading strategies, they are not always statistically and economically meaningful. These long-short portfolio α^{FF3} ranges from -0.482% for the 1/1/1 strategy to -0.028% per month for the 12/0/12 strategy with corresponding Newey-West robust t-statistics of -2.751 and -0.139. The 1/1/1 results show that the incorporation of a 1 month waiting period even increases the negative long-short portfolio α^{FF3} in absolute magnitude when compared to the reference results and hence contradicts the explanation of the puzzle based on short-term reversal as proposed by W. Huang et al. (2010). Whereas for the 1/1/12 and 1/0/12 strategies, the α^{FF3} are negative but not statistically significant at any reasonable level. When portfolios are built by the 12/0/12 strategy, on the other hand, the long-short portfolio α^{FF3} is very close to zero and statistically insignificant such that no IVOL puzzle or any systematic effect at all is revealed. In contrast, the α^{FF6} predict an opposite pattern where all alphas are positive and only statistically insignificant for the 1/1/1 strategy. For the remaining strategies, the 12 month holding period implies positive long-short portfolio α^{FF6} ranging from 0.333% to 0.556% per month and that are economically meaningful and statistically significant at a level not less than 10%. Hence, when the FF6 model is utilized for risk-correction, a positive relationship between IVOL and expected risk-adjusted returns remains that is statistically significant if the holding period covers 12 months. This result contradicts the finding of an IVOL puzzle and rather matches the results of Spiegel and Wang (2005) as well as Malkiel and Xu (1997).

The EW reference results, however, appear robust to the breakpoint adjustment considered here. The long-short portfolio total return and α^{FF6} are positive on average for all breakpoints, but none of them is statistically significant at a reasonable level which is similar to the EW results of section 5.3. Also similar is that the α^{FF3} is negative and statistically, as well as economically, significant ranging from -0.327% to -0.226% per month such that the IVOL puzzle can again be detected in the EW context from the α^{FF3} only. Thus in contrast to the findings of Bali and Cakici (2008), the choice of the breakpoints has almost no effect on the EW portfolio sort results when compared to the reference setting. However, when comparing the results of both breakpoints, the α^{FF3} is lower and has a higher t-statistic for the NYSE breakpoints, whereas the total return and the α^{FF6} as well as their t-statistics are higher for the equal market share breakpoints implying that the IVOL puzzle is indeed weaker for the equal market share breakpoints as also found by Bali and Cakici (2008).

The EW long-short portfolio results hardly change, compared to the reference results, when I sort stocks each month into 10 or 15 portfolios respectively. Total returns and the α^{FF6} remain positive and statistically insignificant again and just the α^{FF3} delivers evidence on the IVOL puzzle, as it is negative and statistically significant at the 5% level for 10 and 15 portfolios. The α^{FF3} ranges from an average -0.464% for 15 to -0.480% per month for 10 monthly portfolios which are thus also economically meaningful. Nevertheless, the magnitudes of the total return, the α^{FF3} and the α^{FF6} for both numbers of portfolios is very close to the one in section 5.3, which is why choosing a different number of univariate portfolios in the research study is unlikely to affect the study's conclusions.

In the last column of Panel A I depict the alpha estimates from the EW long-short portfolio relative to further risk-correction models including the SY4, the HOU5 and the DANIEL3 model. All these alphas are found to be positive ranging from an average of 0.230% for the SY4 model alpha to 0.595% per month for the HOU5 model alpha. Even though these appear to be economically meaningful, only the one relative to the HOU5 model is also statistically significant at the 10% level. Hence, researchers that base their arguments only on the HOU5 alpha find a positive relationship between risk-adjusted expected returns and IVOL instead of an IVOL puzzle. For the alphas relative to the SY4 and the DANIEL3 model, conclusions are in line with those drawn from the α^{FF6}, implying the same positive relationship that is, however, not significant at a plausible level. Thus, the choice of the risk-correction model used for computation of the time-series alphas likely influences the conclusions drawn from the study on the existence of the IVOL puzzle.

The results from the VW long-short portfolio seem to be overall more sensitive to the adjustments in the way the portfolios are characterized. With the choice of the trading strategy researchers can influence their findings compared to the reference results of section 5.3. As the total returns, α^{FF3} and α^{FF6} have been negative and statistically significant in the VW reference setting, this result does not hold for all trading strategies. Even though the long-short portfolio total returns are all still negative, just the one of the 1/1/1 strategy is also statistically significant at the 10% level with a average monthly total return of -0.493% that is also economically meaningful. All remaining strategies imply total returns between -0.201% and -0.287% per month that are less than half the magnitude of the reference results and are furthermore even statistically insignificant. The α^{FF3}, on the other hand, remains statistically significant at the 1% level for all trading strategies, although it has reduced in magnitude to average risk-adjusted returns between -0.945% and -0.609% per month that are still economically relevant. In addition, similar to the total returns, the α^{FF6} is only statically significant for the 1/1/1 strategy with an average value of -0.308% per month that is economically relevant and statistically significant at the 5% level. However, compared to the EW portfolio sorts of Panel A, the inclusion of the 1 month holding period does not strengthen the IVOL puzzle effect but instead makes it slightly weaker as the magnitude of all measures shrinks in comparison to the reference results from Panel B of table 3. This fact, as well as the insignificant results from the remaining strategies, indicate that at least in the VW context the correction for short-term reversal helps explaining the IVOL puzzle, as also pointed out by W. Huang et al. (2010). In conclusion, researchers that exclude the α^{FF3} from their analysis and use trading strategies other than the 1/0/1 or 1/1/1 would not be able discover an IVOL puzzle that is statistically plausible with their VW portfolio-based analysis. However, Ang et al. (2006) claim that their IVOL puzzle finding is robust to the implementation of further trading strategies. This conclusion can be explained by the fact that they focus on the examination of the α^{FF3} in their study which is the only measure that is indeed robust to such adjustments (see Ang et al. (2006)). Nevertheless, their results would likely change when they compute total returns or the α^{FF6} instead. If researchers shift from a 1/0/1 to a 1/1/1 trading strategy they are able to slightly reduce the magnitude of the IVOL puzzle effect.

As found by Bali and Cakici (2008), the IVOL puzzle is sensitive to the choice of the breakpoints used to subdivide stocks into VW portfolios. The long-short portfolio total returns remain negative for both breakpoint types but at the same time become statistically insignificant as well. In addition, the magnitude of the total returns has decreased to -0.173% for the NYSE breakpoints and to an average of only -0.018% per month for the equal market share breakpoints. Therefore, the total return results from the equal market share breakpoints do not only turn statistically insignificant but also become economically negligible. Even though the α^{FF3} remain statistically significant at the 1% level for both breakpoints, their magnitude more than halved to values around -0.562% and -0.334% per month with t-statistics that decrease to -4.246 and -2.761 when compared to the reference results. Hence, although no change in statistical relevance is observable for the α^{FF3}, the decrease in economic magnitude becomes obvious. Even more noticeable is the influence of the choice of the breakpoints when analyzing the α^{FF6}. For the NYSE breakpoints the long-short α^{FF6} is still negative but not statistically significant at any reasonable level, whereas for the equal market share breakpoints the alpha even turns positive albeit remaining statistically insignificant. Hence, researchers that would use the FF6 model for risk-correction and the equal market share breakpoints can almost reverse the IVOL puzzle effect. Conclusively, the choice of the breakpoints has a strong influence on the findings if researchers use total returns or α^{FF6} in their analysis only. The influence on the study is the strongest in terms of changes relative to the reference results when equal market share breakpoints are used for the VW portfolio analysis as also pointed out by Bali and Cakici (2008).

On the other hand, as I increase the number of univariate VW portfolios used for the analysis, the average long-short portfolio total return, α^{FF3} and α^{FF6} as well as the corresponding robust t-statistics grow in magnitude compared to the reference results. Similar to the reference setting, all these measures are negative and statistically significant at the 1% level which also encloses the total returns. In economic terms, the total returns and all the alphas are also plausible, where the total return and the α^{FF3} for the long-short portfolio using 10 and 15 portfolios is in no case higher than -1% per month. Only the long-short portfolio α^{FF6} is slightly larger and varies between an average of -0.700% in case of 10 portfolios and -0.913% per month when 15 portfolios are constructed. Nevertheless, I find that an increase in the number of monthly univariate portfolios strengthens the negative relationship between IVOL and expected cross-sectional returns. Thus in this VW setting researchers can let their results appear more statistically and economically meaningful by increasing the number of univariate IVOL sorted portfolios they compute in their study.

Lastly, the choice of the risk-correction model used for alpha computation also influences the conclusion a researcher would draw from his study. As in the reference setting, the long-short portfolio α^{FF3} and α^{FF6} have been negative and statistically significant at the 1% level which does not apply to the SY4, the HOU5 and the DANIEL3 alphas. The average long-short portfolio alpha relative to the SY4 and the DANIEL3 model are both negative with values of -0.314% and -0.155% per month respectively but both of them are low in magnitude and not statistically significant at any reasonable level. Thus in these cases an IVOL puzzle is detected in signs but cannot be approved statistically in magnitude. In contrast, the long-short portfolio alpha relative to the HOU5 model is positive but not statistically significant at any plausible level and with an average value of 0.033% per month economically negligible. Consequently, the IVOL puzzle finding is not robust to the choice of the risk-correction model used for alpha computation and researchers that use models like the SY4, the HOU5 or the DANIEL3 for that purpose in the VW portfolio sorts would likely not be able to statistically plausible verify the existence of the puzzle.

Conclusively I found out that the EW portfolio sort results are more robust to adjustments in the way the portfolios are characterized than the VW ones. Only the choice of the trading strategy has a notable influence on the results of the EW sorts. Here especially the usage of a longer holding period than 1 month can lead to the conclusion that no systematic effect in the total returns and alphas is found. Conversely, the findings from the VW portfolio sorts are effected by all the adjustments I consider for the category of portfolio characterization. As in the EW setting, a holding period longer than 1 month lets the magnitude of the IVOL puzzle appear mostly statistically insignificant when excluding the analysis of the α^{FF3}. Thus in contrast to the arguments of Ang et al. (2006), the choice of the trading strategy is not irrelevant to the IVOL puzzle finding in the VW context. As stated by Bali and Cakici (2008), the decision on the breakpoints also affects the VW portfolio results because both breakpoints analyzed here show no evidence on the existence of the IVOL puzzle when investigating the total return or α^{FF6}. On the other hand, increasing the number of monthly univariate IVOL portfolios leads to a monotonic increase in the magnitude of the IVOL puzzle in the VW context. Finally, when long-short portfolio alphas are computed towards the SY4, the HOU5 or the DANIEL3 model, no IVOL puzzle can be discovered that is also statistically significant.

7.2.2. Bivariate Portfolio Sorts

Ultimately I investigate if controlling for further firm characteristic effects by bivariate portfolio sorts enables researchers to explain the IVOL puzzle and then also try to determine the sorts they would choose for that purpose. For this I use the dependent portfolio sorting technique and the control variables introduced in section 3.4 to construct the average IVOL portfolios. I compute the total return, the α^{FF3} and the α^{FF6} for average portfolio 1, 5 and the long-short portfolio as well as the corresponding Newey-West robust t-statistics. In table 11 I depict the EW average portfolio sort results in Panel A and those from the VW analysis in Panel B respectively.

The EW bivariately sorted average portfolios from Panel A show that the finding of an IVOL puzzle is predominantly robust to the effect from most of the control variables considered here. Compared to Panel A of table 3, results remain almost unchanged when I control for $Skew_{1M}$, $Volume$ or $CoSkew$. In these cases, I find that the total returns, as well as the α^{FF6}, increases when moving from portfolio 1 to portfolio 5, such that the corresponding measures are positive for the long-short portfolio. However, these positive long-short portfolio total returns and α^{FF6} are still not statistically significant at any reasonable level. Only the corresponding long-short portfolio α^{FF3} are negative with values of -0.384% per month when controlling for skewness and -0.328% per month for the coskewness control that are both economically plausible and statistically significant at the 5% level. Controlling for volume yields a long-short α^{FF6} of -0.493% per month that is economically meaningful and even statistically significant at the 1% level. Hence, researchers that control for the volume effect by Gervais et al. (2001), the monthly return skewness or the coskewness measure of Harvey and Siddique (2000) are not able to modify their findings related to the IVOL puzzle notably and can only verify its existence from the α^{FF3} analysis.

On the other hand, controlling for variables including size, the past months return, expected idiosyncratic skewness, the Amihud (2002) illiquidity measure and the bid-ask spread in the bivariate sorts have only a limited impact on the EW results and often make the IVOL puzzle even more apparent instead of resolving it. For all the bivariate dependent sorts based on these control variables, the analysis of the total returns and the alphas shows the same pattern across average portfolios. In all these cases, the corresponding measure decreases when moving from portfolio 1 to portfolio 5 and thus results in a negative long-short portfolio value, which is consistent with the IVOL puzzle finding. However, only the negative long-short α^{FF3} are also statistically significant even at a 1% level for all controls, whereas the related total returns and α^{FF6} are not significant at any meaningful level. In consequence, even though the IVOL puzzle appears throughout all measures when controlling for the variables mentioned above, the effect is only statistically significant for the α^{FF3}. This pattern found in the α^{FF3} is always driven by a statistically significant underperformance of portfolio 5 relative to portfolio 1 which is similar to the findings of the reference analysis. It is on average the lowest in economic terms when controlling for the past month return with an average long-short portfolio α^{FF3} of -0.477% per month, while it is the highest in the bivariate sorts involving the Amihud (2002) illiquidity measure with an average subsequent month α^{FF3} of -0.832% per month. These α^{FF3} all exceed the corresponding alpha from the reference analysis and are economically meaningful. From this fact, as well as the one that long-short portfolio total returns and α^{FF6} are negative even though not statistically significant for the controls including $Size$, R_t, $EIdioSkew$, $Illiq_{1M}$ and $Spread$, I conclude that researchers using these controls in their EW bivariate dependent portfolio analysis can let the IVOL puzzle appear more plausible than without their consideration.

The only variables that indeed seem to resolve the IVOL puzzle when controlled for in the researchers' EW bivariate portfolio analysis are BM and $Maxret$. Starting with the sorts that control for BM, the total returns show an monotonic increase when moving from portfolio 1 to 5, which is why the long-short portfolio total return is also positive again, even though it is still not significant at any plausible level. Although the negative relationship between IVOL and α^{FF3} persists on average, the long-short portfolio α^{FF3} of -0.229% per month is now no longer statistically significant at any meaningful level as the t-statistic has decreased to -1.229. The typical pattern of an IVOL puzzle even reversed when analyzing the α^{FF6}. Here the α^{FF6} increases monotonically from an average -0.080% for portfolio 1 to 0.356% per month for portfolio 5 resulting in a long-short portfolio α^{FF6} of 0.436% per month that is not only statistically meaningful at the 5% level but is also economically relevant. From this I conclude that the value effect might indeed affect the IVOL puzzle which is also argued by Barinov and Chabakauri (2020) and researchers who are aware of this fact can influence their results by including it in their study. It is of note that the EW regression results from Panel A of table 2 were not able to uncover this relationship which might hint at a non-linear dependence between the value effect and the IVOL puzzle respectively. Controlling for $Maxret$, on the other hand, overturns the IVOL puzzle almost completely, which proves the findings of Bali et al. (2011). An investigation of the EW average portfolios from the dependent bivariate sorts involving $Maxret$ shows that the total returns and alphas monotonically increase when shifting from portfolio 1 to portfolio 5, such that the corresponding long-short portfolio results are also positive for all the measures. The long-short portfolio total return and the α^{FF6} amount on average to 0.408% and 0.481% per month which are not only economically meaningful but also statistically significant at the 1% level and consequently imply a positive relationship between IVOL and expected returns instead of an IVOL puzzle. Whereas for the total returns from portfolio 1 and 5 both are statistically significant at the 1% level, the pattern in the α^{FF6} seems to be driven by the overperformance of the high IVOL portfolio which has a positive α^{FF6} that is statistically significant at the 5% level. Despite the fact that the long-

Table 11: Portfolio-Based Results - Bivariate Portfolio Sorts

This table presents the average results of quintile portfolio 1, 5 and the long-short portfolio from the bivariate portfolio sorting procedure as explained in section 3.4.3 covering all stocks traded on the NYSE/AMEX/NASDAQ. First all stocks are sorted each month into quintile portfolios with respect to their level of the corresponding control variable and afterwards within each of these quintile portfolios the stocks are again sorted into quintile portfolios based on their idiosyncratic volatility computed relative to the FF3 model using daily data over the past month. Afterwards each IVOL quintile portfolios, as well as the long-short portfolio, is averaged over all the quintiles of the control variables to obtain the univariate IVOL quintile portfolios that have been corrected for the effect of the corresponding control variable. For all these bivariately sorted average portfolios I compute the average monthly total returns, the α^{FF3} and the α^{FF6} of portfolios 1, 5 as well as the long-short portfolio (in percentage terms) and in parenthesis below each of these measures I show the corresponding Newey and West (1987) adjusted t-statistics where I use 7 lags for the adjustment. The α^{FF3} and α^{FF6} denote the monthly average of Jensen's time-series alphas relative to the FF3 and the FF6 model again that were calculated by equation 1 using the monthly portfolio excess returns from the respective bivariate portfolios over the complete sample horizon. Each row corresponds to the results of a bivariate portfolio sort where the effect that was controlled for is used as row name. Here I control for the following variables: size that is the natural logarithm of the stock's MTCAP, the firm's BM, the stock's current month return, the Amihud (2002) illiquidity measure calculated over the current month, return skewness over the current month, the expected idiosyncratic skewness measure of Boyer et al. (2010), the coskewness measure of Harvey and Siddique (2000) computed over the recent past 5 years computed, the trading volume of the stock, the monthly average over the daily bid-ask-spreads and the maximum daily return during the current month computed based on the ideas of Bali et al. (2011). These portfolios are rebalanced monthly according to the 1/0/1 trading strategy. In Panel A I use the equal-weighted portfolio returns for the analysis, where all firms are getting the same weight. On the other hand, Panel B uses value-weighted portfolio returns that were calculated by weighting the stock returns within the portfolio by their MTCAP observable at beginning of the month in order to give higher weights to bigger stocks respectively and therefore diminish the effects that might be explicitly related to small stocks. The complete sample period covers July 1963 to December 2020.

Panel A: Equal-Weighted Portfolio Sorts

	Total Return			α^{FF3}			α^{FF6}		
	1	5	5-1	1	5	5-1	1	5	5-1
Size	0.939	0.589	-0.349	0.085	-0.728	-0.813	0.021	-0.213	-0.235
	(3.947)	(1.611)	(-1.423)	(0.412)	(-4.388)	(-5.225)	(0.087)	(-1.074)	(-1.559)
BM	0.932	1.109	0.177	0.003	-0.226	-0.229	-0.080	0.356	0.436
	(4.231)	(2.817)	(0.659)	(0.021)	(-1.105)	(-1.229)	(-0.713)	(1.417)	(2.044)
R_t	0.914	0.830	-0.085	-0.005	-0.482	-0.477	-0.021	-0.051	-0.030
	(4.241)	(2.074)	(-0.328)	(-0.035)	(-2.559)	(-3.199)	(-0.131)	(-0.240)	(-0.198)
$Skew_{1M}$	0.870	0.973	0.103	0.012	-0.372	-0.384	-0.080	0.204	0.283
	(3.920)	(2.460)	(0.363)	(0.059)	(-1.945)	(-2.225)	(-0.355)	(0.841)	(1.388)
EIdioSkew	1.091	0.847	-0.245	0.121	-0.545	-0.666	0.058	-0.028	-0.086
	(4.105)	(2.032)	(-0.937)	(0.768)	(-2.364)	(-3.929)	(0.433)	(-0.086)	(-0.470)
CoSkew	0.955	1.069	0.115	0.053	-0.275	-0.328	-0.016	0.222	0.238
	(4.347)	(2.830)	(0.453)	(0.339)	(-1.469)	(-2.056)	(-0.104)	(0.950)	(1.258)
$Illiq_{1M}$	0.960	0.609	-0.351	0.072	-0.759	-0.832	0.003	-0.171	-0.174
	(4.208)	(1.641)	(-1.350)	(0.371)	(-4.427)	(-5.085)	(0.012)	(-0.804)	(-0.993)
Volume	0.877	0.888	0.011	0.000	-0.493	-0.493	-0.086	0.104	0.190
	(4.104)	(2.234)	(0.038)	(0.001)	(-2.431)	(-2.828)	(-0.464)	(0.411)	(0.945)
Spread	0.889	0.773	-0.115	0.023	-0.540	-0.563	-0.030	-0.120	-0.090
	(3.999)	(2.062)	(-0.456)	(0.135)	(-3.027)	(-3.733)	(-0.170)	(-0.580)	(-0.568)
Maxret	0.913	1.322	0.408	-0.101	0.074	0.175	-0.065	0.416	0.481
	(3.643)	(3.917)	(2.759)	(-0.729)	(0.455)	(1.650)	(-0.335)	(2.164)	(3.848)

Panel B: Value-Weighted Portfolio Sorts

	Total Return			α^{FF3}			α^{FF6}		
	1	5	5-1	1	5	5-1	1	5	5-1
Size	0.802	0.565	-0.237	-0.011	-0.552	-0.540	-0.143	-0.215	-0.072
	(4.167)	(1.848)	(-1.113)	(-0.096)	(-3.963)	(-3.873)	(-1.567)	(-1.359)	(-0.604)
BM	0.794	0.261	-0.533	-0.027	-0.989	-0.962	-0.140	-0.461	-0.321
	(4.067)	(0.744)	(-2.118)	(-0.350)	(-5.308)	(-6.425)	(-2.285)	(-1.674)	(-2.275)
R_t	0.769	0.235	-0.533	-0.092	-1.005	-0.913	-0.143	-0.625	-0.482
	(4.060)	(0.681)	(-2.223)	(-1.203)	(-5.767)	(-6.542)	(-2.282)	(-3.415)	(-3.877)
$Skew_{1M}$	0.787	0.120	-0.667	-0.048	-1.132	-1.085	-0.166	-0.601	-0.435
	(4.058)	(0.331)	(-2.393)	(-0.423)	(-5.940)	(-6.104)	(-2.245)	(-2.428)	(-2.659)
EIdioSkew	0.896	0.453	-0.443	-0.036	-0.872	-0.836	-0.196	-0.480	-0.284
	(3.839)	(1.214)	(-1.855)	(-0.200)	(-2.880)	(-4.981)	(-1.817)	(-0.712)	(-1.919)
CoSkew	0.792	0.318	-0.474	-0.050	-0.913	-0.863	-0.126	-0.546	-0.420
	(4.046)	(0.975)	(-2.157)	(-0.114)	(-5.279)	(-6.209)	(-0.315)	(-2.338)	(-2.995)
$Illiq_{1M}$	0.800	0.530	-0.269	-0.015	-0.619	-0.604	-0.149	-0.268	-0.118
	(4.165)	(1.684)	(-1.221)	(-0.132)	(-4.183)	(-4.193)	(-1.709)	(-1.540)	(-0.944)
Volume	0.785	0.146	-0.639	-0.047	-1.108	-1.061	-0.165	-0.545	-0.380
	(4.081)	(0.404)	(-2.395)	(-0.462)	(-5.770)	(-6.429)	(-2.079)	(-2.105)	(-2.458)
Spread	0.797	0.232	-0.565	-0.028	-0.989	-0.961	-0.040	-0.722	-0.681
	(4.076)	(0.702)	(-2.416)	(-0.282)	(-5.835)	(-6.599)	(-0.467)	(-3.383)	(-4.400)
Maxret	0.756	0.764	0.007	-0.152	-0.280	-0.129	-0.115	-0.293	-0.178
	(3.397)	(2.991)	(0.065)	(-0.991)	(-1.260)	(-1.411)	(-0.672)	(-0.806)	(-1.910)

short portfolio α^{FF3} is also positive with a value of 0.175% per month, it has to be noted that it is rather low in economic terms and only statistically significant at the 10% level. Again the relationship between IVOL and the maximum return is most likely non-linear as the IVOL risk premium from the EW regressions is almost unaffected by the inclusion of $Maxret$ as a further control variable as found in Panel A of table 8.

Panel B displays the results from the VW average dependent sorted portfolios for the same controls as considered in Panel A. When computing the VW average portfolios from bivariate dependent sorts that involve the BM, the past month return, the monthly return skewness, the coskewness measure of Harvey and Siddique (2000), trading volume or the bid-ask spread as control variables, the results from total returns and alphas remain almost unchanged compared to the reference results form Panel B of table 3. The analysis of the long-short portfolio performance in all these sorts shows that portfolio 5 performs worse than portfolio 1, as the long-short portfolio total returns, as well as the alphas relative to both risk-correction models, are negative and statistically significant. The long-short portfolio returns are the lowest in case of the α^{FF3} when controlling for $Skew_{1M}$ with a value of -1.085% per month the highest when controlling for BM with an average α^{FF6} of -0.321% per month. Therefore, the evidence on the IVOL puzzle prevails in all these sorts respectively and is still economically meaningful. It is of note, however, that for all average portfolios considered here the measures are lower in magnitude than compared to their level from the reference analysis of section 5.3. The only exception is the long-short α^{FF6} when controlling for R_t and $Spread$ that are now bigger in absolute terms. Interesting is also that the bivariate dependent sort based on BM was able to resolve the IVOL puzzle in the EW context but is no longer able to do so here. An explanation of that might be that the return weighing by MTCAP resolves the value effect, if the assumption holds that the small firms also have a high BM.
In contrast, controlling for $Size$, $EIdioSkew$ or $Illiq_{1M}$ has a limited, albeit notable, effect on the results. All bivariate sorts related to these control variables show the same pattern consisting of a negative long-short portfolio return, regardless of the return type or risk-correction model considered. Just the long-short portfolio α^{FF3} is also always statistically significant. While the long-short portfolio total return and α^{FF6} of these sorts is mostly statistically insignificant at any reasonable level, the α^{FF3} remains statistically significant at the 1% level. Only the bivariate dependent sorts on $EIdioSkew$ also have long-short portfolio total returns and α^{FF6} that are both statistically significant but just at the 10% level. However, compared to the reference result, the economic magnitude of the long-short α^{FF3} decreases to values ranging from -0.836% per month for the sort that controls for $EIdioSkew$ to -0.540% per month when controlling for $Size$. Nevertheless, these values are still economically meaningful and again driven by the low performance of portfolio 5 as the corresponding α^{FF3} is the only one that is statistically significant at a level not less than 1%. Additionally, it is of note that expected idiosyncratic skewness only marginally helps to explain the IVOL puzzle and does not resolve it, as argued by Boyer et al. (2010). In conclusion, researchers who use $Size$, $EIdioSkew$ or $Illiq_{1M}$ as controls in their VW portfolio-based study, are not able to resolve the IVOL puzzle but can achieve a reduction in its magnitude and statistical relevance.

The only variable that is indeed able to resolve the IVOL puzzle almost completely is the maximum daily return measure over the past month. Controlling for $Maxret$ leads to an increasing total return pattern across IVOL portfolios when moving from portfolio 1 to 5 and causes the long-short portfolio total return to be positive with an average value of 0.007% per month, which is statistically insignificant and economically negligible. Despite the fact that the alphas are still decreasing when moving from IVOL quintile portfolio 1 to 5 also yielding negative long-short portfolio alphas, they are only statistically significant at the 10% level in terms of the α^{FF6}. The long-short portfolio alpha amounts to an average value of -0.129% per month for the α^{FF3} and to -0.178% per month for the α^{FF6} that are both economically small. In contrast to the reference results, the alphas from portfolio 1 and 5 are both not statistically significant at any plausible level regardless of the model chosen for risk-correction which is thus another evidence on the explanatory power of $Maxret$. After all, similar to the EW sorts of Panel A, if researchers would use $Maxret$ as a control variable, they are be able to change their findings such that they conclude that the IVOL puzzle found by Ang et al. (2006) is rather implausible.

To sum up the results, it can be said that bivariate depended sorts hardly help researchers to explain the IVOL puzzle. Overall, as also found by Bali et al. (2011), the control associated with the maximum daily return measure is the only variable that could be used by researchers who try to argue against the existence of the IVOL puzzle, regardless of the weighting scheme they would use for the portfolio-based analysis. Researchers that compute EW portfolio returns only could additionally use the book-to-market ratio of equity as a control variable to offset the IVOL puzzle effect. All remaining control variables are not at all or only partly able to explain the IVOL puzzle where the respective conclusion strongly depends on the return type computed and presented in the corresponding analysis. Here it is mostly the α^{FF3} that discovers the IVOL puzzle irrespective of the control or weighting scheme used.

8. Conclusion

With this study I try to shed light on the dispute over the existence of the IVOL puzzle originally found by Ang et al. (2006). I take on an econometric perspective and analyze in how far researchers can draw different conclusions on the existence of the IVOL puzzle only by adjusting their researcher design in a specific manner. As this anomalous finding is most often analyzed by either a regression-based approach that follows the ideas of Fama and MacBeth (1973) or a portfolio-based methodology which involves sorting stocks into portfolios based on their idiosyncratic volatility, I use these two concepts alongside which I analyze how findings evolve when the research design is modified. As an anchor point I have used the study of Ang et al. (2006) for the portfolio-based and the one of Ang et al. (2009) for the regression-based method as these were among the first to find an IVOL puzzle while using the corresponding research concepts. For the regression-based approach I replicated the IVOL puzzle finding by means of equal-weighted as well as value-weighted Fama-MacBeth regressions. Here both weighting-schemes showed a statistically significant negative IVOL risk premium that is robust when controlling for the current month return skewness, the expected idiosyncratic skewness measure by Boyer et al. (2010), the coskewness measure of Harvey and Siddique (2000) and the Amihud (2002) illiquidity measure. The replication analysis of the portfolio-based approach already showed that the IVOL puzzle finding is more likely to occur when researchers use the value-weighted instead of the equal-weighted portfolio returns in their study. While all measures including excess and total returns as well as risk-adjusted returns relative to the FF3 and the FF6 model prove the existence of the IVOL puzzle in a value-weighted portfolio context, the equal-weighted portfolio returns are solely able to deliver this finding when researchers compute the time-series alphas relative to the FF3 model only. Based on these results I have started the adjustment analysis with the aim to find possible modifications that change the researchers conclusions on the existence of the IVOL puzzle. As general adjustments, that I applied to both research concepts, I have started by modifying the way IVOL is estimated and afterwards adjusted the sample used in the respective analysis. Changing the risk-correction model relative to which IVOL is estimated hardly changes the results researchers would obtain from their study regardless of the research concepts employed. However, those researchers that use the portfolio-based method or the value-weighted Fama-MacBeth regression are able to let the finding of an IVOL puzzle appear statistically insignificant only by switching the data frequency from daily to monthly and using a longer estimation window covering 1 or 5 years when estimating IVOL. The investigation of subsamples constructed based on the stock's price or market capitalization is also shown to influence research findings. Results appear to be particularly sensitive to the sample used when they are obtained from the VW Fama-MacBeth regressions or the VW portfolio sorts, as in these cases the IVOL puzzle effect seems to gradually weaken when stocks with a higher price or bigger market capitalization are used. After implementation of the general adjustments, I start with the analysis of adjustments that focus on the methodological peculiarities of each research concept and reveal how such modifications influence the research findings accordingly. For the regression-based approach I have focused on adjustments in the risk-related control variables as well as the estimation procedure. Results from the inclusion of different systematic risk-correction models or additional firm-specific risk variables appear to be almost similar to the reference results, which is why researchers are typically not able to modify their results by conducting such modifications. Only in the VW Fama-MacBeth regression the usage of a risk-correction model consisting of the FF3 model augmented with a short and long-term reversal factor allows the researchers to slightly mitigate the statistical significance of the puzzle. On the other hand, researchers that modify the estimation procedure of the regression-based approach have only limited ability to change their results. While using different past estimation windows for the computation of risk factor loadings used in the cross-sectional regression does not influence the findings in either the equal- nor the value-weighed context, changing the estimation technique of the cross-sectional regression step by implementation of the GLS technique with a diagonal weighting matrix consisting of the inverse of the stock variances estimated from rolling windows over the current month allows researchers to reverse the IVOL puzzle and obtain a positive risk premium instead that is, however, not statistically significant. Nevertheless, using other windows for the estimation of the stock variance used in the weighting matrix does not change results notably. Then shifting to the portfolio-related adjustments, I modify the way portfolios can be characterized and lastly also use bivariate dependent portfolio sorts to disentangle the effect of the IVOL puzzle that might be due to other firm-related effects. The portfolio characterization adjustments cover modifications in the trading strategy used for portfolio formation, the breakpoints used to subdivide stocks into portfolios, the number of portfolios that are computed as well as further risk-correction models used for alpha computation. Except for the trading strategies that involve a longer holding period than 1 month, researchers are not able to change their results when using EW portfolio sorts. However, the VW portfolio sorts are more sensitive to similar adjustments. Researchers that use trading strategies with longer holding periods than 1 month or breakpoints based on the NYSE stocks or equal market shares are only able to find an IVOL puzzle when they analyze the FF3 model alphas. No IVOL puzzle is found at all, however, by researchers that would compute alphas relative to the SY4, HOU5 or the DANIEL3 model for their VW portfolio sorts. Conversely, an increase in the number of portfolios lets the puzzle appear more present in the VW portfolio analysis. Lastly, I discovered that when researchers control for the maximum daily return over the past month by means of bivariate dependent portfolios sorts, they can conclude that the IVOL puzzle is not statistically relevant or existent at all, regardless of the weighting scheme they use in their study. In addition, those researchers that focus on EW portfolios can also resolve the puzzle when they control for the value effect in their study captured by the BM. All in all, I need to remark that my analysis was only able to investigate a fraction of adjustments that could possibly be employed by researchers to bias their results towards a specific direction. My aim is to draw attention to the fact that the configuration of the researcher design can be important to understand results that contradict theoretically sophisticated predictions and hence trigger a debate in the literature. Furthermore, this study should help to understand why researchers are then also able to provide empirical evidence on contradictory results, even though they appear to use the same data and methodology.

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