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Analysis of Green Bonds

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Abstract

Issuing its first green federal security in 2020, Germany pioneered a unique twin bond concept to address potential liquidity risks compared to their conventional counterparts. A switch mechanism between green and conventional bonds was introduced that allows debt-neutral sale-and-purchase (switch) transactions by the issuing authority. The main goal of this dissertation is to provide a theoretical model that is capable to explain the effects of this twin bond concept on the pricing of green bonds. For this purpose, a stochastic liquidity premium following a Vasicek (1977) process, a constant green premium and a switch option, which is executed when the green bond price falls below the price of its conventional twin bond, are assumed. The model results confirm that this twin bond concept is a viable solution to mitigate illiquidity-induced costs for the green bonds. The main learning from the model is a potential positive value of the switch option before its execution. This implies that issuers adopting this concept could benefit from lower costs of capital compared to ordinary green bonds without a switch mechanism. For investors holding the green instruments, this implies a reduced exposure to liquidity risks.

Keywords: Green bonds; German twin bonds; green premium; liquidity premium; switch transactions.

1. Introduction

Most nations have acknowledged the risks of climate change and pledged to pursue mitigating measures. As of today, 193 Parties adopted the 2015 Paris Agreement on climate change with the commitment to limit global temperature increase to 1.5°C above pre-industrial levels, and all United Nations Member States committed to the 17 Sustainable Development Goals (United Nations, n.d., 2022). This transition to a more sustainable global economy requires a substantial amount of new investments. For example, the European Commission anticipates additional annual investment needs of approximately 2.3% of GDP (i.e., 336 bn. EUR) for necessary energy system investments (exclusive transport) in light of its 2030 Climate Plan and 1.6% of GDP thereafter, aiming to become climate neutral by 2050 (European Commission, 2020). Similarly, the German Federal Government may need to increase its annual expenses from 200 bn. EUR to about 240 bn. EUR to become climate neutral until 2045, which is an additional percentage point of its 2019 GDP, cumulating to approximately 7% of GDP in total (Helmcke, Heuss, Hieronimus, & Engel, 2021). In addition to funding from the public sector, private investments can play a crucial role to provide the required financial resources (European Commission, 2020).

One instrument to raise funding for this transition is the emission of green bonds. Such fixed income debt securities differ from conventional bonds as their proceeds are entirely dedicated to the financing of environmental or climate related projects (Ehlers & Packer, 2017). Ehlers and Packer (2017) show that there is no single global definition for projects that fall into this category, but a range of different established standards and external verification procedures instead. For example, one widely accepted industry standard that is also adopted by the German Green Bond Framework are the Green Bond Principles issued by the International Capital Market Association (ICMA) (Finanzagentur GmbH, 2020). Other external validation concepts are second-party opinions by independent research institutes such as the Center for International Climate Research (CICERO), verification by auditors such as KPMG, certifications by organizations such as the Climate Bond Initiative (CBI) or green ratings, by rating agencies such as Standard & Poor's and Moody's (Dorfleitner, Utz, & Zhang, 2021). Finally, there also exist regional standards such as the EU Green Bond Standard or China's Green Bond Endorsed Project Catalogue (European Commission, 2021; People's Bank of China, 2021).

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The potential benefit of the above-outlined references is the reduction of asymmetric information and increase of transparency (Dorfleitner et al., 2021). The relevance of such additional information is supported by a CBI (2019) survey, which identified green credentials as one of the main drivers for green investment decisions.

The global market for green bonds has strongly increased over the past years. Based on figures published by the CBI (2021), their issuance volume already surpassed the former annual maximum of 2020 (i.e., 294.4 billion (bn.) US Dollar (USD)) in Q3 of 2021, with 354.2 bn. USD Year to Date (YTD). Further, they forecast the annual volume to exceed one trillion USD by 2023. To put these numbers into perspective, SIFMA (2021) reports for 2020 a global long-term bond issuance volume of 27.3 trillion (tn.) USD, indicating a green bond market share of about 1% that year.

While green bonds can help to finance sustainable investments, they may also incur additional expenses for their issuers compared to conventional bonds. These can be caused by internal costs due to screening, managing and reporting on their use of proceeds, as well as external costs for their certification and second-party opinion (Hachenberg & Schiereck, 2018). However, these additional costs might be compensated if investors require a lower return (i.e., yield to maturity) for holding the green assets. In terms of prices, a lower yield means that a green bond can be issued at a higher price in comparison to a conventional twin bond with the same nominal value, which reduces funding costs. In fact, evidence for lower yields of green bonds is found by the majority of empirical studies (MacAskill, Roca, Liu, Stewart, & Sahin, 2021). This implies that in spite of additional expenditures, issuers could even benefit from lower financing costs for their sustainable investments by issuing green bonds instead.

One characteristic that potentially influences the yield of green bonds is their degree of liquidity (e.g., Zerbib (2019)). This is because investors may require a higher return for holding an illiquid asset (Kempf & Uhrig-Homburg, 2000). The German Finance Agency (Finanzagentur) (FA), which is responsible for the issuance of German green sovereign bonds, argues that an excessive volume of green bonds may impede the liquidity of conventional bonds, while a deficient volume may impede their own liquidity (Finanzagentur GmbH, 2021a). In light of this trade-off, they pioneered a unique green bond concept to solve this issue in 2020, which in 2022 was adopted by Denmark as well (Dutch State Treasury Agency, 2019; Finanzagentur GmbH, 2020). In summary, this concept bases on the issuance of green bonds as twins to conventional bonds that coincide in almost all characteristics. This allows the introduction of a switch mechanism between both twins that has the function to secure a superior value of green bonds, which differ to conventional twins in its more restricted use of proceeds. Or in terms of yields, the yield of conventional bonds can serve as an upper limit for the yield of green bonds. For investors, this can imply additional certainty to sell a green bond for at least the price of its conventional counterpart. For issuers, this may imply more favourable financing conditions, as the green bonds can possess a lower yield.

The goal of this dissertation is the derivation of a theoretic model that is capable to explain the mechanisms of the German green bond concept. In detail, it aims to provide a decomposition of the yield differential between both twins into its individual components. Namely, a green premium, a liquidity premium and the added value of the switch mechanism. A successful disentanglement of the yield differential can provide issuers as well as investors with crucial information for evaluating this concept. From an issuers' perspective, this may answer the question if the framework is a viable approach to mitigate undesired illiquidity-induced costs and thus secure more favourable costs of capital. From an investors' perspective, this information can also be relevant to correctly account for their exposure to potential liquidity risks. The added value of this dissertation is therefore viewed as a theoretical contribution to improve the understanding of the implications of the German twin bond approach with a focus on its most defining feature, the switch mechanism between green and conventional twin bonds.

The remainder of this paper is structured as follows. First, an overview of relevant literature is provided. Then an evaluation of the German green bond concept and a comparison to their green sovereign peer bonds is conducted. In the following part, the theoretical model is derived and a calibration of the model parameters is performed. Finally, the model implications are evaluated, including a sensitivity analysis and a discussion of its limitations.

2. Literature Review

2.1. Green Premium

There exists an increasing body of literature with the goal to explain and quantify the potential yield premium for green bonds. Such a green premium can be defined as the difference in yield to a conventional bond, which shares otherwise the same characteristics (Zerbib, 2019). A negative premium implies that investors require a lower return when investing into a green asset.

Fama and French (2007) find that the taste for an asset, expressed by additional utility from holding it beyond its financial payoff, can help to explain its prices. In line with this result, Dorfleitner et al. (2021) argue in the context of green bonds that a lower yield for holding a green asset may be compensated by a non-financial utility component. This is also supported by findings from Riedl and Smeets (2017) who observe that social preferences and signalling outweighs financial motives for explaining socially responsible investment decisions based on a survey conducted in 2011 with Dutch investors. The impact of non-pecuniary factors is also supported by Hartzmark and Sussman (2019), who evaluated the effect of the first introduction of sustainability ratings by Morningstar for the U.S. mutual fund market in 2016, which supported the evaluation of a funds' sustainability. They found that fund flows for more sustainable funds were positively affected, while the flows to less sustainable funds decreased.

In light of this, there is a growing branch of studies evaluating the size of a potential green premium. Reviewing 15 publications that have been published in this area between 2007 and 2019, MacAskill et al. (2021) report a lack of consensus regarding the existence of such premium, which they attribute to different methodological approaches. Nevertheless, its presence is reported in the majority of the studies for both, the primary market (56%) and the secondary market (70%). For the latter market, the reviewed studies observe an average green premium of -1 to -9 basis points (bps). Further, MacAskill et al. (2021) highlight that the premium is generally more profound for green bonds that are "government issued, investment grade, and that follow defined green bond governance and reporting procedures". For the latter, they provide recognized green bond certification standards and third-party verification for the use of proceeds as main drivers. Hachenberg and Schiereck (2018) argue that such an enhanced reporting is necessary to mitigate information asymmetries between the issuers and investors. This aligns with the results of a survey conducted by the CBI (2019) with 48 of the largest Europe-based fixed income asset managers, which showed that they view green credentials, next to pricing, as one of the most important factors for their investment decisions. This is also consistent with Dorfleitner et al. (2021), whose findings support the positive effect of external validation on the green premium. Moreover, Immel, Hachenberg, Kiesel, and Schiereck (2021) and Hachenberg and Schiereck (2018) find the issuer's Environmental, Social and Governance (ESG) rating to influence the yield differential between green and conventional bonds. Finally, Kapraun, Latino, Scheins, and Schlag (2019) identify a bond's "Green credibility" as a main driver for the green premium. The German Green Federal Securities, which are the focus of this dissertation, seem to fulfil the above-mentioned driving factors. However, the evidence for the existence of a green premium for sovereign green bonds considering both, the primary market and the secondary market, is not conclusive. For example, while Doronzo, Siracusa, and Antonelli (2021) find no definite evidence for such a premium based on bond data from 14 countries that have been issued between end-2016 and 2020, Kapraun et al. (2019) find a significant green premium between 5 and 18.5bp for bonds that are issued by government entities.

A feasible methodological approach to evaluate the premium of green bonds is the comparison with a counterfactual brown (i.e., non-green) bond that otherwise exhibits the same characteristics (Bachelet, Becchetti, & Manfredonia, 2019). As such a security is in general not available, one viable alternative is to find a proxy based on a matching method. For example, Bachelet et al. (2019) identify brown nearest-neighbours that have the same currency, issuer, rating coupon type and a similar maturity date, coupon rate and amount issued. Doronzo et al. (2021) and Zerbib (2019) also use a direct matching approach by constructing a synthetic brown bond based on other bonds that have similar properties. Alternatively, two-step matching procedures such as propensity score matching (e.g., Gianfrate and Peri, 2019) or coarsened exact matching (see Löffler, Petreski, and Stephan, 2021) are applied to obtain estimates for the "untreated" brown bonds. However, in this study it is not necessary to rely on proxies for a counterfactual brown bond, as the German Green Federal securities are issued as twins to conventional bonds that share most of their characteristics.

2.2. Liquidity Premium

One property that differs is that German Green bonds have a lower issuance volume than their conventional counterparts (Finanzagentur GmbH, 2021b). The Finance Agency (2021a) argues that a sufficiently high amount outstanding is necessary to ensure that they can be traded in large quantities and at any time. This is because a low volume can imply a lower liquidity due to less owners and thus higher search costs (Helwege, Huang, & Wang, 2014). Therefore, investors may require a higher return to compensate for the additional risk of holding an illiquid asset (Kempf & Uhrig-Homburg, 2000). This understanding of liquidity is based on Fisher (1959), who views it as the ability to sell a bond quickly and without a discount on its value.

The impact of illiquidity on bond prices in general is widely researched (e.g., Chen, Lesmond, and Wei (2007); Dick-Nielsen, Feldhütter, and Lando (2012); Helwege et al. (2014); Kempf and Uhrig-Homburg (2000); Schestag, Schuster, and Uhrig-Homburg (2016)). A main advantage of understanding and measuring illiquidity costs is that it can help investors to improve the management of their exposure to risks. For example, investors who hold a bond until maturity (i.e., no need to sell it early) are not affected by liquidity disadvantages and thus may favour a premium for holding an illiquid asset (Wegener, Basse, Sibbertsen, & Nguyen, 2019). However, if the premium is attributed to other factors (e.g., credit risk) instead, this may not be the optimal investment alternative for such investors.

Nevertheless, it is not straightforward to derive a feasible proxy for the liquidity, which can be attributed to the lack of a universal definition (Díaz & Escribano, 2020). Díaz and Escribano (2020) provide an overview on the various dimensions of liquidity and the selection of proxies that measure its different characteristics. In the context of this dissertation, a proxy that indicates the size of illiquidity costs over time is required. One viable approach to estimate this liquidity premium is the comparison of yields of bonds that only differ in their degree of liquidity. While Schwarz (2019), Monfort and Renne (2014) and Schuster and Uhrig-Homburg (2012) compare the liquid German Federal Securities with less liquid bonds from the German state-owned investment and development bank, Kreditanstalt für Wiederaufbau (KfW), Kempf, Korn, and Uhrig-Homburg (2012) compare the German Federal Securities with less liquid Pfandbrief bonds (Covered Bonds) and Wegener et al. (2019) compare less liquid traditional Pfandbrief bonds with Jumbo Pfandbrief bonds that have a larger issuance volume. To relate the liquidity premium to different investment horizons, Kempf et al. (2012)

model the premium based on the Nelson and Siegel (1987) approach, while Koziol and Sauerbier (2007) use the Svensson (1994) method. Both parametric models provide the term structure of the current spot rate for zero coupon bonds.

The impact of differences in liquidity is also considered in the context of green bonds. For example, Zerbib (2019) remarks the explanatory power of a liquidity proxy for the yield differential between green bonds and counterfactual conventional bonds, when estimating the green premium. Further, Bachelet et al. (2019) find evidence for a higher liquidity of green bonds issued by public institutions in relation to their brown (i.e., conventional) counterparts. Finally, Wulandari, Schäfer, Stephan, and Sun (2018) find a negligible impact of liquidity risk on green bonds.

Finally, liquidity risks can affect the financing costs for issuers of bonds. The costs of capital are determined by the primary market yields issuers can secure at issuance. However, the return investors require from holding a bond may be affected by its expected performance on the secondary market. For example, Goldstein, Hotchkiss, and Pedersen (2019) find evidence based on corporate bonds that the expected aftermarket liquidity at issuance can have an economically large impact on the financing costs. A viable explanation for this finding is that the initial investors have a lower perceived risk in case they need to sell the asset before its maturity, and are thus willing to pay a premium. From an issuer's perspective, it can therefore be advantageous to ensure a liquid secondary market for its bonds.

2.3. Term Structure Models

To derive a structural model for the effect of illiquidity on the German green bonds, we assume a stochastic model that can reflect the development of the liquidity premium until maturity. For this purpose, we apply a term structure model of the short rate, which provides their development over time. In other words, we use a stochastic process to model a sequence of interest rates (i.e., a liquidity premium) each for an infinitely small period of time. This type of models are widely applied to value interest rate derivatives, such as European bond options (Hull, 2018). Further, they have also been used in studies that model bond illiquidity (e.g., Kempf and Uhrig-Homburg (2000); Koziol and Sauerbier (2007)).

In general, the various approaches can be divided into equilibrium models and no-arbitrage models (Hull, 2018). The drift of the short rate in equilibrium models is no function of the time, whereas the drift in no-arbitrage models is time-dependent. While the latter approach allows an exact fit to the current term structure, this is not required for the present application (Hull, 2018). This is because the used liquidity proxy is not calibrated to the actual (il-)liquidity of German green bonds, which only allows a relative evaluation of the effects. Hull (2018) presents the Rendleman and Bartter (1980) model, the Vasicek (1977) model and the Cox, Ingersoll, and Ross (1985) model as possible equilibrium models. The Rendleman and Bartter model differs in a way from the other models that it does not assume a mean reversion for the short rate, while the Cox, Ingersoll, and Ross model excludes negative interest rates by construction. The Vasicek model assumes a mean-reverting process for the short rate and allows for negative rates.

3. Green Sovereign Bonds

The CBI reports that 22 national governments have issued sovereign Green, Social, and Sustainability (GSS) bonds until November 2020 with a total amount of 96 bn. USD (Harrison & Muething, 2021). In the same study that was published in January 2021, Harrison and Muething (2021) report for the majority of GSS bonds a relatively higher imbalance between their supply and demand compared to their conventional counterparts, which was suggested by a mostly higher book cover (i.e., oversubscription). This indicates market growth potential for the green sovereign bonds. Moreover, they report that based on 23 issuances between 2017 and November 2020, ten bonds priced on the yield curve of conventional peers, nine priced below and four above. As the green bond issuance at a yield below the yield curve of conventional (i.e., vanilla) bonds implies more favourable financing costs, the observed sovereign bonds provide no clear evidence for such a potential yield advantage. In the following, we first provide an insight into the German green bond framework. Then we compare it to a small peer group of sovereign green bonds with a focus on how potential liquidity issues are addressed.

3.1. German Green Federal Securities

Since September 2020, the German federal government issued Green federal securities with a total volume of 24 bn. EUR (see Table 1). In 2021, it issued 12.5 bn. EUR of Green bonds, which accounted for 2.6% of the total issuance volume (482.7 bn. EUR) of tradable government debt that year (Finanzagentur GmbH, 2021d). For 2022, it anticipates a similar annual volume (Finanzagentur GmbH, 2021a).

The German Finance Agency ("Finanzagentur"), which administers the issuance of green bonds in the primary market, acknowledges the need to account for sustainability in financial decisions in light of economic risks as well as investment opportunities that result from climate change and transition towards a "more sustainable global ecosystem" (Finanzagentur GmbH, 2020). It concludes that this requires an enhanced transparency and development of the market for green and sustainable investments, to which the Green Federal securities are a key driver.

On the one hand, the enhanced transparency can be attributed to the chosen evaluation, selection, and reporting process. The criteria to identify eligible budget items align with the Green Bond Principles by the ICMA, the EU Charter of Fundamental Rights and consider elements of the draft EU Green Bond Standard (European Commission, 2012; Finanzagentur GmbH, 2020; International Capital Market Association, 2021).

In its Green Bond Investor Presentation (2021b), the Finance Agency provides an overview of the use of proceeds of the German Green Bonds. For example, it attributes the eligible expenditures in 2019 to the following sectors: Transport

Table 1: Overview of German Twin Federal Securities

Name	Issuance	Maturity Date	Outstanding	Type	ID
2021 (2050) Bund/g	18.5.2021	15.8.2050	6.0 bn. EUR	Green	G2050
2019 (2050) Bund	23.8.2019	15.8.2050	29.0 bn. EUR	Conventional	C2050
2021 (2031) Bund/g	10.9.2021	15.8.2031	6.5 bn. EUR	Green	G2031
2021 (2031) II Bund	18.6.2021	15.8.2031	26.5 bn. EUR	Conventional	C2031
2020 (2030) Bund/g	9.9.2020	15.8.2030	6.5 bn. EUR	Green	G2030
2020 (2030) II Bund	19.6.2020	15.8.2030	30.5 bn. EUR	Conventional	C2030
Bobl/g	6.11.2020	10.10.2025	5.0 bn. EUR	Green	G2025
Bobl	10.7.2020	10.10.2025	25.0 bn. EUR	Conventional	C2025

The data in this table is based on Finanzagentur GmbH (2021a) and Refinitiv Eikon (Accessed: 21.11.2021). Table 16 in the Appendix shows an extended version of this table.

(57.9%), International cooperation (24.2%), Energy (9.7%), Research (5.1%) and Agriculture (3.1%). Moreover, it highlights amongst other eligible expenditures the upgrade and electrification of the railway between Ulm and Lindau with total costs of approx. 225 million (mn.) Euro (EUR) and the development loan for a renewable power plant (i.e., solar PV) in India amounting to 89.3 mn. EUR as examples from the infrastructure and international cooperation sector, respectively.

The evaluation criteria are only applied to expenditures that are already accrued (Finanzagentur GmbH, 2020). This process enhances the transparency of the use of proceeds, as the projects precede the issuance of the securities. However, it also restricts the issuance amount of green bonds as, for example pending expenditures are not eligible. In addition to the selection criteria, the agency provides a Second Party Opinion on the Green Bond Framework (see ISS ESG, 2020), an external Third Party Verification of the Allocation Report by the auditing firm Deloitte, and impact reporting (Finanzagentur GmbH, 2021c).

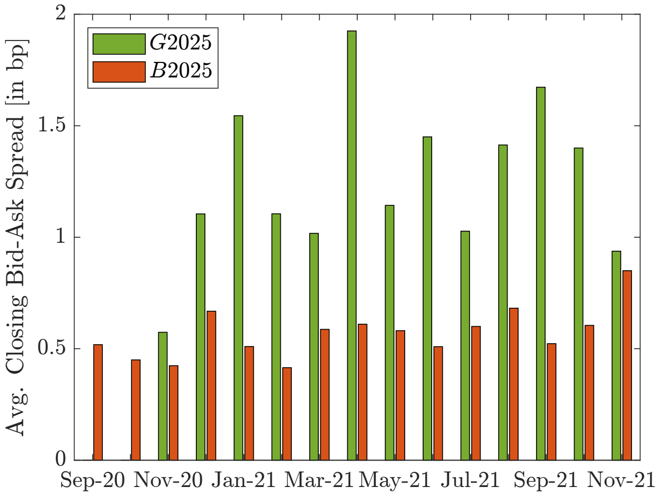
On the other hand, the German Green Bonds aim to support the development of the European green fixed income market by establishing a new interest rate benchmark for such assets, a green yield curve (Finanzagentur GmbH, 2020). While conventional government bonds with a high credit rating can be used to serve as a benchmark return for risk-free investments only, the green yield curve can provide the term structure of interest rates for riskless and green assets as they are ranked pari-passu (i.e., equally) to the conventional bonds from the same issuer. This means that they could provide a reference for the required future payoff of a risk-free green investment with a specific time horizon.

To provide this information to potential investors and quantify their preference for green investments, the German Green Bonds are issued based on a unique twin bond concept (Finanzagentur GmbH, 2020). As summarized in Table 16 in the Appendix, the green bonds and their conventional twins share the same coupon rate and time to maturity, but differ in their issuance volume and are traded separately (i.e., they have different ISIN codes).

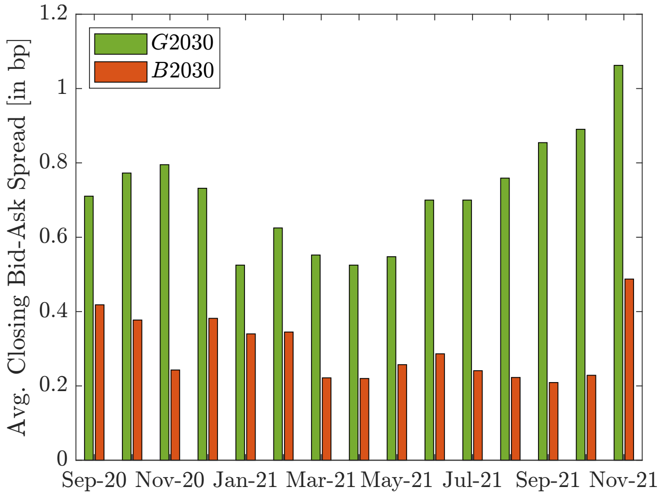
In addition to the high credit quality, another requirement of German Federal securities to serve as benchmark is sufficient liquidity (Finanzagentur GmbH, 2021a). This is to mitigate risks for bondholders that can be induced by illiquidity, for example, the inability to sell the bond rapidly or only for a lower transaction price (e.g., see Kempf and Uhrig-Homburg, 2000). In the context of green bonds, their issuance can entail liquidity risks for both, themselves and conventional twins. This is because a sufficiently high amount outstanding of each type of bond is necessary to ensure that they can be traded in large quantities and at any time (Finanzagentur GmbH, 2021a). While a low issuance volume of green bonds may impede their own liquidity, a high volume can have detrimental effects on the liquidity of conventional bonds, if the total outstanding volume of Federal securities is maintained (Finanzagentur GmbH, 2021a). As a consequence, this potential trade-off has to be solved in order to provide an interest rate benchmark for both, the green as well as the conventional European fixed income market.

The figures in Table 1 indicate that the volume of each green bond is significantly smaller than its conventional counterpart. In fact, the average amount outstanding of a German green bond is approximately one fifth (22%) of the amount of the average conventional twin bond. This suggests that the green bonds may be less liquid than their conventional twins. To test this hypothesis, we evaluate the bid-ask spread (BAS) of the daily closing bond yields, as this measure is frequently used to derive a proxy for the liquidity of bonds (e.g., Dick-Nielsen et al. (2012); Kapraun et al. (2019); Zerbib (2019)). A higher BAS represents higher transaction costs, which can indicate a lower liquidity. Figure 1 shows that since the issuance of the first German green bond, the average monthly BAS of the green bonds is almost consistently larger than the spread of the conventional counterparts.

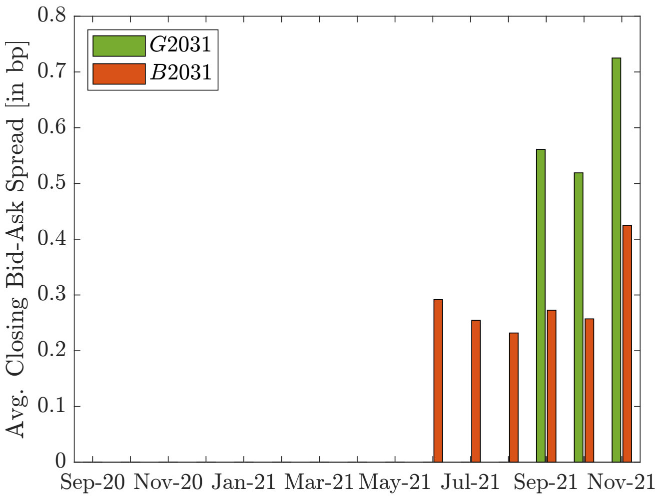
To verify this visual impression, a paired t-test is performed whether the average bid-ask spread ($\overline{BAS}$) since issuance of each green bond $\overline{BAS}_G$ coincides with the same measure for the respective conventional twin $\overline{BAS}_C$. The test results in Table 2 show that we can reject the null hypothesis



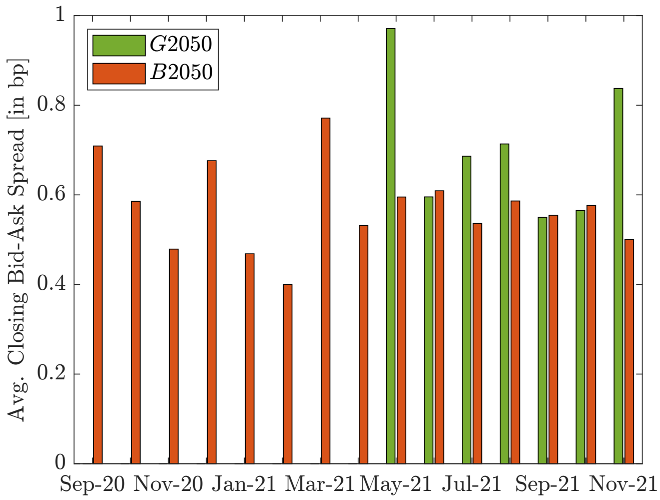
(a) Maturity in 2025



(b) Maturity in 2030



(c) Maturity in 2031



(d) Maturity in 2050

Figure 1: Average monthly closing bid-ask spreads for German twin bonds

The figure is based on data from Refinitiv Eikon (Accessed: 10.11.2021) and shows the average monthly closing bid-ask spread in basis points (bp) for German green bonds and their conventional twins displayed in Table 1 for the period from 09.09.2020 to 10.11.2021. Summary statistics are displayed in Table 11 in the Appendix.

H_0: $\overline{BAS}_G = \overline{BAS}_C$ on a significance level $\alpha = 0.05$ for all twins, in favour of the alternative hypothesis H_A: $\overline{BAS}_G \neq \overline{BAS}_C$. Therefore, we can conclude that the green bonds are traded during the observed period from 09.09.2020 to 10.11.2021 on average at a statistically significant wider spread than the conventional twin bonds.

The absolute difference between the average spreads Δ ranges from 0.1 to 0.7 basis points. To evaluate the economic significance of this difference, we assume values that align with the twin bonds that mature in the year 2031. Assuming a bid yield $y_{bid} = -40$bp for both twins, a narrower spread for the conventional bond with $y^C_{ask} = -39.7$bp and a wider spread for the green bond $y^G_{ask} = -39.3$bp, we have $\Delta = 0.4$bp representing the additional transaction costs TC. Further, we assume a time to maturity of $T = 10$ years. To compute the present value P of a zero-coupon bond, we assume continuous compounding to discount its nominal value N and thus use $P = N \cdot e^{-yT}$. We obtain the trading costs as $TC = N \cdot (P_{ask} - P_{bid})$. Based on this specification, an investor, which acts as a price taker and executes a round trip trade with both, a German green bond and its conventional twin, by buying at the ask price P_{ask} and selling at the bid price P_{bid} for an investment of $N = 1$ mn. EUR and $T = 10$ years, would incur additional trading costs for the green bond amounting to $\Delta TC = TC_G - TC_C = N \cdot (P^G_{ask} - P^C_{ask}) = 385$ EUR or about 4bp that are caused by its wider spread. From an economical perspective, this amount is relatively small, which aligns with the objective of the German approach to address liquidity risks. However, in relative terms, $\overline{BAS}_G$ is on average almost twice the size (+94%) of $\overline{BAS}_C$ for the observed data. It should be noted that these transaction costs are different to the liquidity premium in the model that de-

Table 2: Closing bid-ask spreads for German twin bonds

	BAS_{2050}	BAS_{2031}	BAS_{2030}	BAS_{2025}
$\overline{BAS}_C$ [in bp]	0.573	0.294	0.290	0.572
$\overline{BAS}_G$ [in bp]	0.669	0.570	0.703	1.273
$\Delta = \overline{BAS}_G - \overline{BAS}_C$	0.096	0.276	0.413	0.701
t-statistic	2.9223	5.5221	17.8084	12.4163
p-value	0.0041	0.0000	0.000	0.0000
N	131	47	305	262

The table shows the results of a paired t-test to determine whether the mean bid-ask spread for the closing yields of German green bonds $\overline{BAS}_G$ coincides with the same measure for the conventional twins $\overline{BAS}_C$. The null hypothesis H_0: $\overline{BAS}_G = \overline{BAS}_C$ is tested against the H_A: $\overline{BAS}_G \neq \overline{BAS}_C$. The data is retrieved from Refinitiv Eikon (Accessed: 10.11.2021) and covers the period from 09.09.2020 to 10.11.2021. Summary statistics are displayed in Table 11 in the Appendix.

scribes a premium to the yield (i.e., a higher yield) for illiquid assets instead.

The German green bond concept is designed to mitigate possible liquidity disadvantages for both twins. To ensure the liquidity of the conventional bonds, the Finance Agency issues the same amount as the green counterparts in its own stock, which can be used on the secondary market for repurchase agreements (i.e., repo transactions) and lending activities (Finanzagentur GmbH, 2021a). Therefore, the total amount of conventional securities and thus their liquidity remains unchanged. To ensure the liquidity of the green bonds, the German Finance Agency declares to engage in secondary market activities (Finanzagentur GmbH, 2020). In their Investor Presentation from September 2021, the Finance Agency categorises them as (1) Outright ("one-way") sales and purchases, (2) Repurchase agreements and securities lending, using the Federal Government's own stock of green bonds and (3) Combined and debt-neutral sale-and-purchase (switch) transactions conducted with banks that are members of the Bund Issues Auction Group (Finanzagentur GmbH, 2020, 2021b). This means that it can influence the supply and thus the price of the green bonds on the secondary market. From the issuer's perspective, green bonds are more valuable than the conventional twins. Although both zero-coupon bonds have the same face value and thus the same cash flows, this is because the green bonds provide an additional documentation for the usage of their proceeds. Even in situations, where investors would not attribute a higher value to the green bonds, this would still hold for the issuer, who sustains the associated added costs and more limited use of proceeds. Therefore, the switch, which is the simultaneous and debt-neutral sale of a conventional bond and purchase of a green bond, would be economically viable for the issuer at a yield spread of zero. Further, it can execute such transactions until the green securities are completely in their own holdings. In this case, the respective amount of conventional twins that was initially held back by the Finance Agency is then traded in the secondary market.

Figure 2 shows all available closing ask yields until 04.11.2021 of the German twin bonds displayed in Table 1 retrieved via Refinitiv Eikon. It shows that the yields of the respective twins are closely related for the observed period of time. Further, the data supports an upward sloping yield curve for both bond types. This means that investors with a longer time to maturity require a higher rate of return, ceteris paribus. Figure 3 shows the yield differential between German green bonds y_G and their respective conventional twin y_C (i.e., $\Delta y = y_G - y_C$). For most of the observed period, the data shows a negative trending spread with an average of around -5bp. This implies that investors are increasingly willing to sacrifice return in favour of investing into the German Green Federal securities. However, it should be noted that the historical data covering a period of one year is relatively scarce and the future size of the spread may change.

3.2. Addressing Liquidity Risks

This section aims to provide a brief insight into how selected sovereign issuers different to Germany address the possible risk of illiquidity. These issuers are France, the Netherlands, and Belgium. France issued its first green sovereign bond in 2017 for 7 bn. EUR, which was since then increased to a total amount outstanding of 28.9 bn. EUR République Française (2021). In the French framework for green Obligations assimilables du Trésor (OAT) (2017), their liquidity is emphasized on its first page. Also, the respective investor presentation covers the liquidity as one of six main topics (République Française, 2021). This underlines the relevance of liquidity concerns. In the same document, they show that the average monthly bid-ask spread is consistently lower for their green bond (RIC: FR0013234333=) than a conventional bond (RIC: FR0013515806=) which matures one year later in 2040. Further, both bonds show a similar ownership structure with a share of long-term investors of 37% and 38%, respectively. They also highlight that its amount outstanding is with 31 bn. EUR similar to neighbouring (in terms of time to maturity) conventional bonds and argue that this supports its liquidity (République Française,

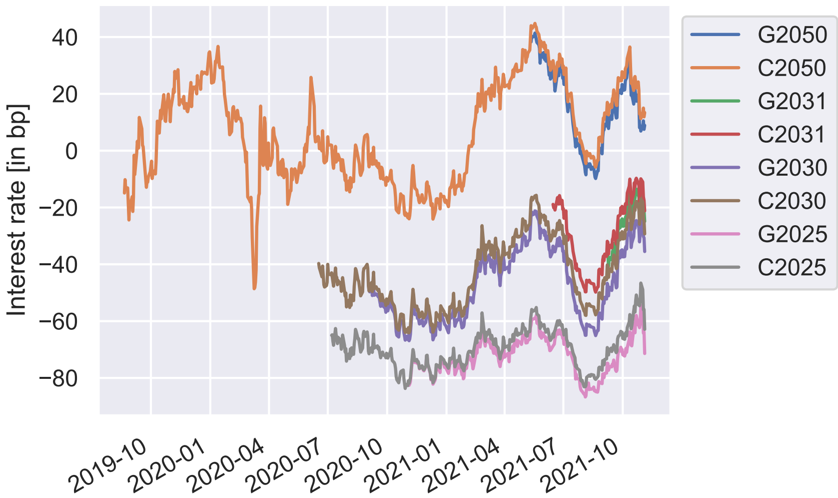


Figure 2: Yield of German government bonds and green twins

The figure is based on data from Refinitiv Eikon (Accessed: 04.11.2021) and shows the closing ask yield in basis points for the German Federal securities displayed in Table 1. The summary statistics are displayed in Table 12 in the Appendix.

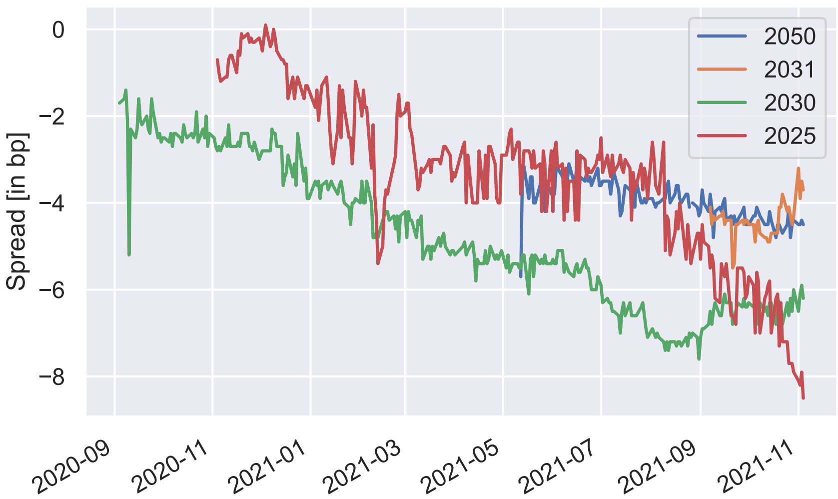


Figure 3: Spread between German government bonds and green twins

The figure is based on data from Refinitiv Eikon (Accessed: 04.11.2021) and shows the yield differential of German green bonds y_G and their respective conventional twin y_C (i.e., $y_G - y_C$) in basis points. The summary statistics are displayed in Table 13 in the Appendix.

2017, 2021). The Dutch State Treasury Agency (2019) justifies the liquidity of the Dutch sovereign green bonds with a minimum issuance volume of 10 bn. EUR within several years, a quotation obligation for Primary Dealers to ensure the availability of tradable prices and a Repo facility available to Primary Dealers that serves as a lender of last resort. Belgium reports that its green bonds have no liquidity disadvantages and a similar issuance volume as conventional government bonds (The Kingdom of Belgium, 2018).

So far, there exists only one other country that decided to adopt the German approach that was introduced in 2020. The national bank of Denmark 2022 reported following the

German twin bond concept with the first Danish green government bond that was issued on January, 19th 2022 as a 10-year zero coupon bond. Further, they announce that a switch of the green bond to its corresponding more liquid conventional twin bond will be possible for investors "at any time" to support its liquidity. Before opting for the twin bond concept, the Danish Debt Management Office also considered issuing green certificates in addition to the conventional bonds instead (Bongaerts & Schoenmaker, 2020). Bongaerts and Schoenmaker (2020) recommend such green certificates as a viable approach to meet the demand for environmentally-friendly debt, while avoiding potential drawbacks of green government bonds. Namely, impeding the liquidity of both, green and conventional bonds, making the price of green certificates more suitable to adequately reflect environmental fundamentals.

The above examples suggest that possible liquidity concerns are a relevant factor, which is generally addressed by issuers of green bonds. The issuing institutions of French, Dutch and Belgian sovereign green bonds all emphasize a sufficiently high amount outstanding as one mitigating measure of liquidity disadvantages. However, a large amount of green bonds can potentially have adverse effects on the liquidity of conventional bonds and might thus not be desired (e.g., see Finanzagentur GmbH, 2021a). An evaluation of the potential post-issuance liquidity effects of green bonds on conventional bonds from those countries is not pursued in this dissertation due to its limited scope. In the case of Germany, the largest currently traded German green bond, with an issuance size of 6.5 bn. EUR, is relatively small compared to the Dutch benchmark of 10 bn. EUR. Therefore, it is reasonable that the German twin bond approach aims to offer an alternative approach to address such risks.

4. Methodology and Data

In this section, we derive a non-closed form solution for the yield differential (i.e., spread) between German green bonds y_G and its conventional twin bonds y_C. To achieve this, we decompose the yield into three effects of the green bond relative to the conventional bond: A liquidity premium LP, a green premium GP, and the effect of the secondary market interventions (i.e., switch transactions) by the German Finance Agency, in the following denoted as ST. Therefore, we write the decomposition of the yield differential Δy as

$$\begin{aligned}\Delta y &= y_G - y_C \\ &= LP - GP - ST.\end{aligned} \tag{1}$$

For any additional degree of illiquidity of the green bond, investors require a higher return (i.e., a higher LP), which increases the spread. Further, investors may accept a lower yield for investing into a “green” asset (i.e., a higher GP), which reduces the spread. Finally, the market interventions (i.e., switch transactions ST) by the Finance Agency increase the liquidity of the green bonds and thus have a negative effect on the spread as well. It should be noted that a negative green premium (e.g., as in Zerbib (2019)) is in Equation 1 defined as a positive value for GP and thus subtracted. In the same fashion as Kempf and Uhrig-Homburg (2000), this model assumes perfect and arbitrage-free markets except for illiquidity costs. However, the bonds are traded in discrete time and only the liquidity premium is subject to change, which is modelled as a stochastic short rate. The green premium and the interest rate of the conventional bond are assumed to be constant.

4.1. Trinomial Tree Model

We consider a stochastic liquidity premium LP_t that follows an Ornstein-Uhlenbeck process, as suggested by Vasicek (1977). Based on this approach, the increment dLP_t is defined as

$$dLP_t = a(b - LP_t)dt + \sigma dz, \tag{2}$$

where a, b are non-negative constants and denote the mean reversion rate and the long term level reversion level, respectively. σ denotes the local volatility and dz follows a standard Wiener process with $dz = \varepsilon \cdot \sqrt{dt}$ and $\varepsilon \sim \mathcal{N}(0,1)$. From a theoretical perspective, it is plausible to assume a mean-reverting process for the liquidity premium, as a lower level of liquidity leads to a higher premium, which may attract new investors. This increase in demand for the bond can positively affect its traded volume on the secondary market and thus increase its liquidity. A geometric Brownian motion would not coincide with this theoretic argumentation, as the liquidity premium could increase (or decrease) indefinitely. Nevertheless, a Dickey-Fuller test is performed during the calibration of the model to confirm if the discrete data for the selected liquidity proxy supports a random walk or not.

In the next step, we derive a discrete trinomial tree representation of the stochastic process $\tilde{LP}$. This non-closed form solution is required to incorporate the effects of ST into the model for the green bond yields. We can re-write the liquidity premium as $\tilde{LP} = b + \tilde{s}$, where b denotes the long term mean as in Equation 2 and $\tilde{s}$ denotes the stochastic part of the premium. Using this, we can rewrite Equation 2 as

$$\begin{aligned}dLP_t &= a\left(b - (b + s_t)\right)dt + \sigma dz \\ &= -as_t dt + \sigma dz,\end{aligned} \tag{3}$$

with $\mathbb{E}[dLP_t] = -as_t dt$ and $Var[dLP_t] = \sigma^2 dt$. We use this result to derive the trinomial tree representation, where the change in $\tilde{s}$ for each time step is indicated by Figure 4.

The spacing between the nodes in the time-dimension i (e.g., $s_{j,i}$ and $s_{j,i+1}$) is denoted as $\Delta t = \frac{T}{N}$, for a tree with an investment period of T years and N equidistant discrete time steps. The spacing between the state-dimension j with $s_{j+1,i} - s_{j,i} = s_{j,i} - s_{j-1,i}$ is denoted as Δs. This means that node (j,i) describes the possible states that the liquidity premium can assume in time $t = i\Delta t$ with $LP_{j,t} = b + s_t = b + j\Delta s$. The probabilities for the up-state, mid-state and down-state in the next period are denoted as p_u , p_m & p_d, respectively.

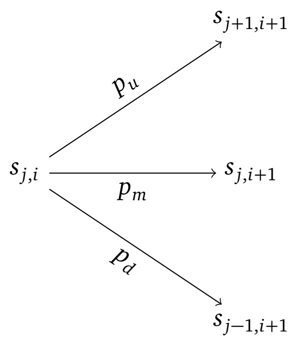


Figure 4: Trinomial tree of stochastic part of liquidity premium $\tilde{s}$

Following Hull (2018), we set the spacing Δs to

$$\Delta s = \sigma \sqrt{3\Delta t}, \tag{4}$$

which was found to minimize the model error. Further, we restrict the branching structure to ensure positive probabilities in the tree (Hull, 2018). For this, we set the limits j_{max} and $j_{min} = -j_{max}$, where the branching changes from the form displayed in Figure 4 to the respective structure displayed in Figure 5. In the same fashion as Hull (2018), we set j_{max} as the smallest larger integer than $\frac{0.184}{a\Delta t}$. While these limits restrict the maximum and minimum size of the mean-reverting liquidity premium, they do not impede the fitting of the discrete trinomial tree model to the continuous-time Vasicek process from Equation 2. The calibrated tree is still able to match the first two moments of the observed process.

We derive the discrete solution for the liquidity premium by setting three restrictions on the three time-independent, but state-dependent tree probabilities p_u, p_m and p_d (Hull, 2018). In detail, for each time-step, we match the first two moments of ds_t, using the expected change $\mathbb{E}[dLP_t] = -as_t dt$ and the variance $Var[dLP_t] = \sigma^2 dt$. Further, we require the probabilities to add to one.

For the default branching method (i.e., $j_{min} < j < j_{max}$) the condition for the expected change notates as,

$$\begin{aligned} p_u \cdot \Delta s + p_m \cdot 0 + p_d \cdot (-\Delta s) &= \mathbb{E}[dLP_t] \\ &= -a \cdot j \cdot \Delta s \cdot \Delta t \end{aligned} \tag{5}$$

For the condition for the variance we use $Var[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2$ and thus obtain

$$\begin{aligned} \mathbb{E}[dLP_t^2] &= Var[dLP_t] + \mathbb{E}[dLP_t]^2 \\ p_u \cdot \Delta s^2 + p_m \cdot 0^2 + p_d \cdot \Delta s^2 &= \sigma^2 \cdot \Delta t + a^2 \cdot j^2 \cdot \Delta s^2 \cdot \Delta t^2 \end{aligned} \tag{6}$$

The final condition is for all branching structures the same and denotes as

$$p_u + p_m + p_d = 1. \tag{7}$$

The expressions for the probabilities for each branching structure are derived in Appendix **??** and coincide with the solution provided by Hull (2018).

4.2. Extension to Twin Bond Approach

In Section 4.1, we derived a discrete trinomial tree model that provides the respective probability weights for the change of the stochastic liquidity premium at each time step. In the following, we use this result to derive a solution for the initial bond price at time $t = 0$. For this, we assume no default risk. This implies that the price of each bond at time $t = T$ is set equal to its nominal value $P_T = 1$. To obtain the fair value of the bond in the periods before, we need to discount the expected bond price with the correct discount rate.

For example, for a liquid and non-green zero coupon bond C with a constant interest rate r we obtain its value at time t as,

$$P_t^C = \frac{P_{t+1}^C}{e^{r\Delta t}}. \tag{8}$$

This implies a present value at time $t = 0$ of $P_0^C = P_T^C \cdot e^{-rT} = e^{-rT}$, where $T = N\Delta t$.

This expression for the bond price changes considering the stochastic liquidity premium in addition to the interest rate r. In this case, the trinomial tree allows the derivation of state-dependent results based on the respective value of the premium. As before, we assume that the value of the bond must equal its nominal value at maturity in all states j of the liquidity premium. Therefore, the expected bond price at time $t = T$ is set equal to

$$\mathbb{E}[P_T | j] = P_T = 1. \tag{9}$$

To obtain the fair bond price at time t, we first need to compute its expected bond price at time $t + 1$ and then discount it with the correct interest rate. Following this approach, we can recursively obtain all bond prices until time $t = 0$. To compute the expected bond price we use the derived tree probabilities of the liquidity premium. Therefore, we can write the expected bond price at time $t = i\Delta t$ and state j, with $j_{min} < j < j_{max}$, in the trinomial tree as,

$$\mathbb{E}[P_{t+1} | j] = p_{u,j} \cdot P_{j+1,t+1} + p_{m,j} \cdot P_{j,t+1} + p_{d,j} \cdot P_{j-1,t+1}. \tag{10}$$

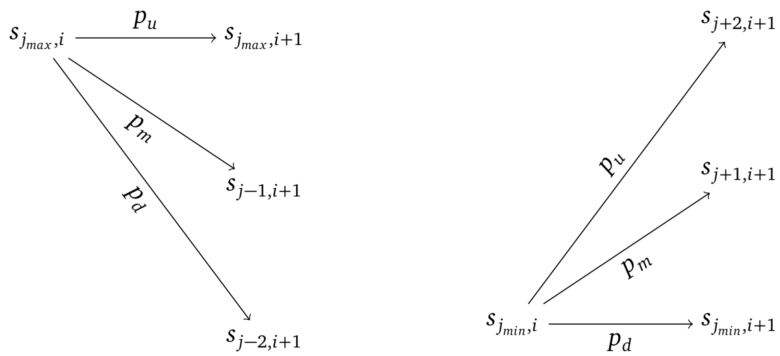


Figure 5: Restriction of trinomial tree branching structure

The figure is based on Hull (2018) and shows the upper and lower limits of the branching structer for $j = j_{max}$ (left) and $j = j_{min}$ (right), respectively.

For $j = j_{max}$ this equation changes to

$$\mathbb{E}[P_{t+1}|j_{max}] = p_{u,j_{max}} \cdot P_{j_{max},t+1} + p_{m,j_{max}} \cdot P_{j_{max}-1,t+1} + p_{d,j_{max}} \cdot P_{j_{max}-2,t+1} \tag{11}$$

and for $j = j_{min}$ we use

$$\mathbb{E}[P_{t+1}|j_{min}] = p_{u,j_{min}} \cdot P_{j_{min}+2,t+1} + p_{m,j_{min}} \cdot P_{j_{min}+1,t+1} + p_{d,j_{min}} \cdot P_{j_{min},t+1}. \tag{12}$$

For a counterfactual bond that only differs from the bond C in its liquidity, we need to consider in addition to r the respective liquidity spread $LP_{j,t}$. We obtain

$$P^I_{j,t} = \frac{\mathbb{E}[P^I_{t+1}|j]}{e^{(r+LP_{j,t})\Delta t}} \tag{13}$$

and use this expression to obtain the bond prices for all states and time steps in the tree via Backward-Induction. This is possible because we have the final value of the bond $P^I_T = 1$. By applying this procedure, we obtain one single value for the bond at time $t = 0$.

Based on this result, we can easily modify the expression in Equation 13 to additionally account for a constant green premium. This additional assumption implies for the model that the expected value that investors attribute to investing into a green asset does not change over time. We obtain

$$P^{IG}_{j,t} = \frac{\mathbb{E}[P^{IG}_{t+1}|j]}{e^{(r+LP_{j,t}-GP)\Delta t}}. \tag{14}$$

The negative sign implies that given a non-negative green premium GP, investors accept a lower yield to maturity. Discounting with a smaller value yields a higher price for the green and illiquid bond, which is therefore inversely related to the interest rate.

However, the expression for P^{IG} does not coincide with the theoretical value of the German green bond P^G. This is because it neglects the impact of the secondary market interventions by the FA. Section 3.1 outlines why it is not only possible but also rational for the FA to perform secondary market transaction, when the yield spread (i.e., $\Delta y = y_G - y_C$) between the twins assumes a non-negative value. In short, from the point of view of the FA, a green twin is due to the additional documentation for its use of proceeds always more valuable than the corresponding conventional twin. Therefore, if the yield of a green bond y_G notates above the yield of the conventional twin y_c, it is economically viable for them to execute combined and debt-neutral sale-and-purchase transactions. Those have a positive impact on the price of the green bond by reducing its supply, and thus negatively affect its yield and yield spread. We therefore assume that the price of a green bond cannot notate for a prolonged period of time below the price of its conventional twin. We can account for this additional characteristic by adding another condition to the model. Namely, we can restrict the prices of the German green bond to always assume values equal or higher than the corresponding conventional twin P^B_t. For the price at time t and in state j, this denotes as

$$P^G_{j,t} = \max\left[P^G_{j,t}, P^C_t\right]. \tag{15}$$

Further, this additional feature of the German green bonds can be interpreted as a call option on the illiquidity of the bond, assuming a constant green premium. When the liquidity premium becomes large, ceteris paribus, the switch transactions prevent P^G to fall below P^C. In this case, the value of the switch transactions need to compensate the lower price that would be implied by an illiquid and green bond alone. Therefore, above a certain threshold, increasing illiquidity leads to a higher value of the switch transactions. If the liquidity premium is sufficiently smaller than the green premium, intervention by the Finance Agency is unlikely to be required, and its value is equal to zero. Based on this comparison, we use in the following the terms “switch option” and “switch transactions” interchangeably to refer to the same mechanism of the German green bonds.

Building on the above results, we can use the model to derive the initial bond prices for a conventional bond, a counterfactual bond with a liquidity premium, a counterfactual with a liquidity premium and a green premium and for the German green bond by accounting for the switch option. In general, we can define the initial by the model implied bond price as $P_{j=0,t=0} = P_0$ with

$$P_0 = P_N \cdot e^{-y_0 T}, \tag{16}$$

which we can reformulate to

$$\begin{aligned} y_0 &= \frac{\ln\left(\frac{P_N}{P_0}\right)}{T} \\ &= \frac{\ln\left(\frac{1}{P_0}\right)}{T}. \end{aligned} \tag{17}$$

Further, we can derive of the implied size for the liquidity premium LP, the green premium GP and the value of the switch transactions ST. We obtain the liquidity premium by subtracting the model yield of the conventional bond from the yield of the illiquid bond, which denotes as $LP = y_0^I - y_0^C$. Further, we obtain the value of the green premium by subtracting y_0^I from the yield of the illiquid and green bond, denoted as $GP = y_0^{IG} - y_0^I$. Finally, we can compute the value of the switch transactions by subtracting y_0^{IG} from the yield of the German green bond, denoted as $ST = y_0^G - y_0^{IG}$. As r is constant and has the same value for all bond types, it does not affect the implied values in the decomposition.

4.3. Model Calibration

In the following section, we calibrate the model parameters. For this, we need to estimate the parameters of the Vasicek model for the stochastic liquidity premium $\tilde{LP}$ and find a viable value for the green premium GP. The objective of this work is to evaluate the impact of secondary market interventions on the price formation of the green bond. For this purpose, it is not required to use exact estimates for the liquidity and green premium, but to focus on the size difference between both effects. This is sufficient because there exist infinite many combinations that yield the same result for the spread Δy. This can be shown by adding a constant m to the liquidity premium LP, and to the green premium GP to Equation 1, which cancel each other out.

To obtain an estimate for the development of the liquidity premium over time, we compare the yield differential of German Bundesanleihen (i.e., conventional bonds) and German Pfandbriefe (i.e., covered bonds). This is possible because they exhibit the same characteristics, but only differ in terms of liquidity. To adjust the results to the time horizons of the respective green bonds, we follow the approach suggested by Svensson (1994) to obtain estimates for the daily spot rate for an investment over T years, $y_{t,T}$. The required (daily) parameters are estimated and published by the Deutsche Bundesbank (2021) for both, conventional bonds and covered bonds. Following this method, the yield to maturity y_T at time t can be estimated by

$$\begin{aligned} y_T = \beta_0 + \beta_1\left(\frac{1-e^{-\frac{T}{\tau_1}}}{\frac{1}{\tau_1}}\right) + \beta_2\left(\frac{1-e^{-\frac{T}{\tau_1}}}{\frac{1}{\tau_1}} - e^{-\frac{T}{\tau_1}}\right) \\ + \beta_3\left(\frac{1-e^{-\frac{T}{\tau_2}}}{\frac{1}{\tau_2}} - e^{-\frac{T}{\tau_2}}\right), \end{aligned} \tag{18}$$

where β_0, β_1, β_2, β_3, τ_1 and τ_2 denote the daily estimated and published parameters by the Deutsche Bundesbank. Based on Equation 18, we obtain the daily estimates for the liquidity premium LP_t as,

$$LP_t = y_{t,T}^{Covered} - y_{t,T}^{Conventional}, \tag{19}$$

where $y_{t,T}^{Covered}$ denotes the estimated spot rate based on the daily parameters for the German Pfandbriefe, while $y_{t,T}^{Conventional}$ denotes the same measure for the German Bundesanleihen.

The Svensson method provides a daily measure for the historical development of the yield differential between German Pfandbriefe and Bundesanleihen for time to maturity T and serves as a proxy for the liquidity premium in the model. This assumption implies that the process for the long-rate coincides with the short-rate process of the liquidity premium that is modelled in Equation 2. This causes an estimation error, as the instantaneous liquidity premium can differentiate from the premium of longer maturities. For example, Kempf and Uhrig-Homburg (2000) found a higher liquidity premium in the longer-maturity segment. Further, this might also affect the mean-reversion and volatility characteristics of the assumed stochastic process. In terms of the model, a possible overestimation of the size of the premium does not affect the evaluation of the switch option. This is because the absolute difference between the GP and LP determines its value, which are evaluated for a range of spreads. Moreover, a sensitivity analysis of the model results to changes in the parameters is performed. To derive a viable proxy for LP, it requires two counterfactual interest rates that show a different liquidity premium. Other approaches to obtain such instantaneous proxy are to use shorter maturity times for the Svensson approach or follow Kempf and Uhrig-Homburg (2000) and compare the yield differences of two bonds that only differ in its liquidity and mature within the next year. In contrast to these measures, the chosen approach provides a liquidity proxy with the time horizon of the German green bonds. This has the advantage that it would be possible to remove the liquidity effects from the observed yield spread, by matching its size to the German green bonds.

Figure 6 shows the yield differential between German covered bonds and German conventional bonds for the same maturity times as the German green bonds, which serves as a proxy for the liquidity premium. The data aligns with the finding of Kempf and Uhrig-Homburg (2000) that the premium is larger for longer times to maturity. Further, the premium increases in 2Q2020 for the short to medium term bonds (i.e., 2025, 2030 & 2031), which may be attributed to

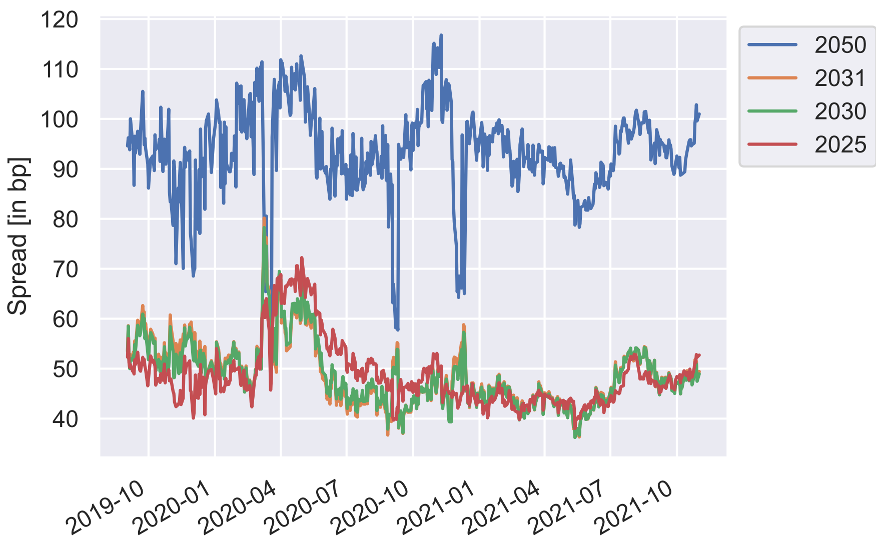


Figure 6: Yield spread between German government bonds and covered bonds

The figure displays the yield differential between German Pfandbriefe and Bundesanleihen. It is based on the estimated yield curves following the Svensson method and uses parameters published by Deutsche Bundesbank (2021). The summary statistics are displayed in Table 14 in the Appendix.

the effects of the Covid-19 crisis. A higher degree of uncertainty in this period could have increased the preference of investors to hold liquid assets, and thus the liquidity costs. While the volatility clusters in the data cannot be explained by the Ornstein-Uhlenbeck process in Equation 2 as it has a constant volatility term, the visualization in Figure 6 supports a mean-reverting process.

To test the adequacy of the underlying process for the given data, a Dickey-Fuller test is performed. This test can help to decide whether the data-generating process is stationary or has a unit root. As shown by Hayashi (2000), we need an ergodic stationary process to derive consistent parameter estimates for the population parameters. This is because a historical time series is only one possible realization of the underlying process. To obtain consistent estimates from the sample moments, we need to assume that all single observations over time result from the same process (i.e., stationarity) and that the memory of the process is not too persistent (i.e., ergodicity) (Hayashi, 2000). Further, a stationary process is also suggested by theory (see Section 4.1). Following Hamilton (1994), we estimate a random walk with drift and time trend,

$$\begin{aligned} \Delta LP_t &= LP_t - LP_{t-1} \\ &= \alpha + \gamma LP_{t-1} + \delta t + u_t, \end{aligned} \tag{20}$$

where α denotes the constant for the drift, γ is the coefficient for the unit root, δ denotes the slope of a linear time trend and u_t denotes independent white noise (i.e., an independent and identically distributed zero-mean error term with constant variance). To evaluate whether the data supports a unit root process, and thus does not support a mean-reverting process, we test the null hypothesis for the unit root $H_0 : \gamma = 0$ against the alternative hypothesis $H_A : \gamma < 0$. The relevant value of the test statistic τ is computed as,

$$\tau = \frac{\hat{\gamma}}{s.e.(\hat{\gamma})}, \tag{21}$$

which follows a non-standard distribution under the H_0. Therefore, we use the simulated critical values provided by Fuller (2009). The estimation results of the Dickey-Fuller test for the unit root parameter γ are summarized in Table 3. Based on the data, we can reject the H_0 of a unit root on a significance level $\alpha = 0.05$ for all time series, but for LP_{2025}. As a non-rejection of the H_0 contradicts the assumption for the data-generating process of the liquidity premium in the model, the results support the assumption in three of the four samples. We therefore proceed with the estimation of the model parameters specified in Equation 2.

We estimate the model parameters of Equation 2 to coincide with the Maximum Likelihood solution. This means that the estimated parameters b, a and σ maximize the joint probability that the estimated process yields the observed sample. For this, we follow Brigo, Dalessandro, Neugebauer, and Triki (2009) and estimate the parameters of the explicit solution for Equation 2 in discrete time by an Ordinary Least Squares (OLS) estimation,

$$LP_t = c + \phi LP_{t-1} + \delta \varepsilon_t, \tag{22}$$

where ε denotes Gaussian white noise. The estimation results of Equation 22 are summarized in the Appendix in Table 15. As suggested by Brigo et al. (2009), we use the

Table 3: Results of Dickey-Fuller Test

	LP_{2050}	LP_{2031}	LP_{2030}	LP_{2025}
$\hat{\gamma}$	−0.230	−0.136	−0.125	−0.047
$s.e.(\hat{\gamma})$	0.0304	0.022	0.0216	0.016
τ	−7.565	−6.114	−5.773	−2.935
$\tau_{0.05}^{crit}$	−3.423	−3.423	−3.423	−3.423
p-value	0.000	0.000	0.000	0.151
N	432	432	432	432

The table shows a summary of the Dickey-Fuller test results as specified in Equation 20. Observations with gaps due to missing data (i.e. weekends) are omitted.

following solution to obtain the parameters for Equation 2,

$$
\begin{aligned}
a &= -\frac{\ln(\phi)}{\Delta t} \\
b &= \frac{c}{1-\phi} \\
\sigma &= \frac{\delta}{\sqrt{(\phi^2-1)\Delta t/2\ln(\phi)}},
\end{aligned}
\tag{23}
$$

where we use $\Delta t = \frac{T}{N} = \frac{1}{250}$ due to daily observations. Further, we use a sample size of $N = 548$, which is lower than the sample of Kempf and Uhrig-Homburg (2000) that estimated the parameters of a term structure model using $N = 755$ observations. The sample period is chosen because it covers the complete period since the first emission of a German green bond. While a larger sample size might allow for a higher estimation precision, historical data that is too far in the past might not reflect current market conditions. The estimation results are summarized in Table 4.

The results in Table 4 for the fitted Vasicek process of the liquidity premium LP show significant differences based on their maturity time. The values for the mean reversion a and volatility parameter σ are larger for the curve that represents the long-term segment of the yield curve, namely 2050. This might be caused by the three drops shown in Figure 6 that are less pronounced and inversely seen for the short- and medium-term segments of the yield curves (i.e., 2025, 2030 & 2031). This means that during these very short periods, short-term liquidity became more expensive, while the long-term liquidity premium briefly declined in value but then returned to its initial level. As the model cannot accommodate such jumps and is fully described by the first two moments of the stochastic process $\tilde{LP}$, we rely on the estimated parameters for the 2025 time series for the further evaluation of the model.

Finally, the model requires a value for the green premium GP as an additional input parameter. In section 4.3 it was shown that the absolute difference between the long term mean of the liquidity premium b, in the following interpreted as the expected liquidity premium LP, and GP is sufficient to derive the size of ST, which is implied by the model. Therefore, for the purpose of evaluating the value of the switch option ST, it is only necessary to set the absolute difference between both premiums. For example, to reflect current market situations, we can set this difference so that the resulting yield spread coincides with the observed yield spread on the secondary market. Further, we can evaluate how the value of the switch option changes for different values of this spread between b and GP. For evaluating the effect of the switch option, it is thus not necessary to know the absolute value of the long term mean of the liquidity premium b, nor the value of the green premium GP, but only the difference $\Delta = LP - GP$. Similarly, the constant interest rate r affects both, the conventional bond C and the German green bond G, in the same fashion. Therefore, for this evaluation, an arbitrary value can be assumed as well.

5. Model Results

The evaluation of the model shows that it is able to reflect the main characteristic of the German twin bond approach. Namely, that the price of a German green bond P_G cannot fall below the value of its conventional twin P_C. Consequently, its maximum yield y_G is capped by an upper threshold equal to the yield of the conventional twin y_C. Furthermore, the model indicates additional potential advantages of the twin bond approach. Due to the additional value of the switch option, investors tolerate a higher degree of illiquidity until the value of the green bond assumes the threshold value of its conventional twin. In the same fashion, given a fixed level of illiquidity, a lower green premium is required that issuers can achieve a yield advantage, compared to a green and illiquid bond without a switch option.

5.1. Green Bond Yields

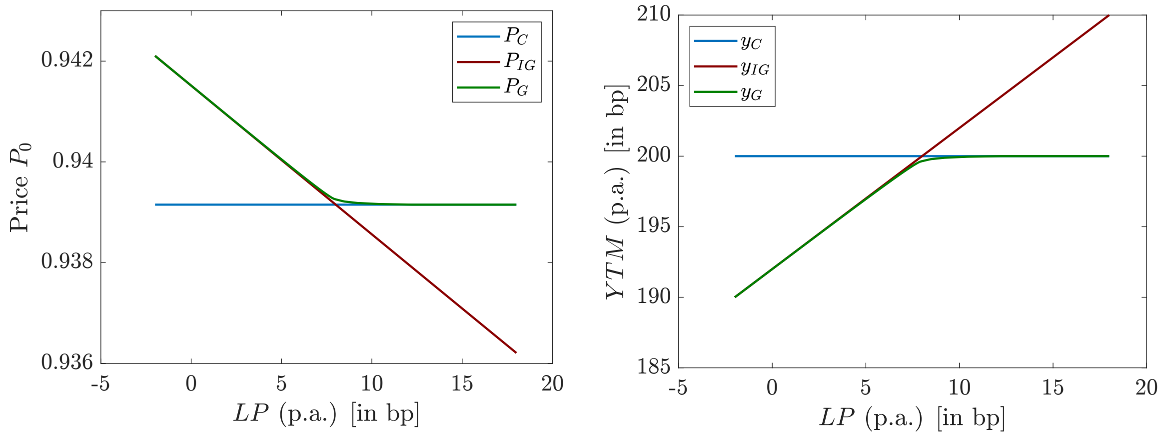
Using the parameters from the model calibration in section 4.3, we can evaluate the impact of changes in the expected liquidity premium LP for the different bond types. These are a conventional bond C, an illiquid and green bond IG, as well as a German green bond G, which is, in addition to being green and illiquid, also affected by the switch option.

Figure 7 shows the initial model bond prices and yields at $t = 0$, which are inversely related. Given a fixed face value

Table 4: Summary of ML estimates for $\tilde{LP}$

Vasicek model parameters		LP_{2050}	LP_{2031}	LP_{2030}	LP_{2025}
Mean-reversion rate	a	66.588	25.693	24.176	11.919
Long-term mean	b [in bp]	93.1	48.7	48.5	48.7
Instantaneous volatility	σ	0.0111	0.0046	0.0044	0.0031
Sample size	N	548	548	548	548

The table shows the estimation results for the process of the liquidity premium. The data is based on published yield curves by the Deutsche Bundesbank (2021) and covers the period from 02.09.2019 until 01.11.2021.



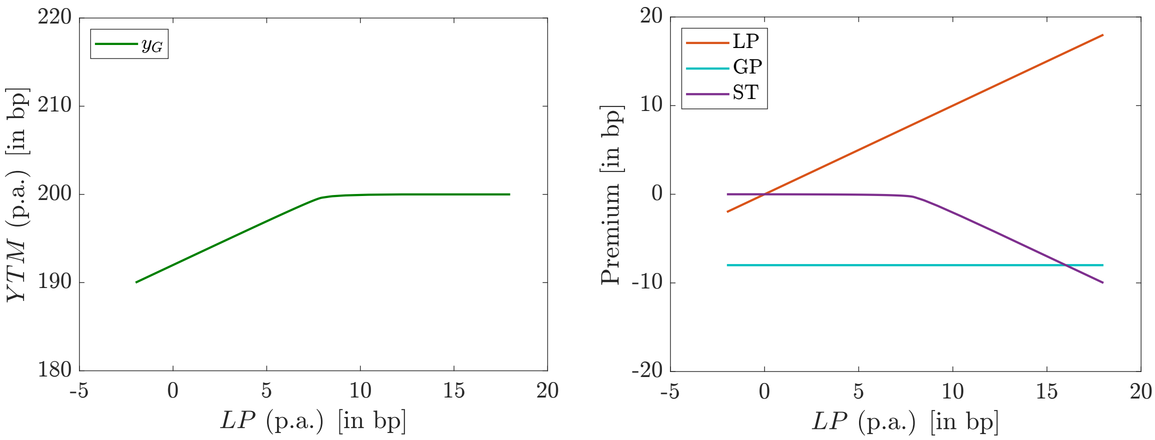
(a) Bond prices **(b)** Bond yields

Figure 7: Model results for different LP

The model results displayed in the figures above are based on a green premium of $GP = 8$bp, a risk-free rate of $r_f = 200$bp, $\sigma = 0.0031$, $a = 11.919$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

of the zero coupon bonds (i.e., FV=1), a lower price P_t, ceteris paribus, implies a higher yield to maturity y_t, and vice versa. The conventional bond C is assumed to be liquid, and thus not affected by changes in the premium that compensates for illiquidity of the asset. Therefore, the price P_C (see Figure 7a) and yield y_C (see Figure 7b) are unaffected by changes in the expected liquidity premium LP. On the other hand, the value of the illiquid green bond IG is affected as investors require a higher compensation for their liquidity risk and are thus only willing to pay lower prices. The German green bond G differs from the bond IG by having the additional switch option ST. This prevents the bond price P_G from assuming values lower than P_C. In the same fashion, the yields y_G cannot assume values higher than y_C. When the green premium outweighs the liquidity premium, the model yield y_G is smaller than the yield of the conventional bond y_C. This implies for the secondary market that a negative yield differential Δy (i.e., $\Delta y = y_G - y_C$) is observed. Finally, it should be noted that the green premium GP is equal to 8bp for all scenarios in the figure. For the German green bonds, this means that the yield difference Δy can be equal to zero, although there exists a green premium GP larger than zero. In such cases, liquidity effects dominate and the value of the upper threshold for the yield, y_C, is assumed. Moreover, in the case of bonds without a switch option, IG, the yield difference to a conventional twin can even assume positive values. This means that liquidity effects of bonds can potentially compensate the green premium. The model suggests that issuers and investors should therefore incorporate the bonds' exposure to illiquidity in their emission and valuation decision, respectively. This finding aligns with the published investor presentations or Green Bond Frameworks from France, Netherlands and Belgium, who all address liquidity aspects of their bonds (see section 3.2). Moreover, the model also shows that the German approach can prevent the yield spread from becoming positive. Therefore, it can be a viable method for issuers to mitigate by illiquidity induced risks.

Figure 8 displays the yield of a German green bond for different degrees of illiquidity and a decomposition of its yield premium that exists relative to its conventional twin. The green premium GP is constant with $GP = 8$bp for all values of the expected liquidity premium LP. The figure demonstrates that the switch option prevents the yield y_G to become larger than the yield of the conventional bond y_C. Further, its value (in bp) reflects the payoff structure of a short call



(a) Yield of a German green bond

(b) Decomposition of the yield

Figure 8: Model results for German green bonds

The model results displayed in the figures above are based on a green premium of $GP = 8$bp, a risk-free rate of $r_f = 200$bp, $\sigma = 0.0031$, $a = 11.919$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

option on the illiquidity of the bond. Using this analogy, the strike would coincide with the value of GP. If LP assumes a value larger than GP, the switch option ST needs to compensate this difference.

The time series of the yield spreads displayed in Figure 3 indicate that a value of -5bp can be a realistic value for German green bonds. Based on the model results displayed in Figure 7b and Figure 8b, this would imply $LP = 3$bp and $ST = 0$bp, assuming $GP = 8$bp. In words, this model specification indicates that the greenium is sufficiently larger than the liquidity premium so that market intervention by the Finance Agency is very unlikely to be necessary and thus the value of the switch option is equal to zero.

5.2. Maximum Switch Option Value

Figure 9 shows that the yield of the German green bond y_G is capped by y_C at an expected liquidity premium LP that is larger than the green premium GP. This is because the stochastic liquidity premium might still assume a lower value, in which case the execution of the switch option, (i.e., the execution of switch transactions) is not optimal. Based on Equation 1, we know that at this point the difference $LP - GP$ coincides with the maximum value of the switch option $ST^{\max}$, as Δy is equal to zero. This maximum value is relevant as it indicates how much additional liquidity costs in excess of a greenium the holders of a German-type green bond can bear until they assign the same value to it as to a conventional government bond. In comparison, in the case of an illiquid green bond without the switch option, this value would be zero.

In light of the above, the maximum value of the switch option $ST^{\max}$ can be defined as,

$$ST^{\max} = \max_{LP} \{ST \mid y_G \leq y_C\}. \tag{24}$$

In the following, we provide an overview of how this measure changes for different model specifications and an estimation precision of 0.01bp. Table 5 shows the value of the switch option at execution, $ST^{\max}$ for different levels of GP. The model results show that $ST^{\max}$ is unaffected by the size of GP, ceteris paribus. This is because a higher GP increases the expected illiquidity LP that can be tolerated before the switch option is executed. From an issuers' perspective, this implies that by adopting the German approach, they can compensate an additional liquidity premium of 4.1bp compared to conventional green bonds until the yield differential Δy assumes a value equal to zero.

To put the value of 4.1bp into perspective, we assume a total issuance volume of 5 bn. EUR which equals the size of the smallest currently issued German green bond. This implies a potential maximum value of approximately 2 mn. EUR for the switch option, given an issuance volume of 5 bn. EUR. However, the Green bond from this example currently (01.11.2021) trades at a spread Δy of -8bp. Based on the model calibration displayed in Figure 7b, this would imply a $LP < 5bp$, for which the value of the switch option ST is equal to zero (see Figure 8b).

Table 6 shows the value of $ST^{\max}$ for different local volatilities of the underlying process for the liquidity premium. The model results indicate that a higher σ increases the maximum value of the switch option $ST^{\max}$. This is plausible, as a higher volatility of the stochastic liquidity premium increases the chance of realizing very low values, while larger values do not change the outcome once the threshold is y_C is reached. This means that the switch option is executed later, which implies a higher value for LP and $ST^{\max}$. The model also accommodates a special case assuming a non-stochastic liquidity premium. In this case, the option is executed for $LP = GP$. As the liquidity premium cannot change over time, the option is executed as soon as liquidity effects and the green premium cancel each other

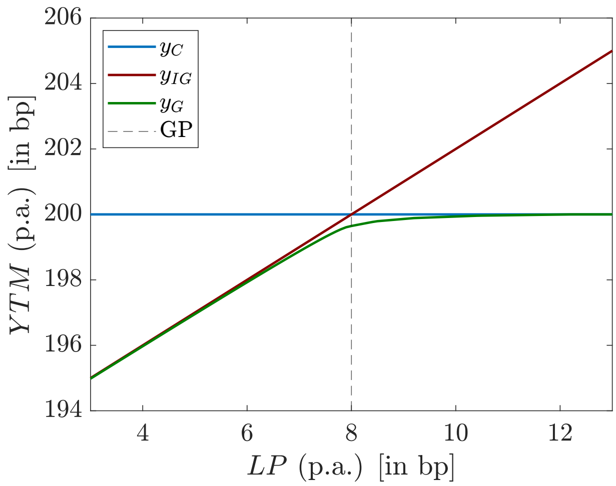


Figure 9: Bond yields for different LP

The model results displayed in the figures above are based on a green premium of $GP = 8$bp, a risk-free rate of $r_f = 200$bp, $\sigma = 0.0031$, $a = 11.919$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

Table 5: Option value at execution for different GP

GP	LP	$ST^{\max}$
0	4.06	4.06
10	14.06	4.06
20	24.06	4.06
30	34.06	4.06

The table shows the values of $ST^{\max}$ for different GP based on a risk-free rate of $r_f = 200$bp, $\sigma = 0.0031$, $a = 11.9$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

out. The resulting maximum value of the switch option before execution, $ST^{\max}$, is equal to zero in this scenario. For $LP < GP$, there is no chance that the option is executed as it implies a certain negative yield differential Δy. Therefore, the value of the option is equal to zero in this case as well.

Finally, Table 7 shows $ST^{\max}$ for different times to maturity T. In the model, this increases the length of the trinomial tree because $\Delta t = \frac{T}{N} = \frac{1}{250}$ is held constant. The results indicate a lower maximum value of the switch option $ST^{\max}$ for longer maturities T. This is explained by the decreasing likelihood of the stochastic liquidity premium realizing an outcome lower than LP. Therefore, the switch option is executed for a lower expected liquidity premium LP reducing its maximum value $ST^{\max}$.

5.3. Sensitivity Analysis of Bond Yields

The sensitivities of the initial yield to maturity to changes in the model parameters are estimated using finite differences that is motivated by a Taylor approximation. This approximation is required because a closed-form solution is not available due to the non-closed form of the model. Following Brandimarte (2006), a symmetric approximation of the first partial derivative of the yield y_0 with regard to the model parameters is computed, as this approach yields a lower order truncation error compared to forward or backward approximation. In its general form, the first derivative can be estimated using,

$$\frac{\partial y_0(x)}{\partial x} \approx \frac{y_0(x+h) - y_0(x-h)}{2h}, \tag{25}$$

where h denotes a small and constant value and x the parameter of interest, while the other model parameters are hold constant. The resulting sensitivities are displayed in Figure 10. The figures indicate that the sensitivity of the German green bond G has a continuous part, and a discontinuous part with jumps when LP assumes values above a certain threshold. The number of observed jumps in the figures for G coincide with $j_{max} = 4$ (or $-j_{min}$) of the calibrated model. One viable explanation might be that nodes in the tree switch to the value of the conventional bond, if the liquidity premium assumes a high enough value so that $P^G < P^C$ (see Equation 15). This also explains the continuous part on the left-hand side of the figures, as a switch scenario does not occur for low values of LP.

Figure 10a describes how much units the yield changes, if LP changes by one unit. The yield of the illiquid green bond y^{IG} changes by one basis point, if LP increases by one basis

Table 6: Option value at execution for different σ

σ	LP	$ST^{\max}$
0	8	0
0.002	10.53	2.53
0.004	13.28	5.28
0.008	18.81	10.81
0.010	21.26	13.26

The table shows the values of $ST^{\max}$ for different σ based on $GP = 8$bp, a risk-free rate of $r_f = 200$bp, $a = 11.9$, $T = 3.1$ years and a trinomial tree length of $N = 791$.

Table 7: Option value at execution for different T

T	LP	$ST^{\max}$
1	12.35	4.35
5	12.06	4.06
10	11.51	3.51
20	10.76	2.76
30	10.31	2.31

The table shows the values of $ST^{\max}$ for different T (constant Δt) based on $GP = 8$bp, a risk-free rate of $r_f = 200$bp, $\sigma = 0.0031$ and $a = 11.9$. Changes in T affect the tree length N, as Δt is hold constant with $\Delta t = \frac{T}{N} = \frac{1}{250}$.

point, while y^C is unaffected by changes in LP. The sensitivity of y^G ranges between 1 and 0. This aligns with the notion that the German green bond is valued as a conventional bond if LP is sufficiently high and valued as a counterfactual bond without switch option, if LP is sufficiently low, assuming a constant GP. In those cases, the stochastic process for LP either cannot assume values where y^G is lower than y^C, or where the switch option is executed. Figure 10b implies that a higher instantaneous volatility σ decreases y^G. This is because the downside potential is restricted by the switch option, while a lower realized liquidity premium reduces y^G. The parameter a describes the mean reversion rate of the stochastic process. Therefore, this sensitivity is inversely related with the sensitivity of y^G to σ. Finally, an increase in T, increases the yield y^G as well. Based on the absolute size of the sensitivities, the evaluation suggests that changes in σ and LP have the strongest impact on the model results. In light of the evaluation, it should be noted that the sensitivities only reflect the impact of small changes in the parameters. Further, their changes and thus the effect on the model results is restricted by their plausible range. Nevertheless, the model outcome might be significantly larger or smaller, if different estimates for those parameters are chosen.

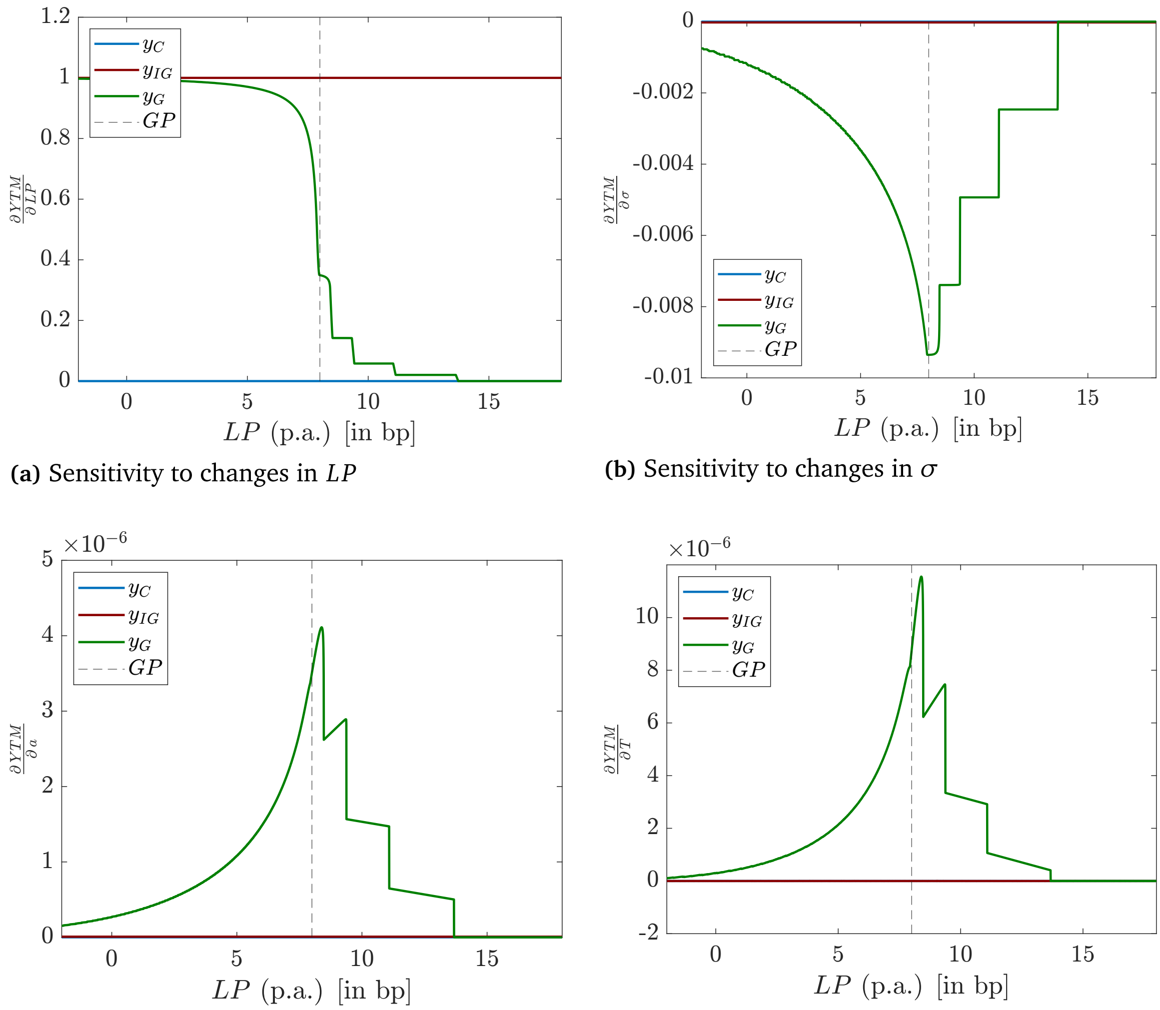
5.4. Limitations

The above discussed model for the green bond yields provides a first insight into the potential effects of the switch option between green and conventional bonds, which was pioneered by the German twin bond approach. However, the model is subject to some limitations that are discussed in the following.

First, the model cannot decompose observed green bond yields $\hat{y}_G$ into the different components suggested by the model. Namely, the observed yield of the respective conventional twin $\hat{y}_C$, the liquidity premium LP, the green premium GP and the added-value of the switch option ST. This means that a calibration of the model parameters is not straightforward and proxies need to be applied instead. Moreover, this impedes the validation of the model results based on actual observations.

Another possible limitation can be the assumed process for the liquidity premium and its translation into a trinomial tree representation. For example, the Vasicek process in Equation 2 assumes a constant volatility and is, in addition to a mean-reversion parameter, defined by its first two moments. This means that it cannot accommodate possible volatility clusters or skewness that is introduced by jumps in the liquidity premium, as shown in Figure 6. Moreover, deriving the trinomial tree representation, we assume a maximum range from LP_{jmin} to $LP_{j_{max}}$ for the liquidity premium to ensure positive tree probabilities. This creates an upper and lower threshold that the liquidity premium cannot exceed. However, increasing the volatility of the process may provide a first idea of the possible implications when accounting for these effects, as it increases the overall dispersion of the stochastic premium.

Finally, the model assumes a constant risk-free rate r and green premium GP. While adding additional complexity to the model by introducing more flexible (e.g., stochastic or



(a) Sensitivity to changes in LP

(b) Sensitivity to changes in σ

(c) Sensitivity to changes in a

(d) Sensitivity to changes in T

Figure 10: Model sensitivities

The model results displayed in the figures above are based on a green premium of $GP = 8$bp, a risk-free rate of $r_f = 200$bp, $\sigma = 0.0031$, $a = 11.9$, $T = 3.1$ years, a trinomial tree length of $N = 791$ and $h = 0.00001$

time-dependent) components might improve the calibration to observed yield spreads, this is not relevant for the main objective of this dissertation to better understand the potential impact of the switch option.

6. Conclusion

The goal of this dissertation is to provide a theoretical model for the pricing of green bonds that are based on the German twin bond approach. The focus here is on improving the understanding of the potential effects of introducing a switch mechanism between green bonds and their conventional counterparts. For this purpose, a non-closed form solution was derived that decomposes the yield differential into three effects: A liquidity premium, a green premium and the added value of the switch option. The model assumes a stochastic liquidity premium that follows a Vasicek process in discrete time, a constant green premium as well as a constant risk-free rate. The switch mechanism is modelled by assuming the theoretical value of conventional bonds as a lower limit for the green bond prices. For the model calibration the term structures of German Bundesanleihen and Pfandbriefen are used to obtain a proxy for the stochastic liquidity premium.

The main learning from the model is that the switch option can in certain conditions increase the value of the green bonds, which corresponds to a lower yield. Based on the calibration of the model, a maximum added-value of 4.1 bp before the execution of the option was identified. This translates to a maximum value of about 2 mn. EUR assuming a green bond with a 5 bn. EUR issuance volume. This means that issuers adopting the twin bond concept may be able to secure lower costs of capital compared to a traditional green bond concept that does not provide the switch option. For investors the concept reduces their exposure to potential liquidity risks by using the liquid conventional bonds to create a lower limit for the green bond price. The model improves the understanding of the twin bond concept and thereby fills a gap in the literature. From a practical perspective, the model implications may assist issuers in the design choice of their green bond framework. For example, Denmark decided to adopt the twin bond concept, including a switch mechanism, which supports the potential benefits of this approach.

Green bonds are one important instrument to finance the transition to a more sustainable economy. In light of the significant growth of the green bond market in recent history and the competing frameworks, it is crucial to elaborate on their respective advantages and disadvantages. While this work contributes to the understanding of the twin bond switch mechanism, the current model can be further developed. On the one hand, an improved proxy for the liquidity premium and a larger sample of historic data may affect the calibration results, which can impact the size of the evaluated effects. On the other hand, a more sophisticated stochastic process for the liquidity premium and less restrictive assumptions in its discrete representation may increase the precision of the model results. In a broader context, one should evaluate if a high issuance volume of green bonds can affect the liquidity of similar conventional bonds, and whether a potential effect vanishes for lower volumes. If such effects are found, this would support the relevance of the twin bond approach with switch option to mitigate liquidity risks, as lower overall issuance volumes may be required. Otherwise, ensuring a critical volume that is high enough to avoid liquidity costs may be a viable alternative to this concept.

References

Almeida, M., Filkova, M., Harrison, C., & Sette, P. (2019). *Green bond european investor survey.* Retrieved from https://www.climatebonds.net/resources/reports/green-bond-european-investor-survey-2019 (Last accessed: 19.01.2022)

Bachelet, M. J., Becchetti, L., & Manfredonia, S. (2019). The green bonds premium puzzle: The role of issuer characteristics and third-party verification. *Sustainability*, *11*(4), 1098.

Bongaerts, D., & Schoenmaker, D. (2020). Green certificates: a better version of green bonds. *Policy Contribution 2020/20, Bruegel*.

Brandimarte, P. (2006). *Numerical methods in finance and economics*. Hoboken, NJ, USA: John Wiley & Sons, Inc.

Brigo, D., Dalessandro, A., Neugebauer, M., & Triki, F. (2009). A stochastic processes toolkit for risk management: Geometric brownian motion, jumps, garch and variance gamma models. *Journal of Risk Management in Financial Institutions*, *2*(4), 365–393.

Chen, L., Lesmond, D., & Wei, J. (2007). Corporate yield spreads and bond liquidity. *The Journal of Finance*, *62*(1), 119–149.

Climate Bonds Initiative. (2021). *Sustainable debt market summary q3 2021.* Retrieved from https://www.climatebonds.net/resources/reports/sustainable-debt-summary-q3-2021 (Last accessed: 19.01.2022)

Cox, J. C., Ingersoll, J. E., & Ross, S. A. (1985). A theory of the term structure of interest rates. *Econometrica*, *53*(2), 385.

Danmarks Nationalbank. (2022). *Green bonds.* Retrieved from https://www.nationalbanken.dk/en/governmentdebt/green_bonds/Pages/Default.aspx (Last accessed: 19.01.2022)

Deutsche Bundesbank. (2021). *Term structure of interest rates in the debt securities market - estimated values: Listed federal securities and pfandbriefe.* Retrieved from www.bundesbank.de (Last accessed: 19.01.2022)

Díaz, A., & Escribano, A. (2020). Measuring the multi-faceted dimension of liquidity in financial markets: A literature review. *Research in International Business and Finance*, *51*, 101079.

Dick-Nielsen, J., Feldhütter, P., & Lando, D. (2012). Corporate bond liquidity before and after the onset of the subprime crisis. *Journal of Financial Economics*, *103*(3), 471–492.

Dorfleitner, G., Utz, S., & Zhang, R. (2021). The pricing of green bonds: external reviews and the shades of green. *Review of Managerial Science*.

Doronzo, R., Siracusa, V., & Antonelli, S. (2021). Green bonds: The sovereign issuers' perspective. *Bank of Italy Markets, Infrastructures, Payment Systems Working Paper No. 3*.

Dutch State Treasury Agency. (2019). *Investor presentation green dsl.* Retrieved from https://english.dsta.nl/binaries/dsta-english/documents/publication/2019/05/06/investor-presentation-green-dsl/Investor+presentation+Green+DSL.pdf (Last accessed: 19.01.2022)

Ehlers, T., & Packer, F. (2017). Green bond finance and certification. *BIS Quarterly Review September*.

European Commission. (2012). *Charter of fundamental rights of the european union: 2012/c 326/02.* Retrieved from https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:12012P/TXT (Last accessed: 19.01.2022)

European Commission. (2020). *Stepping up europe's 2030 climate ambition: Swd/2020/176 final.* Retrieved from https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52020SC0176 (Last accessed: 19.01.2022)

European Commission. (2021). *Proposal for a regulation of the european parliament and of the council on european green bonds: Com/2021/391 final.* Retrieved from https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52021PC0391 (Last accessed: 19.01.2022)

Fama, E. F., & French, K. R. (2007). Disagreement, tastes, and asset prices. *Journal of Financial Economics*, *83*(3), 667–689.

Finanzagentur GmbH. (2020). *Green bond framework.* Retrieved from https://www.deutsche-finanzagentur.de/fileadmin/user_upload/institutionelle-investoren/pdf/GreenBondFramework.pdf (Last accessed: 19.01.2022)

Finanzagentur GmbH. (2021a). *The bund's green twins: Green federal securities.* Retrieved from https://www.deutsche-finanzagentur.de/en/institutional-investors/federal-securities/green-federal-securities/ (Last accessed: 19.01.2022)

Finanzagentur GmbH. (2021b). *Green bond investor presentation september 2021.* Retrieved from https://www.deutsche-finanzagentur.de/fileadmin/user_upload/institutionelle-investoren/pdf/Green_Bond_Investor_Presentation_2021_II.pdf (Last accessed: 19.01.2022)

Finanzagentur GmbH. (2021c). *Joint press release 20 april 2021: Federal government publishes first allocation report for green federal securities.* Retrieved from https://www.deutsche-finanzagentur.de/fileadmin/user_upload/pressemeldungen/en/2021/2021-04-20_pm03_allocation_report_2020_en.pdf (Last accessed: 19.01.2022)

Finanzagentur GmbH. (2021d). *Primary market.* Retrieved from https://www.deutsche-finanzagentur.de/en/institutional-investors/primary-market/ (Last accessed: 19.01.2022)

Fisher, L. (1959). Determinants of risk premiums on corporate bonds. *Journal of political economy*, *67*(3), 217–237.

Fuller, W. A. (2009). *Introduction to statistical time series*. John Wiley & Sons.

Gianfrate, G., & Peri, M. (2019). The green advantage: Exploring the convenience of issuing green bonds. *Journal of Cleaner Production*, *219*, 127–135.

Goldstein, M. A., Hotchkiss, E. S., & Pedersen, D. J. (2019). Secondary market liquidity and primary market pricing of corporate bonds. *Journal of Risk and Financial Management*, *12*(2), 86.

Hachenberg, B., & Schiereck, D. (2018). Are green bonds priced differently from conventional bonds? *Journal of Asset Management*, *19*(6), 371–383.

Hamilton, J. D. (1994). *Time series analysis*. Princeton, NJ: Princeton Univ. Press.

Harrison, C., & Muething, L. (2021). *Sovereign green, social, and sustainability bond survey.* Retrieved from https://www.climatebonds.net/files/reports/cbi-sovereign-green-social-sustainability-bond-survey-jan2021.pdf (Last accessed: 19.01.2022)

Hartzmark, S., & Sussman, A. (2019). Do investors value sustainability? a natural experiment examining ranking and fund flows. *The Journal of Finance*, *74*(6), 2789–2837.

Hayashi, F. (2000). *Econometrics*. Princeton, NJ: Princeton Univ. Press.

Helmcke, S., Heuss, R., Hieronimus, S., & Engel, H. (2021). *Net-zero deutschland: Chancen und herausforderungen auf dem weg zur klimaneutralität bis 2045.* Retrieved from https://www.mckinsey.de/de/news/presse/studie-net-zero-deutschland-klimaneutralitaet-chancen-herausforderungen (Last accessed: 19.01.2022)

Helwege, J., Huang, J.-Z., & Wang, Y. (2014). Liquidity effects in corporate bond spreads. *Journal of Banking & Finance*, *45*, 105–116.

Hull, J. (2018). *Options, futures, and other derivatives* (Ninth edition, global edition ed.). Harlow, England: Pearson.

Immel, M., Hachenberg, B., Kiesel, F., & Schiereck, D. (2021). Green bonds: shades of green and brown. *Journal of Asset Management*, *22*(2), 96–109.

International Capital Market Association. (2021). *The green bond principles: Voluntary process guidelines for issuing green bonds.* Retrieved from https://www.icmagroup.org/sustainable-finance/the-principles-guidelines-and-handbooks/green-bond-principles-gbp/ (Last accessed: 19.01.2022)

ISS ESG. (2020). *Second party opinion (spo): Sustainability quality of the issuer and the green german federal securities.* Retrieved from https://www.deutsche-finanzagentur.de/fileadmin/user_upload/institutionelle-investoren/pdf/SPO.pdf (Last accessed: 19.01.2022)

Kapraun, J., Latino, C., Scheins, C., & Schlag, C. (2019). (in)-credibly green: Which bonds trade at a green bond premium? *Proceedings of Paris December 2019 Finance Meeting EUROFIDAI - ESSEC*.

Kempf, A., Korn, O., & Uhrig-Homburg, M. (2012). The term structure of illiquidity premia. *Journal of Banking & Finance*, *36*(5), 1381–1391.

Kempf, A., & Uhrig-Homburg, M. (2000). Liquidity and its impact on bond prices. *Schmalenbach Business Review*, *52*(1), 26–44.
Koziol, C., & Sauerbier, P. (2007). Valuation of bond illiquidity. *The Journal of Fixed Income*, *16*(4), 81–107.
Löffler, K. U., Petreski, A., & Stephan, A. (2021). Drivers of green bond issuance and new evidence on the "greenium". *Eurasian Economic Review*, *11*(1), 1–24.
MacAskill, S., Roca, E., Liu, B., Stewart, R. A., & Sahin, O. (2021). Is there a green premium in the green bond market? systematic literature review revealing premium determinants. *Journal of Cleaner Production*, *280*, 124491.
Monfort, A., & Renne, J.-P. (2014). Decomposing euro-area sovereign spreads: Credit and liquidity risks*. *Review of Finance*, *18*(6), 2103–2151.
People's Bank of China. (2021). *Green bond endorsed projects catalogue (2021 edition)*. Retrieved from https://climatebonds.net/market/country/china/green-bond-endorsed-project-catalogue (Last accessed: 19.01.2022)
Rendleman, R. J., & Bartter, B. J. (1980). The pricing of options on debt securities. *The Journal of Financial and Quantitative Analysis*, *15*(1), 11.
République Française. (2017). *Framework for the green oat.* Retrieved from https://www.aft.gouv.fr/files/archives/attachments/25562.pdf (Last accessed: 19.01.2022)
République Française. (2021). *Investors presentation - march 2021.* Retrieved from https://www.aft.gouv.fr/files/medias-aft/3_Dette/3.2_OATMLT/3.2.2_OATVerte/Investor%20presentation%20AFT%20GIC%20March%202021_UK.pdf (Last accessed: 19.01.2022)
Riedl, A., & Smeets, P. (2017). Why do investors hold socially responsible mutual funds? *The Journal of Finance*, *72*(6), 2505–2550.
Schestag, R., Schuster, P., & Uhrig-Homburg, M. (2016). Measuring liquidity in bond markets. *Review of Financial Studies*, *29*(5), 1170–1219.
Schuster, P., & Uhrig-Homburg, M. (2012). The term structure of bond market liquidity conditional on the economic environment: An analysis of government guaranteed bonds. *No 45, Working Paper Series in Economics, Karlsruhe Institute of Technology (KIT), Department of Economics and Management*.
Schwarz, K. (2019). Mind the gap: Disentangling credit and liquidity in risk spreads. *Review of Finance*, *23*(3), 557–597.
SIFMA. (2021). *2021 capital markets fact book.* Retrieved from https://www.sifma.org/resources/research/fact-book/ (Last accessed: 19.01.2022)
Svensson, L. (1994). Estimating and interpreting forward interest rates: Sweden 1992-4. *C.E.P.R. Discussion Papers*, *1051*.
The Kingdom of Belgium. (2018). *Green olo framework.* Retrieved from https://www.debtagency.be/sites/default/files/content/download/files/green_olo_framework.pdf (Last accessed: 19.01.2022)
United Nations. (n.d.). *United nations sustainable development goals.* Retrieved from https://sdgs.un.org/goals (Last accessed: 19.01.2022)
United Nations. (2022). *Chapter xxvii environment 7. d paris agreement.* Retrieved from https://treaties.un.org/Pages/ViewDetails.aspx?src=TREATY&mtdsg_no=XXVII-7-d&chapter=27&clang=_en (Last accessed: 19.01.2022)
Vasicek, O. (1977). An equilibrium characterization of the term structure. *Journal of Financial Economics*, *5*(2), 177–188.
Wegener, C., Basse, T., Sibbertsen, P., & Nguyen, D. K. (2019). Liquidity risk and the covered bond market in times of crisis: empirical evidence from germany. *Annals of Operations Research*, *282*(1-2), 407–426.
Wulandari, F., Schäfer, D., Stephan, A., & Sun, C. (2018). The impact of liquidity risk on the yield spread of green bonds. *Finance Research Letters*, *27*, 53–59.
Zerbib, O. D. (2019). The effect of pro-environmental preferences on bond prices: Evidence from green bonds. *Journal of Banking & Finance*, *98*, 39–60.