Online-Appendix zu

„Implications of the Creditors' Influence on Corporate Decisions"

Carsten S. Ruhnke
Leibniz Universität Hannover

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Appendix for the model from Douglas (2009)

A.1. Managerial wage payment in the first-best-case with risk-free debt

With the participation constraints (5) in place, the incentive compatibility constraints (6) hold as long as $u_0 \geq w\left(v(x^i, a)\right) - A(a)$. The shareholders ensure this by setting $w\left(v(x^i, a)\right) = 0$ for all $a \neq a^i$, because this leads to $-A(a) \leq 0 < u_0$. They seek to minimize the wage payment to the manager, so that the participation constraints (5) bind in the optimum:

$$w(v^i) - A(a^i) = u_o$$
$$\leftrightarrow \quad w(v^i) = u_o + A(a^i) \tag{A1}$$

The manager receives the wage payment $w(v^i)$ if he chooses the desired action a^i and nothing otherwise. With $a^i = a^{FB}$, this leads to $w^{FB}(v)$, as given in (7).

A.2. Binding constraints in the second-best-case with risk-free debt

Firstly, consider the participation constraints, given by (9). Because of $A'(a) > 0$ it follows with $\Delta x > 0$ that $w^L - A(a^L - \Delta x) > w^L - A(a^L)$ and with (11) and (12) this results in:

$$w^H - A(a^H) > w^L - A(a^L) \tag{A2}$$

Hence, only the participation constraint (PC_L) binds.

As explained in Appendix A.1, the incentive compatibility constraints (IC_i) automatically hold as long as the participation constraints (PC_i) hold, but they do not bind. The incentive compatibility constraints (IC_1) and (IC_2), given by (11) and (12), can be rearranged:

$$w^H - w^L \leq A(a^H + \Delta x) - A(a^L) \tag{A3}$$

$$w^H - w^L \geq A(a^H) - A(a^L - \Delta x) \tag{A4}$$

Both of the constraints can only bind in case of $A(a^H + \Delta x) - A(a^L) = A(a^H) - A(a^L - \Delta x)$. However, with $a^H \geq a^L$, $A'(a) > 0$ and $A''(a) > 0$, it follows that:

$$A(a^H + \Delta x) - A(a^L) > A(a^H) - A(a^L - \Delta x) \tag{A5}$$

Thus, only one of the two constraints can bind. The binding constraint has to be (IC_2), because otherwise the shareholders could decrease w^H in order to increase their expected payoff, which would not represent the optimum.

A.3. Results of the second-best-case with risk-free debt

With (PC_L) and (IC_2) binding, the Lagrange function of the problem in (9) to (12) is given by:

$$\mathcal{L} = \sigma(x^H + a^H - w^H) + (1-\sigma)(x^L + a^L - w^L) - F + \theta_1[w^L - A(a^L) - u_0] \\ +\theta_2[w^H - A(a^H) - w^L + A(a^L - \Delta x)] \tag{A6}$$

$\mathcal{L}$ is maximized with respect to w^H, w^L, a^H, a^L by calculating the first-order-conditions:

$$\frac{\partial \mathcal{L}}{\partial w^H} = -\sigma + \theta_2 = 0 \tag{A7}$$

$$\frac{\partial \mathcal{L}}{\partial w^L} = \sigma - 1 + \theta_1 - \theta_2 = 0 \tag{A8}$$

$$\frac{\partial \mathcal{L}}{\partial a^H} = \sigma - \theta_2 A'(a^H) = 0 \tag{A9}$$

$$\frac{\partial \mathcal{L}}{\partial a^L} = 1 - \sigma - \theta_1 A'(a^L) + \theta_2 A'(a^L - \Delta x) = 0 \tag{A10}$$

From (A7) it follows directly that $\theta_2 = \sigma$. Inserting this into (A8) leads to $\theta_1 = 1$. With $\theta_2 = \sigma$, (A9) leads to $A'(a^H) = 1$. Douglas (2009) assumes that $A'(a^{FB}) = 1$ and $A''(a) > 0$, so that $a^H = a_{SB}^H = a^{FB}$. With the above solutions for θ_1 and θ_2, (A10) can be rearranged as follows:

$$(1-\sigma)\big(1 - A'(a^L)\big) = \sigma(A'(a^L) - A'(a^L - \Delta x)) \tag{A11}$$

As the right side of (A11) is positive, the left side needs to be positive, too, so that $A'(a^L) < 1$. With $A'(a^{FB}) = 1$ and $A''(a) > 0$, it follows that $a^L = a_{SB}^L < a^{FB} = a_{SB}^H$.

Using the disutility function $A(a) = \frac{k}{2}a^2$, given in (1), the result for a_{SB}^H is as follows:

$$A'(a_{SB}^H) = k a_{SB}^H = 1 \\ \leftrightarrow \quad a_{SB}^H = \frac{1}{k} = a^{FB} \tag{A12}$$

Moreover, a_{SB}^L is calculated by:

$$1 - \sigma - A'(a_{SB}^L) + \sigma A'(a_{SB}^L - \Delta x) = 0 \\ \leftrightarrow \quad 1 - \sigma - k a_{SB}^L + \sigma k (a_{SB}^L - \Delta x) = 0 \\ \leftrightarrow \quad k a_{SB}^L (1 - \sigma) = 1 - \sigma - \sigma k \Delta x \\ \leftrightarrow \quad a_{SB}^L = \frac{1}{k} - \frac{\sigma}{1-\sigma}\Delta x = a^{FB} - \frac{\sigma}{1-\sigma}\Delta x \tag{A13}$$

With $K_1(\sigma) = \sigma/(1-\sigma)$, it follows that $a_{SB}^L = a^{FB} - K_1(\sigma)\Delta x$, as given in (14). Note that $\sigma > 0$ leads to $K_1 > 0$, so that $a_{SB}^L < a^{FB}$. For $K_1 > a^{FB}/\Delta x$, it directly follows that $a_{SB}^L = 0$, as the action cannot be negative.

A.4. Agency costs in the second-best-case with risk-free debt

The agency costs for the high state are calculated as follows:

$$
\begin{aligned}
AC^H(a_{SB}^L) &= \left[x^H + a^{FB} - w^{FB}\left(v(x^H, a^{FB})\right)\right] - [x^H + a^{FB} - w_{SB}^H] \\
&= w_{SB}^H - w^{FB}\left(v(x^H, a^{FB})\right) \\
&= u_0 + A(a^{FB}) + A(a_{SB}^L) - A(a_{SB}^L - \Delta x) - A(a^{FB}) - u_0 \\
&= A(a_{SB}^L) - A(a_{SB}^L - \Delta x)
\end{aligned}
\quad \text{(A14)}
$$

The agency costs for the low state are given by:

$$
\begin{aligned}
AC^L(a_{SB}^L) &= \left[x^L + a^{FB} - w^{FB}\left(v(x^L, a^{FB})\right)\right] - [x^L + a_{SB}^L - w_{SB}^L] \\
&= x^L + a^{FB} - A(a^{FB}) - u_0 - x^L - a_{SB}^L + A(a_{SB}^L) + u_0 \\
&= [a^{FB} - A(a^{FB})] - [a_{SB}^L - A(a_{SB}^L)]
\end{aligned}
\quad \text{(A15)}
$$

The derivatives of the agency costs with respect to a_{SB}^L are:

$$\frac{\partial AC^H}{\partial a_{SB}^L} = A'(a_{SB}^L) - A'(a_{SB}^L - \Delta x) \quad \text{(A16)}$$

$$\frac{\partial AC^L}{\partial a_{SB}^L} = -1 + A'(a_{SB}^L) \quad \text{(A17)}$$

As $A''(a) > 0$, it follows that $\partial AC^H / \partial a_{SB}^L > 0$. Also, $a_{SB}^L < a^{FB}$, so that $A'(a_{SB}^L) < A'(a^{FB}) = 1$, which leads to $\partial AC^L / \partial a_{SB}^L < 0$.

The action a_{SB}^L that minimizes the total agency costs AC, given in (19), is calculated by the first-order-condition:

$$\frac{\partial AC}{\partial a_{SB}^L} = \sigma[A'(a_{SB}^L) - A'(a_{SB}^L - \Delta x)] + (1-\sigma)[-1 + A'(a_{SB}^L)] = 0 \quad \text{(A18)}$$

Using the disutility function $A(a) = \frac{k}{2}a^2$, this leads to:

$$
\begin{aligned}
&\sigma[k a_{SB}^L - k(a_{SB}^L - \Delta x)] + (1-\sigma)[-1 + k a_{SB}^L] = 0 \\
\leftrightarrow \quad & k a_{SB}^L(\sigma - \sigma + 1 - \sigma) = 1 - \sigma - \sigma k \Delta x \\
\leftrightarrow \quad & a_{SB}^L = \frac{1}{k} - \frac{\sigma}{1-\sigma}\Delta x
\end{aligned}
\quad \text{(A19)}
$$

Hence, the solution for a_{SB}^L derived in (A13) minimizes the agency costs.

A.5. Results of the second-best-case with risky debt

With (PC_L) and (IC_2) binding, the Lagrange function of the problem with the objective function (20) and the constraints (9) to (12) is given by:

$$\begin{aligned}\mathcal{L} = &(1-m)\big[(1-\beta)\sigma(x^H + a^H - w^H - F) + \beta\big((1-\sigma)(x^L + a^L - w^L) + \sigma F\big)\big] \\ &+m\big[(1-\sigma)\big(w^L - A(a^L)\big) + \sigma\big(w^H - A(a^H)\big)\big] + \theta_1[w^L - A(a^L) - u_0] \\ &+\theta_2[w^H - A(a^H) - w^L + A(a^L - \Delta x)]\end{aligned} \quad \text{(A20)}$$

This function is maximized with respect to w^H, w^L, a^H, a^L by calculating the first-order-conditions as follows:

$$\frac{\partial \mathcal{L}}{\partial w^H} = -(1-m)(1-\beta)\sigma + m\sigma + \theta_2 = 0 \quad \text{(A21)}$$

$$\frac{\partial \mathcal{L}}{\partial w^L} = -(1-m)\beta(1-\sigma) + m(1-\sigma) + \theta_1 - \theta_2 = 0 \quad \text{(A22)}$$

$$\frac{\partial \mathcal{L}}{\partial a^H} = (1-m)(1-\beta)\sigma - m\sigma A'(a^H) - \theta_2 A'(a^H) = 0 \quad \text{(A23)}$$

$$\frac{\partial \mathcal{L}}{\partial a^L} = (1-m)\beta(1-\sigma) - m(1-\sigma)A'(a^L) - \theta_1 A'(a^L) + \theta_2 A'(a^L - \Delta x) = 0 \quad \text{(A24)}$$

From equation (A21) it follows directly that $\theta_2 = \sigma[(1-m)(1-\beta) - m]$. With this result for θ_2, equation (A22) leads to $\theta_1 = (1-m)(\sigma(1-\beta) + (1-\sigma)\beta) - m$. The equation (A23) can now be solved as follows:

$$\begin{aligned}(1-m)(1-\beta)\sigma &= A'(a^H)(\sigma(1-m)(1-\beta) - \sigma m + \sigma m) \\ \leftrightarrow \quad A'(a^H) &= 1 = A'(a^{FB}) \\ \leftrightarrow \quad a^H &= a^{FB} = a^H_{SB,RD}\end{aligned} \quad \text{(A25)}$$

Given the values of θ_1 and θ_2, the equation (A24) can be rearranged, as shown below:

$$(1-m)\beta(1-\sigma)\big(1 - A'(a^L)\big) = \sigma(A'(a^L) - A'(a^L - \Delta x))((1-m)(1-\beta) - m) \quad \text{(A26)}$$

With $A''(a) > 0$, it follows that $A'(a^L) - A'(a^L - \Delta x) > 0$ and $m < (1-\beta)/(2-\beta)$ leads to $(1-m)(1-\beta) - m > 0$, so that the right side of the equation is positive.[275] Hence, the left side needs to be positive as well, leading to $1 - A'(a^L) > 0$ and therefore $A'(a^L) < 1$. Because of $A'(a^{FB}) = 1$ and $A''(a) > 0$, this means that $a^L = a^L_{SB,RD} < a^{FB}$.

[275] The condition $m < (1-\beta)/(2-\beta)$ is needed to obtain the results shown below. It is similar to not giving the manager too much control on the board of directors, as mentioned in footnote 77.

Using the disutility function $A(a) = \frac{k}{2}a^2$, given in (1), leads to the following result for $a_{SB,RD}^H$:

$$A'\left(a_{SB,RD}^H\right) = ka_{SB,RD}^H = 1$$

$$\leftrightarrow \quad a_{SB,RD}^H = \frac{1}{k} = a^{FB} \tag{A27}$$

The result for $a_{SB,RD}^L$ is calculated as follows:

$$(1-m)\beta(1-\sigma) - m(1-\sigma)A'\left(a_{SB,RD}^L\right) - \theta_2 A'\left(a_{SB,RD}^L\right) + \theta_1 A'\left(a_{SB,RD}^L - \Delta x\right) = 0$$

$$\leftrightarrow \quad (1-m)\beta(1-\sigma) - m(1-\sigma)ka_{SB,RD}^L - \theta_2 ka_{SB,RD}^L + \theta_1 k\left(a_{SB,RD}^L - \Delta x\right) = 0$$

$$\leftrightarrow \quad ka_{SB,RD}^L[m(1-\sigma) + (1-m)(\sigma(1-\beta) + (1-\sigma)\beta) - m - \sigma(1-m)(1-\beta) + \sigma m]$$

$$= (1-m)\beta(1-\sigma) - \sigma\big((1-m)(1-\beta) - m\big)\Delta x$$

$$\leftrightarrow \quad a_{SB,RD}^L = \frac{1}{k} - \frac{\sigma\big((1-m)(1-\beta) - m\big)}{(1-m)(1-\sigma)\beta}\Delta x = a^{FB} - \frac{\sigma\big((1-m)(1-\beta) - m\big)}{(1-m)(1-\sigma)\beta}\Delta x \tag{A28}$$

With $K_2(\sigma, \beta, m) = \frac{\sigma((1-m)(1-\beta)-m)}{(1-m)(1-\sigma)\beta}$ it follows:

$$a_{SB,RD}^L = a^{FB} - K_2(\sigma, \beta, m)\Delta x \tag{A29}$$

Note that $m < (1-\beta)/(2-\beta)$ leads to $K_2 > 0$, so that $a_{SB,RD}^L < a^{FB}$. If $K_2 > a^{FB}/\Delta x$, the managerial action is set as $a_{SB,RD}^L = 0$ in order to avoid any negative action. For further investigation, the derivatives of $a_{SB,RD}^L$ with respect to m and β are calculated:

$$\frac{\partial a_{SB,RD}^L}{\partial m} = \frac{\partial a^{FB}}{\partial m} - \frac{\partial(K_2\Delta x)}{\partial m} = 0 - \frac{\partial K_2}{\partial m}\Delta x$$

$$= -\frac{(-\sigma(1-\beta) - \sigma)(1-m)(1-\sigma)\beta + (1-\sigma)\beta\sigma\big((1-m)(1-\beta) - m\big)}{(1-m)^2(1-\sigma)^2\beta^2}\Delta x$$

$$= \frac{\sigma(1-\beta)(1-m) + \sigma(1-m) - \sigma(1-m)(1-\beta) + \sigma m}{(1-m)^2(1-\sigma)\beta}\Delta x$$

$$= \frac{\sigma}{(1-m)^2(1-\sigma)\beta}\Delta x > 0 \tag{A30}$$

$$\frac{\partial a_{SB,RD}^L}{\partial \beta} = \frac{\partial a^{FB}}{\partial \beta} - \frac{\partial(K_2\Delta x)}{\partial \beta} = 0 - \frac{\partial K_2}{\partial \beta}\Delta x$$

$$= -\frac{-\sigma(1-m)^2(1-\sigma)\beta - (1-m)(1-\sigma)\sigma\big((1-m)(1-\beta) - m\big)}{(1-m)^2(1-\sigma)^2\beta^2}\Delta x$$

$$= \frac{\sigma(1-m)\beta + \sigma(1-m)(1-\beta) - \sigma m}{(1-m)(1-\sigma)\beta^2}\Delta x = \frac{\sigma(1-m) - \sigma m}{(1-m)(1-\sigma)\beta^2}\Delta x \tag{A31}$$

With $m < (1-\beta)/(2-\beta) \leq 1/2$, it follows that $\partial a_{SB,RD}^L/\partial \beta > 0$.

A.6. Graphical presentation of $a_{SB,RD}^{L}$

Figure I shows $a_{SB,RD}^{L}$, given by (A28), as a function of β for different values of m.

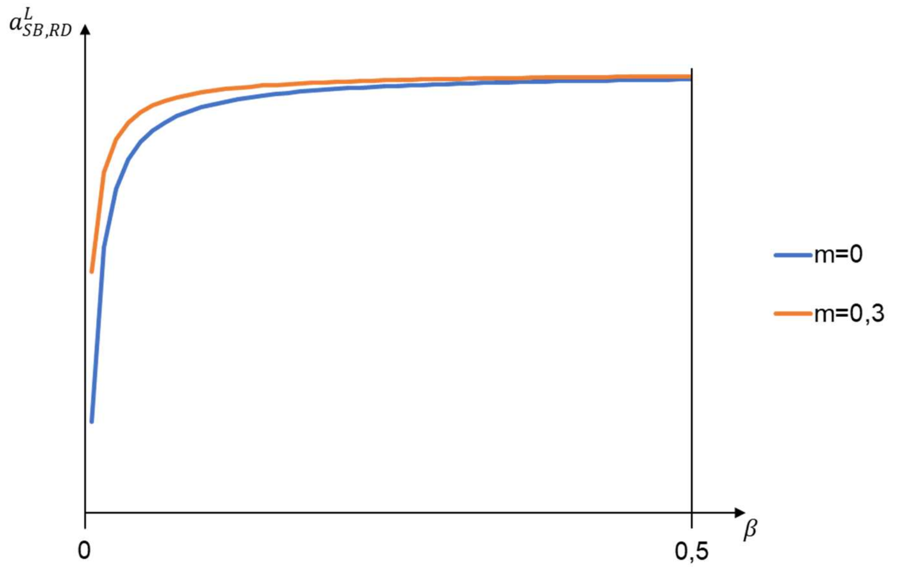

Figure I: Managerial action $a_{SB,RD}^{L}$ as function of β. Own graphic, functions calculated according to Douglas (2009), pp.173-174.[276]

As $a_{SB,RD}^{L}$ is almost constant for values of $\beta > 0{,}5$, figure I only focuses on $\beta \leq 0{,}5$.

A.7. Mean preserving variations of Δx

With the constant expected value of the investment $\pi \geq 0$, the following equation holds:

$$\sigma x^{H} + (1-\sigma)x^{L} = \pi \qquad \text{(A32)}$$

With $\Delta x = x^{H} - x^{L}$, this can be rewritten as:

$$\sigma x^{H} + (1-\sigma)(x^{H} - \Delta x) = \pi$$
$$\leftrightarrow \quad x^{H} = (1-\sigma)\Delta x + \pi \qquad \text{(A33)}$$

Calculating the derivative with respect to Δx leads to:

$$\frac{\partial x^{H}}{\partial \Delta x} = 1 - \sigma > 0 \qquad \text{(A34)}$$

With $\Delta x = x^{H} - x^{L}$, (A32) can also be written as:

$$\sigma(\Delta x + x^{L}) + (1-\sigma)x^{L} = \pi$$
$$\leftrightarrow \quad x^{L} = -\sigma \Delta x + \pi \qquad \text{(A35)}$$

[276] The values used for the calculation of the functions are: $\sigma = 0{,}5;\ \Delta x = 0{,}1;\ k = 0{,}08$.

The derivative with respect to Δx leads to:

$$\frac{\partial x^L}{\partial \Delta x} = -\sigma < 0 \tag{A36}$$

A.8. Calculation and analysis of the stakeholders' payoff functions

With the disutility function $A(a) = \frac{k}{2}a^2$, as in (1), and therefore $a^L_{SB,RD}$, as in (A29), the specific payoff function of each stakeholder can be derived, as shown below.

For the manager's utility function M, given by (23), this leads to:

$$\begin{aligned}
M(\Delta x) &= \sigma\left(u_0 + A\left(a^L_{SB,RD}(\Delta x)\right) - A\left(a^L_{SB,RD}(\Delta x) - \Delta x\right)\right) + (1-\sigma)u_0 \\
&= u_0 + \sigma\left(A\left(a^L_{SB,RD}(\Delta x)\right) - A\left(a^L_{SB,RD}(\Delta x) - \Delta x\right)\right) \\
&= u_0 + \sigma\frac{k}{2}\left(\left(\frac{1}{k} - K_2\Delta x\right)^2 - \left(\frac{1}{k} - K_2\Delta x - \Delta x\right)^2\right) \\
&= u_0 + \sigma\frac{k}{2}\left(\frac{1}{k^2} - \frac{2K_2\Delta x}{k} + K_2^2(\Delta x)^2 - \frac{1}{k^2} + \frac{2(1+K_2)\Delta x}{k} - (1+K_2)^2(\Delta x)^2\right) \\
&= u_0 + \sigma\frac{k}{2}\left(\frac{2\Delta x}{k} - (\Delta x)^2(1+2K_2)\right) \qquad \text{(A37)}
\end{aligned}$$

For the debtholders' payoff function D, given by (24), it follows:

$$\begin{aligned}
D(\Delta x) &= \sigma\mathrm{F} + (1-\sigma)\left(x^L(\Delta x) + a^L_{SB,RD}(\Delta x) - u_0 - A(a^L_{SB,RD}(\Delta x))\right) \\
&= \sigma\mathrm{F} + (1-\sigma)\left(\pi - \sigma\Delta x + \frac{1}{k} - K_2\Delta x - u_0 - \frac{k}{2}\left(\frac{1}{k} - K_2\Delta x\right)^2\right) \\
&= \sigma\mathrm{F} + (1-\sigma)\left(\pi - \sigma\Delta x - u_0 + \frac{1}{k} - K_2\Delta x - \frac{k}{2k^2} + \frac{2kK_2\Delta x}{2k} - \frac{k}{2}K_2^2(\Delta x)^2\right) \\
&= \sigma\mathrm{F} + (1-\sigma)\left(\pi - u_0 - \sigma\Delta x - K_2\Delta x + \frac{1}{k} - \frac{1}{2k} + K_2\Delta x - \frac{k}{2}K_2^2(\Delta x)^2\right) \\
&= \sigma\mathrm{F} + (1-\sigma)\left(\pi - u_0 + \frac{1}{2k}\right) + (1-\sigma)\left(-\frac{k}{2}K_2^2(\Delta x)^2 - \sigma\Delta x\right) \qquad \text{(A38)}
\end{aligned}$$

The shareholders' expected payoff, given by (25), is specified as follows:

$$S(\Delta x) = \sigma\left(x^H(\Delta x) + a^{FB} - u_0 - A(a^{FB}) - A\left(a^L_{SB,RD}(\Delta x)\right) + A\left(a^L_{SB,RD}(\Delta x) - \Delta x\right) - F\right)$$

$$= \sigma\left(\pi + (1-\sigma)\Delta x + \frac{1}{k} - u_0 - \frac{1}{2k} - \frac{k}{2}\left[\left(\frac{1}{k} - K_2\Delta x\right)^2 - \left(\frac{1}{k} - K_2\Delta x - \Delta x\right)^2\right] - F\right)$$

$$= \sigma\left(\pi + \frac{1}{2k} - u_0 - F + (1-\sigma)\Delta x\right.$$

$$\left. - \frac{k}{2}\left[\frac{1}{k^2} - \frac{2K_2\Delta x}{k} + K_2^2(\Delta x)^2 - \frac{1}{k^2} + \frac{2(1+K_2)\Delta x}{k} - (1+K_2)^2(\Delta x)^2\right]\right)$$

$$= \sigma\left(\pi + \frac{1}{2k} - u_0 - F + (1-\sigma)\Delta x - \frac{k}{2}\left[\frac{2\Delta x}{k} + (\Delta x)^2(K_2^2 - 1 - 2K_2 - K_2^2)\right]\right)$$

$$= \sigma\left(\pi + \frac{1}{2k} - u_0 - F - \sigma\Delta x + \frac{k}{2}(\Delta x)^2(1 + 2K_2)\right) \tag{A39}$$

All of the payoff functions have the form of a parabola. The functions $D(\Delta x)$ and $M(\Delta x)$ are facing downwards and have a maximum point, while $S(\Delta x)$ is facing upwards with a minimum point. For further investigation, the maximum and minimum points are derived:

$$\frac{\partial M(\Delta x)}{\partial \Delta x} = -2\sigma\frac{k}{2}(1+2K_2)\Delta x + \sigma = 0$$

$$\leftrightarrow \quad \Delta x = \frac{1}{k(1+2K_2)} \tag{A40}$$

$$\frac{\partial D(\Delta x)}{\partial \Delta x} = -(1-\sigma)kK_2^2\Delta x - \sigma(1-\sigma) = 0$$

$$\leftrightarrow \quad \Delta x = -\frac{\sigma}{kK_2^2} \tag{A41}$$

$$\frac{\partial S(\Delta x)}{\partial \Delta x} = -\sigma^2 + k(1+2K_2)\sigma\Delta x = 0$$

$$\leftrightarrow \quad \Delta x = \frac{\sigma}{k(1+2K_2)} \tag{A42}$$

A.9. Calculation of the boundaries for Δx

The maximum point of $D(\Delta x)$ lies at a value $\Delta x < 0$, as shown in (A41), so that the debtholders prefer to minimize Δx. For all values of Δx that are smaller than the minimum point of $S(\Delta x)$, given by (A42), the shareholders prefer to decrease Δx, just like the debtholders. As these two groups always have the majority of the votes on the board, any Δx below the minimum point of $S(\Delta x)$ would lead to the minimum value of Δx, which would mean that the investment and therefore the debt would be practically riskless. Hence, setting $\underline{\Delta x} = \sigma/k(1+2K_2)$ ensures that the shareholders always prefer to increase the risk and the case of a practically riskless investment and debt does not occur.[277]

The upper bound $\overline{\Delta x}$ is set, so that the assumption $a^L \geq \Delta x$, introduced in section 3.1.2, holds. With $a^L_{SB,RD}$ as in (A29), it follows:

$$\frac{1}{k} - K_2 \Delta x \geq \Delta x$$

$$\leftrightarrow \quad \Delta x \leq \frac{1}{k(1+K_2)} \tag{A43}$$

Hence, the upper bound is $\overline{\Delta x} = 1/k(1+K_2)$

A.10. Increase in the shareholders' payoff without managerial influence

As the risk $\Delta x(\beta)$ of the investment is modelled as function of β, the only term in the shareholders' payoff function $S(\Delta x)$, given by (A39), that is contingent on the managerial influence m is the term K_2. The derivative of the function $S(\Delta x)$ with respect to K_2 is calculated as follows:

$$\frac{\partial S(\Delta x)}{\partial K_2} = k(\Delta x)^2 > 0 \tag{A44}$$

K_2 can be rearranged:

$$K_2 = \frac{\sigma\big((1-m)(1-\beta)-m\big)}{(1-m)(1-\sigma)\beta}$$

$$= \frac{\sigma(1-\beta)}{(1-\sigma)\beta} - \frac{\sigma m}{(1-m)(1-\sigma)\beta} \tag{A45}$$

The derivative of K_2 with respect to m is given by:

$$\frac{\partial K_2}{\partial m} = 0 - \frac{\sigma(1-m)(1-\sigma)\beta + (1-\sigma)\beta\sigma m}{(1-m)^2(1-\sigma)^2\beta^2} = -\frac{\sigma(1-m)+\sigma m}{(1-m)^2(1-\sigma)\beta}$$

$$= -\frac{\sigma}{(1-m)^2(1-\sigma)\beta} < 0 \tag{A46}$$

As $\partial K_2/\partial m < 0$ and $\partial S(\Delta x)/\partial K_2 > 0$, it follows that $\partial S(\Delta x)/\partial m < 0$. Thus, $m = 0$ maximizes $S(\Delta x)$, so that:

$$S(\Delta x, m=0) > S(\Delta x, m) \quad \forall \; m > 0 \tag{A47}$$

[277] This is what footnote 83 in section 3.1.3.1 refers to.

A.11. Derivative of agency costs AC with respect to Δx

Firstly, consider AC^L, as in (18). As $\partial a_{SB,RD}^L/\partial\Delta x = -K_2 < 0$ and $\partial AC^L(a_{SB,RD}^L)/\partial a_{SB,RD}^L < 0$, similar to (A18), it follows that $\partial AC^L/\partial\Delta x > 0$. As per assumption, the condition $\Delta x < \Delta x^M$ holds, which means that the manager's expected payoff increases with increasing values of Δx. Hence, his information rent AC^H, as in (17), increases with Δx. With $\partial AC^L/\partial\Delta x > 0$ and $\partial AC^H/\partial\Delta x > 0$, it follows that $\partial AC/\partial\Delta x > 0$.

A.12. Proof for $\Delta x < \Delta x^M$ in both cases of control allocation

<u>*Initial manager control*</u>

Firstly, calculate the Lagrange function of the problem in (26) and (27):

$$\begin{aligned}\mathcal{L} = u_0 + \sigma\left(A\left(a_{SB,RD}^L(\Delta x(\beta),\beta,m)\right) - A\left(a_{SB,RD}^L(\Delta x(\beta),\beta,m) - \Delta x(\beta)\right)\right)\\ +\theta\left[S\left(\Delta x(\beta), a_{SB,RD}^L(\Delta x(\beta),\beta,m)\right) - S\left(\Delta x(\beta), a_{SB,RD}^L(\Delta x(\beta),\beta,m=0)\right) + R\right]\end{aligned} \tag{A48}$$

The first-order-condition with respect to m is given by:

$$\begin{aligned}\frac{\partial\mathcal{L}}{\partial m} = \sigma\left[\left(\frac{\partial\left(A(a_{SB,RD}^L) - A(a_{SB,RD}^L - \Delta x)\right)}{\partial a_{SB,RD}^L}\right)\left(\frac{\partial a_{SB,RD}^L}{\partial m}\right)\right]\\ + \theta\left[\left(\frac{\partial S(\Delta x, a_{SB,RD}^L)}{\partial a_{SB,RD}^L}\right)\left(\frac{\partial a_{SB,RD}^L}{\partial m}\right)\right] = 0\end{aligned} \tag{A49}$$

Several terms of (A49) can be specified and simplified. With $A(a)$, given by (1), it follows:

$$\frac{\partial\left(A(a_{SB,RD}^L) - A(a_{SB,RD}^L - \Delta x)\right)}{\partial a_{SB,RD}^L} = k a_{SB,RD}^L - k(a_{SB,RD}^L - \Delta x) = k\Delta x \tag{A50}$$

Consider the shareholders' payoff function, given by (25), with $w_{SB,RD}^H$, as given in (15) with $a_{SB,RD}^L$ instead of a_{SB}^L:

$$S(\Delta x, a_{SB,RD}^L) = \sigma\left(x^H(\Delta x) + a^{FB} - \left(u_0 + A(a^{FB}) + A(a_{SB,RD}^L) - A(a_{SB,RD}^L - \Delta x)\right) - F\right) \tag{A51}$$

For the derivative it follows:

$$\frac{\partial S(\Delta x, a_{SB,RD}^L)}{\partial a_{SB,RD}^L} = \sigma\frac{\partial\left(-A(a_{SB,RD}^L) + A(a_{SB,RD}^L - \Delta x)\right)}{\partial a_{SB,RD}^L} = -\sigma k\Delta x \tag{A52}$$

Inserting (A50) and (A52) into (A49) leads to:

$$\sigma k\Delta x\left(\frac{\partial a_{SB,RD}^L}{\partial m}\right) - \theta\sigma k\Delta x\left(\frac{\partial a_{SB,RD}^L}{\partial m}\right) = 0 \tag{A53}$$

With $\partial a_{SB,RD}^{L}/\partial m > 0$, as shown in (A30), it follows from (A53) that $\theta = 1$ and the constraint (27) binds.

With $\theta = 1$ the first-order-condition of $\mathcal{L}$ with respect to β is given by:[278]

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial \beta} = \sigma \Bigg[& \left(\frac{\partial \left(A\left(a_{SB,RD}^{L}\right) - A\left(a_{SB,RD}^{L} - \Delta x\right) \right)}{\partial a_{SB,RD}^{L}} \right) \left(\frac{\partial a_{SB,RD}^{L}}{\partial \beta} \right) \\ & + \left(\left(\frac{\partial \left(A\left(a_{SB,RD}^{L}\right) - A\left(a_{SB,RD}^{L} - \Delta x\right) \right)}{\partial a_{SB,RD}^{L}} \right) \left(\frac{\partial a_{SB,RD}^{L}}{\partial \Delta x} \right) \right. \\ & \left. + \frac{\partial \left(A\left(a_{SB,RD}^{L}\right) - A\left(a_{SB,RD}^{L} - \Delta x\right) \right)}{\partial \Delta x} \right) \frac{\partial \Delta x}{\partial \beta} \Bigg] \\ & + \left(\left(\frac{\partial S\left(\Delta x, a_{SB,RD}^{L}(\Delta x)\right)}{\partial \Delta x} \right) - \left(\frac{\partial S\left(\Delta x, a_{SB,RD,0}^{L}(\Delta x)\right)}{\partial \Delta x} \right) \right) \frac{\partial \Delta x}{\partial \beta} \\ & + \left(\frac{\partial S\left(\Delta x, a_{SB,RD}^{L}\right)}{\partial a_{SB,RD}^{L}} \right) \left(\frac{\partial a_{SB,RD}^{L}}{\partial \beta} \right) - \left(\frac{\partial S\left(\Delta x, a_{SB,RD,0}^{L}\right)}{\partial a_{SB,RD,0}^{L}} \right) \left(\frac{\partial a_{SB,RD,0}^{L}}{\partial \beta} \right) = 0\end{aligned} \tag{A54}$$

With $A(a)$, given by (1), and $a_{SB,RD}^{L}$, given by (A29), S can be specified as in (A39). This leads to the following derivative, also used in (A42):

$$\frac{\partial S\left(\Delta x, a_{SB,RD}^{L}(\Delta x)\right)}{\partial \Delta x} = -\sigma^2 + k(1 + 2K_2)\sigma \Delta x \tag{A55}$$

With (A50), (A52) and (A55) the first order-condition (A54) can be simplified as follows:[279]

$$\begin{aligned}& \sigma \left[k\Delta x \left(\frac{\partial a_{SB,RD}^{L}}{\partial \beta} \right) + \left(k\Delta x \left(\frac{\partial a_{SB,RD}^{L}}{\partial \Delta x} \right) + A'\left(a_{SB,RD}^{L} - \Delta x\right) \right) \frac{\partial \Delta x}{\partial \beta} \right] \\ & \quad + \left[-\sigma^2 + k(1 + 2K_2)\sigma \Delta x - \left(-\sigma^2 + k\left(1 + 2K_{2,m=0}\right)\sigma \Delta x \right) \right] \frac{\partial \Delta x}{\partial \beta} \\ & \quad - \sigma k \Delta x \left(\frac{\partial a_{SB,RD}^{L}}{\partial \beta} \right) + \sigma k \Delta x \left(\frac{\partial a_{SB,RD,0}^{L}}{\partial \beta} \right) = 0 \\ \leftrightarrow \quad & \sigma \left[\left(k\Delta x \left(\frac{\partial a_{SB,RD}^{L}}{\partial \Delta x} \right) + A'\left(a_{SB,RD}^{L} - \Delta x\right) \right) \frac{\partial \Delta x}{\partial \beta} \right] + \left[2\sigma k \Delta x \left(K_2 - K_{2,m=0} \right) \right] \frac{\partial \Delta x}{\partial \beta} \\ & \quad + \sigma k \Delta x \left(\frac{\partial a_{SB,RD,0}^{L}}{\partial \beta} \right) = 0\end{aligned} \tag{A56}$$

[278] Note that $a_{SB,RD,0}^{L}$ represents the case with $m = 0$.
[279] Note that $K_{2,m=0}$ represents the case with $m = 0$.

The managerial utility function M is given in (23) and AC^H is given in (17). Thus, with (A50) it follows:

$$\sigma\left[\left(\frac{\partial M}{\partial \Delta x}\right)\left(\frac{\partial \Delta x}{\partial \beta}\right)\right] + \left[2\sigma k \Delta x\left(K_2 - K_{2,m=0}\right)\right]\frac{\partial \Delta x}{\partial \beta} + \sigma k \Delta x\left(\frac{\partial a_{SB,RD,0}^L}{\partial \beta}\right) = 0 \tag{A57}$$

Note that $\partial a_{SB,RD}^L/\partial \beta > 0$, as shown in Appendix A.5, so that $\partial a_{SB,RD,0}^L/\partial \beta > 0$ holds. Moreover, $\partial \Delta x/\partial \beta < 0$ holds as per assumption. With $\partial K_2/\partial m < 0$, as shown in (A46), it follows that $\left(K_2 - K_{2,m=0}\right) \leq 0$. Hence, the second and third term on the left side of (A57) are positive, so that the first one needs to be negative, which leads to $\partial M/\partial \Delta x > 0$. Considering the function of M, displayed in figure 2, this means that $\Delta x < \Delta x^M$.

Initial board control

Firstly, calculate the Lagrange function of the problem in (28) and (29), expressed as a maximization problem, including the constraint that $m \geq 0$:

$$\mathcal{L} = -AC\left(a_{SB,RD}^L(\Delta x(\beta), \beta, m), \Delta x(\beta)\right) + \theta_1 m - \theta_2(\beta - \overline{\beta}) \tag{A58}$$

The first-order-conditions with respect to m and β are given by the following:

$$\frac{\partial \mathcal{L}}{\partial m} = -\left(\frac{\partial AC}{\partial a_{SB,RD}^L}\right)\left(\frac{\partial a_{SB,RD}^L}{\partial m}\right) + \theta_1 = 0 \tag{A59}$$

$$\frac{\partial \mathcal{L}}{\partial \beta} = -\left(\frac{\partial AC}{\partial a_{SB,RD}^L}\right)\left(\left(\frac{\partial a_{SB,RD}^L}{\partial \beta}\right) + \left(\frac{\partial a_{SB,RD}^L}{\partial \Delta x}\right)\left(\frac{\partial \Delta x}{\partial \beta}\right)\right) - \left(\frac{\partial AC}{\partial \Delta x}\right)\left(\frac{\partial \Delta x}{\partial \beta}\right) - \theta_2 = 0 \tag{A60}$$

Given AC from (19) and AC^H from (17) with $a_{SB,RD}^L$ instead of a_{SB}^L, the following derivative can be calculated:[280]

$$\frac{\partial AC}{\partial \Delta x} = \sigma\frac{\partial AC^H}{\partial \Delta x} = \sigma A'(a_{SB,RD}^L - \Delta x) \tag{A61}$$

Inserting this into (A60) leads to:

$$\left(\frac{\partial AC}{\partial a_{SB,RD}^L}\right)\left(\underbrace{\left(\frac{\partial a_{SB,RD}^L}{\partial \beta}\right)}_{>0} + \underbrace{\left(\frac{\partial a_{SB,RD}^L}{\partial \Delta x}\right)}_{<0}\underbrace{\left(\frac{\partial \Delta x}{\partial \beta}\right)}_{<0}\right) + \sigma\underbrace{A'\left(a_{SB,RD}^L - \Delta x\right)}_{>0}\underbrace{\left(\frac{\partial \Delta x}{\partial \beta}\right)}_{<0} + \theta_2 = 0 \tag{A62}$$

If the constraint for m binds, it follows that $\theta_1 > 0$. With $\partial a_{SB,RD}^L/\partial m > 0$, as shown in A.5, this leads to $\partial AC/\partial a_{SB,RD}^L > 0$, so that (A59) holds. This means that the first term in (A.62) is positive, while the second one is negative. As Douglas (2009) states that the effect of Δx on

[280] Note that (A61) is the derivative with respect to Δx, whereby $a_{SB,RD}^L$ is considered as a variable and not as a function of Δx.

the agency costs is greater than the effect of $a^L_{SB,RD}$, the modulus of the second term in (A62) is greater than the one of the first, so that the addition of the two first terms is negative. Hence, $\theta_2 > 0$ and the constraint for β binds.

If the constraint for m does not bind, it follows that $\theta_1 = 0$ and therefore $\partial AC/\partial a^L_{SB,RD} = 0$. Thus, the first term in (A62) is eliminated, while the second term is negative. This means that $\theta_2 > 0$ and the constraint for β binds.

Overall, the condition $\beta = \overline{\beta}$ holds, so that $\Delta x = \underline{\Delta x} < \Delta x^M$.

Appendix for the model from Berkovitch and Israel (1996)

A.13. Replacement policy preferred by the manager

The first-order-condition of (30) with respect to y is calculated as follows:

$$\frac{\partial M}{\partial y} = -\omega \frac{\partial G(y|a)}{\partial y} = 0$$
$$\leftrightarrow \quad -\omega g(y|a) = 0$$
$$\leftrightarrow \quad y = 0 = y^M \tag{A63}$$

A.14. Replacement policy preferred by the shareholders

With $y(P) = y^S > F$, equation (31) is solved as follows:

$$y^S - F = \int_F^\infty (y - F)h(y)dy$$
$$\leftrightarrow \quad y^S - F = \int_0^\infty (y - F)h(y)dy - \int_0^F (y - F)h(y)dy$$
$$\leftrightarrow \quad y^S - F = \int_0^\infty yh(y)dy - \int_0^\infty Fh(y)dy - \int_0^F (y - F)h(y)dy$$
$$\leftrightarrow \quad y^S - F = \overline{y} - [FH(y)]_0^\infty - \int_0^F (y - F)h(y)dy$$
$$\leftrightarrow \quad y^S - F = \overline{y} - F + \int_0^F (F - y)h(y)dy$$
$$\leftrightarrow \quad y^S(F) = \overline{y} + \int_0^F (F - y)h(y)dy \tag{A64}$$

A.15. Characteristics of the critical value y^S

The only term in $y^S(F)$, given by (32), that is contingent on F is the integral, which is simplified via partial integration:

$$\int_0^F (F - y)h(y)dy = [(F - y)H(y)]_0^F - \int_0^F (-1) * H(y)dy = \int_0^F H(y)dy \tag{A65}$$

Without determining the derivative of (A65) with respect to F, it can already be seen that an increasing F raises the upper bound of the integral and therefore its total value, as the integrand $H(y)$ is positive. Hence, $y^S(F)$ increases in F, with the following values for $F = 0$ and for $F \to \infty$:

$$y^S(0) = \overline{y} + \int_0^0 (0 - y)h(y)dy = \overline{y} + 0 = \overline{y} \quad \text{(A66)}$$

$$\lim_{F\to\infty} y^S(F) = \overline{y} + \lim_{F\to\infty} \int_0^F (F - y)h(y)dy = \overline{y} + \int_0^\infty \lim_{F\to\infty} (F - y)h(y)\,dy = \infty \quad \text{(A67)}$$

A.16. Replacement policy preferred by the debtholders with absolute control

Assume that $y(P) < F$. While the debtholders receive $y(P)$ if the manager stays on the job, their payoff in case of the liquidation of the firm is:

$$D = \min\{F, \overline{y}\} \quad \text{(A68)}$$

For $\overline{y} > y(P)$ it follows that $D > y(P)$ and the debtholders prefer the liquidation of the firm.

If $y(P) > F$ holds, the debtholders receive F as long as the manager stays on the job. In case of $\overline{y} > y(P)$, the liquidation of the firm also leads to a payoff of F, so that the debtholders are indifferent between the two options. When being indifferent, they are assumed to always choose the liquidation of the firm.

Overall, the debtholders wish to liquidate the firm if $\overline{y} > y(P)$.

A.17. Replacement policy preferred by the debtholders with partial control

With $y(P) = y^D > F$ equation (33) is solved in the following:

$$y^D = \int_0^F yh(y)dy + \int_F^\infty Fh(y)dy$$

$$\leftrightarrow \quad y^D = \int_0^\infty yh(y)dy - \int_F^\infty yh(y)dy + \int_F^\infty Fh(y)dy$$

$$\leftrightarrow \quad y^D(F) = \overline{y} - \int_F^\infty (y - F)h(y)dy \quad \text{(A69)}$$

A.18. Characteristics of the critical value y^D

Only the integral in $y^D(F)$, given by (34), is contingent on F. Using (A65), it can be rewritten as follows:[281]

$$\begin{aligned}\int_F^{\infty}(y-F)h(y)dy &= \int_0^{\infty}(y-F)h(y)dy - \int_0^F (y-F)h(y)dy \\ &= \overline{y} - F[H(y)]_0^{\infty} + \int_0^F (F-y)h(y)dy \\ &= \overline{y} - F + \int_0^F H(y)dy \\ &= \overline{y} - F + \left[H^{int}(y)\right]_0^F \\ &= \overline{y} - F + H^{int}(F) - H^{int}(0) \end{aligned} \quad \text{(A70)}$$

The derivative with respect to F is calculated by:

$$\frac{\partial}{\partial F}\left[\int_F^{\infty}(y-F)h(y)dy\right] = -1 + H(F) \quad \text{(A71)}$$

As $H(F) < 1$, it follows that the integral decreases in F. Because it is subtracted in the function of $y^D(F)$, the overall function increases in F, with the following values for $F = 0$ and for $F \to \infty$:

$$y^D(0) = \overline{y} - \int_0^{\infty}(y-0)h(y)dy = \overline{y} - \overline{y} = 0 \quad \text{(A72)}$$

$$\lim_{F\to\infty} y^D(F) = \overline{y} - \lim_{F\to\infty}\int_F^{\infty}(y-F)h(y)dy = \overline{y} - \int_{\infty}^{\infty}\lim_{F\to\infty}(y-F)h(y)dy = \overline{y} - 0 = \overline{y} \quad \text{(A73)}$$

A.19. Unique interior solution for problem of managerial effort choice

The assumptions $G_a < 0$ and $G_{aa} > 0$ lead to $-\omega G_a(y^C|a)$ being a strictly decreasing function, which is positive for all values of a. As per assumption $A(a)$ is increasing and convex, so that $A'(a) > 0$ increases strictly, as $A''(a) > 0$. Hence, the functions $-\omega G_a(y^C|a)$ and $kA'(a^*)$ have exactly one intersection at a^*, representing the unique solution of equation (36).

A.20. Changes in the managerial effort choice due to changes in y^C

Generally, the total differential of a function f with respect to y^C and a^* is calculated as follows:

$$df = \frac{\partial f}{\partial y^c}dy^c + \frac{\partial f}{\partial a^*}da^* \quad \text{(A74)}$$

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[281] Note that H^{int} represents the indefinite integral of H with respect to y.

Using (A74) on the first-order-condition, given in (36), leads to:

$$-\omega g_a(y^C|a^*)dy^C - \omega G_{aa}(y^C|a^*)da^* = \tau A''(a^*)da^*$$
$$\leftrightarrow \quad \omega g_a(y^C|a^*)dy^C = \left(-\omega G_{aa}(y^C|a^*) - \tau A''(a^*)\right)da^*$$
$$\leftrightarrow \quad \frac{da^*}{dy^C} = \frac{\omega g_a(y^C|a^*)}{\left(-\omega G_{aa}(y^C|a^*) - \tau A''(a^*)\right)} \tag{A75}$$

As Berkovitch and Israel (1996) assume that $G_{aa} > 0$ and $A'' > 0$, the denominator of the above expression is negative. This leads to the following results:

$$\frac{da^*}{dy^C} < 0, \quad if\ g_a(y^C|a^*) > 0 \tag{A76}$$

$$\frac{da^*}{dy^C} > 0, \quad if\ g_a(y^C|a^*) < 0 \tag{A77}$$

A.21. Optimal replacement policy $[\boldsymbol{y}^*]$

The first-order-condition of χ, given by (37), with respect to the critical value y^C is calculated as follows:

$$\frac{\partial\chi}{\partial y^C} = \frac{\partial}{\partial y^C}\left[\int_{y^C}^{\infty} yg(y|a^*)dy\right] + \bar{y}g(y^C|a^*) + \frac{\partial a^*}{\partial y^C}\left[\int_{y^C}^{\infty} yg_a(y|a^*)dy + \bar{y}G_a(y^C|a^*)\right] \tag{A78}$$

Simplify the first term by using partial integration:[282]

$$\frac{\partial}{\partial y^C}\left[\int_{y^C}^{\infty} yg(y|a^*)dy\right] = \frac{\partial}{\partial y^C}\left[[yG(y|a^*)]_{y^C}^{\infty} - \int_{y^C}^{\infty} G(y|a^*)dy\right]$$
$$= \frac{\partial}{\partial y^C}\left[\lim_{y\to\infty} G(y|a^*) - y^C G(y^C|a^*) - \int_{y^C}^{\infty} G(y|a^*)dy\right]$$
$$= \frac{\partial}{\partial y^C}\left[\lim_{y\to\infty} G(y|a^*) - y^C G(y^C|a^*) - [G^{Int}(y|a^*)]_{y^C}^{\infty}\right]$$
$$= \frac{\partial}{\partial y^C}\left[\lim_{y\to\infty} G(y|a^*) - y^C G(y^C|a^*) - \lim_{y\to\infty} G^{Int}(y|a^*) + G^{Int}(y^C|a^*)\right]$$
$$= -G(y^C|a^*) - y^C g(y^C|a^*) + G(y^C|a^*) = -y^C g(y^C|a^*) \tag{A79}$$

This leads to:

$$\frac{\partial\chi}{\partial y^C} = -y^C g(y^C|a^*) + \bar{y}g(y^C|a^*) + \frac{\partial a^*}{\partial y^C}\left[\int_{y^C}^{\infty} yg_a(y|a^*)dy + \bar{y}G_a(y^C|a^*)\right] \tag{A80}$$

[282] Note that G^{Int} is the indefinite integral of G.

In the following, focus on the point $y^C = \overline{y}$:

$$\frac{\partial \chi}{\partial y^C} = -\overline{y}g(\overline{y}|a^*) + \overline{y}g(\overline{y}|a^*) + \frac{\partial a^*}{\partial y^C}\left[\int_{\overline{y}}^{\infty} y g_a(y|a^*)dy + \overline{y}G_a(\overline{y}|a^*)\right]$$

$$= \frac{\partial a^*}{\partial y^C}\left[\int_{\overline{y}}^{\infty} y g_a(y|a^*)dy + \overline{y}G_a(\overline{y}|a^*)\right] \quad \text{(A81)}$$

Partial integration and the condition $\lim_{y\to\infty} yG_a(y|a) = 0$ lead to:

$$\frac{\partial \chi}{\partial y^C} = \frac{\partial a^*}{\partial y^C}\left[[yG_a(y|a^*)]_{\overline{y}}^{\infty} - \int_{\overline{y}}^{\infty} G_a(y|a^*)dy + \overline{y}G_a(\overline{y}|a^*)\right]$$

$$= \frac{\partial a^*}{\partial y^C}\left[0 - \overline{y}G_a(\overline{y}|a^*) - \int_{\overline{y}}^{\infty} G_a(y|a^*)dy + \overline{y}G_a(\overline{y}|a^*)\right]$$

$$= \frac{\partial a^*}{\partial y^C}\left[-\int_{\overline{y}}^{\infty} G_a(y|a^*)dy\right] \quad \text{(A82)}$$

As $G_a < 0$, the integral is positive.

If $g_a(\overline{y}) < 0$ and therefore $\partial a^*/\partial y^C > 0$, as shown in (A77), it follows that $\partial \chi/\partial y^C > 0$. With $\lim_{y^C\to\infty} \partial \chi/\partial y^C = -\infty$, there must be an optimal $y^* > \overline{y}$ that satisfies the first-order-condition $\partial \chi/\partial y^C = 0$. The monotone likelihood ratio property states that $g_a(y|a)/g(y|a)$ increases in y, so that $g_a(y^C) < 0$ for all $y^C < \overline{y}$, as the conditions $g_a(\overline{y}) < 0$ and $g(y|a) > 0$ hold. Following from this, $\partial \chi/\partial y^C > 0$ for all $y^C < \overline{y}$ and therefore $\chi(\overline{y}) > \chi(y^C)$ for all $y^C < \overline{y}$. Hence, the optimal solution is not $y^* < \overline{y}$, but can only be $y^* > \overline{y}$.

If $g_a(\overline{y}) > 0$, it follows that $\partial a^*/\partial y^C < 0$, as shown in (A76), which leads to $\partial \chi/\partial y^C < 0$. With $\lim_{y^C\to 0} \partial \chi/\partial y^C = \infty$, there exists a $y^* < \overline{y}$, for which the first-order-condition $\partial \chi/\partial y^C = 0$ holds. The monotone likelihood ratio property states that $g_a(y|a)/g(y|a)$ is increasing in y. As $g_a(\overline{y}) > 0$ and also $g(y|a) > 0$ generally holds, it follows that $g_a(y^C) > 0$ for all $y^C > \overline{y}$. Thus, $\partial \chi/\partial y^C < 0$ holds for all $y^C > \overline{y}$, so that $\chi(\overline{y}) > \chi(y^C)$ for all $y^C > \overline{y}$. This means that the optimal solution can only be $y^* < \overline{y}$ and not $y^* > \overline{y}$.

Overall, the optimal solution is $y^* > \overline{y}$ for $g_a(\overline{y}) < 0$ and $y^* < \overline{y}$ for $g_a(\overline{y}) > 0$.

Appendix for the model from John and John (1993)

A.22. Expected value of the investment

The manager invests in the safe project if $q \leq \overline{q}$ and in the risky project otherwise. With q being uniformly distributed over $[0,1]$, $V(\overline{q})$ is calculated by:

$$\begin{aligned} V(\overline{q}) &= -I + Prob(q \leq \overline{q})I + (1 - Prob(q \leq \overline{q}))[qH + (1-q)L] \\ &= -I + \left(\int_0^{\overline{q}} \frac{1}{1-0} dq\right) I + \left(1 - \int_0^{\overline{q}} \frac{1}{1-0} dq\right)[qH + (1-q)L] \\ &= -I + \left(\frac{\overline{q}-0}{1-0}\right) I + \left(1 - \frac{\overline{q}-0}{1-0}\right)[qH + (1-q)L] \\ &= -I + \overline{q}I + (1-\overline{q})[qH + (1-q)L] \end{aligned} \tag{A83}$$

As q is uniformly distributed over $[0,1]$, it follows that $q = (\overline{q}+1)/2$ and therefore:

$$\begin{aligned} V(\overline{q}) &= -I + \overline{q}I + (1-\overline{q})\left[\frac{\overline{q}+1}{2}H + \left(1 - \frac{\overline{q}+1}{2}\right)L\right] \\ &= -I + \overline{q}I + \frac{(1-\overline{q}^2)}{2}H + \frac{(1-\overline{q})^2}{2}L \end{aligned} \tag{A84}$$

A.23. Optimal investment policy

The first-order-condition of $V(\overline{q})$, given by (40), with respect to $\overline{q}$ is:

$$\begin{aligned} \frac{\partial V(\overline{q})}{\partial \overline{q}} &= I - \overline{q}H - (1-\overline{q})L = 0 \\ \leftrightarrow \quad \overline{q}(H-L) &= I - L \\ \leftrightarrow \quad \overline{q} &= \frac{I-L}{H-L} = \hat{q} \end{aligned} \tag{A85}$$

A.24. Investment policy of equity-aligned manager in all-equity firm

The manager chooses the risky project if his expected payoff for this project is higher than for the safe project. With the compensation scheme as in (42), the following inequation holds:

$$\begin{aligned} \varepsilon[qH + (1-q)L] &> \varepsilon I \\ \leftrightarrow \quad qH + (1-q)L &> I \end{aligned} \tag{A86}$$

For the cutoff level $\overline{q}$, (A86) needs to hold as equation:

$$\overline{q}H + (1-\overline{q})L = I$$
$$\leftrightarrow \quad \overline{q} = \frac{I-L}{H-L} = \hat{q} \tag{A87}$$

A.25. Pareto-Optimum

The condition $E[\mu] = w_0$ always holds, as the participation constraint binds. Thus, the manager is not affected by the pareto-optimum, as his expected payoff equals w_0 in any case.

In an all-equity firm the shareholders' expected payoff is given by the following:

$$S(\overline{q}) = V(\overline{q}) - w_0 = -I + \overline{q}I + \frac{\left(1-\overline{q}^2\right)}{2}H + \frac{(1-\overline{q})^2}{2}L - w_0 \tag{A88}$$

The first-order-condition with respect to $\overline{q}$ is the same as in Appendix A.23 and therefore leads to $\hat{q}$, as in (A85). Hence, $\hat{q}$ does not only maximize $V(\overline{q})$ but also $S(\overline{q})$. While $E[\mu] = w_0$, maximizing $S(\overline{q})$ means obtaining the pareto-optimum.

The above derivation continues to hold if debt of the face value $F > 0$ is introduced to the problem. The debtholders break even because they set the bond value B equal to their expected payoff D in $t = 2$. The shareholders' expected payoff in $t = 2$ is then calculated as follows:

$$S(\overline{q}) = V(\overline{q}) + B - D - w_0 \tag{A89}$$

With $B = D$ the direct impact of the debtholders can be eliminated, which leads to $S(\overline{q})$, as in (A88), and therefore to the same results as above.

A.26. Investment policies $[\overline{q}]$ with $\overline{q} \neq \hat{q}$

As shown in Appendix A.25, the policy that maximizes $V(\overline{q})$, as in (40), also represents the pareto-optimum. $V(\overline{q})$ can be rearranged as follows:

$$V(\overline{q}) = -I + \overline{q}I + \frac{H}{2} - \frac{H}{2}\overline{q}^2 + \frac{L}{2} - L\overline{q} + \frac{L}{2}\overline{q}^2$$
$$= \frac{\overline{q}^2}{2}[L-H] + \overline{q}[I-L] + \frac{1}{2}[H+L] - I \tag{A90}$$

With $L - H < 0$ the function has the form of a downward facing parabola with the maximum point at $\hat{q}$, so that all points $\overline{q} \neq \hat{q}$ lead to lower values of $V(\overline{q})$ and cannot be optimal.

Considering the function for $V(\overline{q})$ as in (40), if $\overline{q}$ increases, the factor multiplied with the safe return I increases, while the factors multiplied with the risky returns L and H decrease, so that the return of the investment and therefore the final cash flow is less risky. Analogously, if $\overline{q}$ decreases, the risk of the cash flow increases.

A.27. Investment policy of equity-aligned manager with debt outstanding

Firstly, note that the shareholders receive the constant proportion $(1-\varepsilon)$ of the residual, while the manager receives the proportion ε. Hence, both of them make the decision based on the residual payoff, as displayed by inequation (45).

In case of $\boldsymbol{F} \leq \boldsymbol{L}$, inequation (45) becomes:

$$q(H-F)+(1-q)(L-F) > I-F \tag{A91}$$

F can be eliminated, so that (A91) becomes (43). As shown in Appendix A.23, this leads to $\overline{q} = \hat{q}$, as in (41).

If $\boldsymbol{L} < \boldsymbol{F} < \boldsymbol{I}$, inequation (45) becomes:

$$q(H-F)+(1-q)*0 > I-F \tag{A92}$$

For the cutoff level $\overline{q}$ it follows:

$$\begin{aligned} \overline{q}(H-F) &= I-F \\ \leftrightarrow \quad \overline{q} &= \frac{I-F}{H-F} \end{aligned} \tag{A93}$$

If $\boldsymbol{F} \geq \boldsymbol{I}$, inequation (45) becomes:

$$q\, max\{H-F, 0\} + (1-q)*0 > 0 \tag{A94}$$

For $F \geq H$, it follows:

$$q*0+(1-q)*0 > 0 \tag{A95}$$

There is no solution to the inequation (A95) and it does not make sense for the firm to issue debt in order to make the investment, as there is no residual left in any case.

For $F < H$, it follows:

$$q\,(H-F)+(1-q)*0 > 0 \tag{A96}$$

This leads to the cutoff level $\overline{q}$:

$$\begin{aligned} \overline{q}(H-F) &= 0 \\ \leftrightarrow \quad \overline{q} &= 0 \end{aligned} \tag{A97}$$

Thus, the manager always chooses the risky project.

A.28. Analysis of the cutoff level $\overline{q} = \frac{I-F}{H-F}$

The cutoff level $\overline{q} = \frac{I-F}{H-F}$ can also be written as:

$$\overline{q} = \frac{I-F}{H-F} = \frac{I-L-(F-L)}{H-L-(F-L)} \tag{A98}$$

With $\hat{q} = \frac{I-L}{H-L} < 1$, as given in (41), subtracting the same term $(F-L)$ from the numerator and denominator decreases the fraction, so that $\overline{q} < \hat{q}$.

Also, $\overline{q}$ decreases in F:

$$\frac{\partial \overline{q}}{\partial F} = \frac{-(H-F)+(I-F)}{(H-F)^2} = \frac{I-H}{(H-F)^2} < 0 \tag{A99}$$

A.29. Agency costs in case of equity-aligned manager with risky debt

For $F < L$, the optimal policy with $\hat{q}$, given by (41), is implemented and no agency costs occur. The agency costs for $F > L$ are calculated as follows:

$$\begin{aligned} AC &= V(\hat{q}) - V(\overline{q}(F)) \\ &= -I + \hat{q}I + \frac{(1-\hat{q}^2)}{2}H + \frac{(1-\hat{q})^2}{2}L \\ &\qquad - \left(-I + \overline{q}(F)I + \frac{(1-\overline{q}(F)^2)}{2}H + \frac{(1-\overline{q}(F))^2}{2}L\right) \\ &= I(\hat{q} - \overline{q}(F)) + \frac{H}{2}(\overline{q}(F)^2 - \hat{q}^2) + \frac{L}{2}\left((1-\hat{q})^2 - (1-\overline{q}(F))^2\right) \end{aligned} \tag{A100}$$

If $L < F < I$, the agency costs are a quadratic function in F, as $\overline{q}(F)$ is calculated by (A98).

For $F > I$, the equation (A100) can be simplified:

$$AC = V(\hat{q}) - V(0) = -I + \hat{q}I + \frac{(1-\hat{q}^2)}{2}H + \frac{(1-\hat{q})^2}{2}L - \left(-I + \frac{1}{2}H + \frac{1}{2}L\right) \tag{A101}$$

Thus, the agency costs are not contingent on F but stay constant.

Figure II shows the agency costs AC, given by (A100), graphically as a function of F.

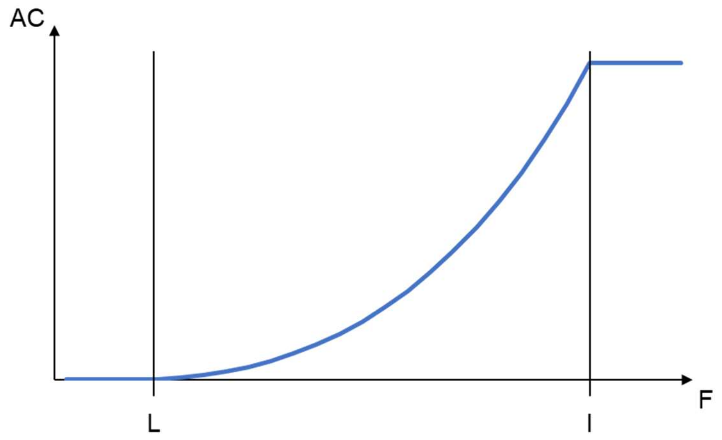

Figure II: Agency costs AC with an equity-aligned manager as function of F. Own graphic, calculations made according to John and John (1993), pp.957-960.[283]

A.30. Investment policy induced by incentive contract with risky debt

For the determination of the cutoff level $\overline{q}$, (51) needs to hold as equation, which leads to:

$$\overline{q}\big(W + \alpha(H - F)\big) + (1 - \overline{q})(W - \vartheta) = W + \alpha(I - F)$$

$$\leftrightarrow \quad \overline{q}[\alpha(H - F) + \vartheta] = \alpha(I - F) + \vartheta$$

$$\leftrightarrow \quad \overline{q} = \frac{\alpha(I - F) + \vartheta}{\alpha(H - F) + \vartheta} = \frac{I - F + \frac{\vartheta}{\alpha}}{H - F + \frac{\vartheta}{\alpha}} = \overline{q}_m \tag{A102}$$

A.31. Optimal value for α

Given (41) and (52), setting $\overline{q}_m = \hat{q}$ leads to:

$$\overline{q}_m = \frac{I - F + \frac{\vartheta}{\alpha}}{H - F + \frac{\vartheta}{\alpha}} = \frac{I - L}{H - L} = \hat{q}$$

$$\leftrightarrow \quad -F + \frac{\vartheta}{\alpha} = -L$$

$$\leftrightarrow \quad \alpha = \frac{\vartheta}{F - L} = \hat{\alpha} \tag{A103}$$

The derivative of $\hat{\alpha}$ with respect to F is calculated as follows:

$$\frac{\partial \hat{\alpha}}{\partial F} = \frac{-\vartheta}{(F - L)^2} < 0 \tag{A104}$$

283 The values used to calculated the function are: $L = 1$; $I = 2$; $H = 5$.

A.32. Investment policy with $\alpha \neq \hat{\alpha}$

The derivative of $\overline{q}_m$, as in (52), with respect to α is calculated as follows:

$$\frac{\partial \overline{q}_m}{\partial \alpha} = \frac{(I-F)(\alpha(H-F)+\vartheta)-(H-F)(\alpha(I-F)+\vartheta)}{(\alpha(H-F)+\vartheta)^2} = \frac{\vartheta[(I-F)-(H-F)]}{(\alpha(H-F)+\vartheta)^2}$$

$$= \frac{\vartheta(I-H)}{(\alpha(H-F)+\vartheta)^2} < 0 \quad \text{(A105)}$$

Thus, a higher α leads to a lower $\overline{q}_m$ and therefore to a riskier final cash flow, while a lower α leads to a safer cash flow, similar to the explication in Appendix A.26.

A.33. Alternative compensation structure

Assume that W is paid out of the return of the investment and is senior to F, whereas the remaining structure of the compensation scheme stays the same. With $L - W < F < I - W$, the manager chooses the risky project if:

$$W + \alpha[q(H-W-F)] - (1-q)\vartheta > W + \alpha(I-W-F) \quad \text{(A106)}$$

The cutoff level is calculated as follows:

$$\alpha[\overline{q}(H-W-F)] - (1-\overline{q})\vartheta = \alpha(I-W-F)$$

$$\leftrightarrow \quad \overline{q}[\alpha(H-W-F)] + \vartheta = \alpha(I-W-F) + \vartheta$$

$$\leftrightarrow \quad \overline{q} = \frac{I-W-F+\frac{\vartheta}{\alpha}}{H-W-F+\frac{\vartheta}{\alpha}} \quad \text{(A107)}$$

The goal is to set $\overline{q} = \hat{q}$, with $\hat{q}$ given by (41):

$$\frac{I-W-F+\frac{\vartheta}{\alpha}}{H-W-F+\frac{\vartheta}{\alpha}} = \frac{I-L}{H-L}$$

$$\leftrightarrow \quad -W-F+\frac{\vartheta}{\alpha} = -L$$

$$\leftrightarrow \quad \alpha = \frac{\vartheta}{W-L+F} \quad \text{(A108)}$$

Thus, the values of W and ϑ are set, so that the manager’s participation constraint binds and α is set according to (A108) in order to induce the ideal investment policy $[\hat{q}]$.

A.34. Calculation of $\overline{q}_{m,alt}$

With the performance-related part of the wage payment being senior to the debt payment and for $(1-\alpha)L < F < (1-\alpha)I$, the manager chooses the risky project if:

$$W + q\alpha H + (1-q)(\alpha L - \vartheta) > W + \alpha I \quad \text{(A109)}$$

For the cutoff level, (A109) needs to hold as equation, which leads to:

$$\overline{q}\alpha H + (1-\overline{q})(\alpha L - \vartheta) = \alpha I$$

$$\leftrightarrow \quad \overline{q}(\alpha H - \alpha L + \vartheta) = \alpha I - \alpha L + \vartheta$$

$$\leftrightarrow \quad \overline{q} = \frac{\alpha I - \alpha L + \vartheta}{\alpha H - \alpha L + \vartheta} = \frac{I - L + \frac{\vartheta}{\alpha}}{H - L + \frac{\vartheta}{\alpha}} = \overline{q}_{m,alt} \quad \text{(A110)}$$

Appendix for the model derived in section 4

A.35. The shareholders' incentive to lie to the debtholders

In the following, I, H and the compensation scheme μ are reported by the shareholders. They can report any value I or H that is higher than F without the debtholders noticing, as the debtholders only receive F in these cases and cannot deduce the actual return of the investment from that. The debtholders are only able to recognize lies concerning the value of L if the return is actually low, as this equates to their payoff. This is why the firm never lies about L in order to avoid the contractual punishment in form of the liquidation.[284]

The debtholders know that their expected payoff $D_1(\overline{q})$ is given by (57). With the information about the general structure of the investment, they can predict that the investment policy $[\hat{q}]$, given by (41), is preferred by the shareholders, as derived in Appendix A.25, and therefore implemented by the firm. Thus, their expected payoff becomes $D_1(\hat{q})$, which is known by the shareholders. The derivative with respect to $\hat{q}$ is calculated as follows:

$$\frac{\partial D_1(\hat{q})}{\partial \hat{q}} = F - \hat{q}F - (1-\hat{q})L = (1-\hat{q})(F-L) > 0 \qquad \text{(A111)}$$

Hence, $D_1(\hat{q})$ and therefore the bond value B_1 increase in $\hat{q}$. For any $\hat{q}_r > \hat{q}$, reported by the shareholders, it follows that $B_1 = D_1(\hat{q}_r) > D_1(\hat{q})$, so that the shareholders gain a profit at the cost of the debtholders without them noticing, as they cannot deduce the actual value of $\hat{q}$ from the realized payoff. Hence, the shareholders report the wrong values of I and H and the according compensation scheme μ to support the highest possible $\hat{q}_r$, so that the information cannot be trusted.

A.36. Worst case for debtholders

The debtholders expect to receive the payoff $D_1(\overline{q})$, given by (57). The worst case for them occurs if $D_1(\overline{q})$ is minimized. For further investigation, calculate the following derivative:

$$\frac{\partial D_1(\overline{q})}{\partial \overline{q}} = F - \overline{q}F - (1-\overline{q})L = (1-\hat{q})(F-L) > 0 \qquad \text{(A112)}$$

Note that this is similar to the derivative in (A111) but with $\overline{q}$ instead of $\hat{q}$.[285] It follows that $D_1(\overline{q})$ is minimized for the minimum value of $\overline{q}$, which is zero. Thus, the worst case occurs if $\overline{q} = 0$.

[284] Also see footnote 199 for this.
[285] The cutoff level $\overline{q}$ is used, because the debtholders do not have any information and cannot predict the cutoff level $\hat{q}$ preferred by the shareholders.

A.37. Calculation of $\Delta B(\overline{q})$

With $D_1(\overline{q})$ from (57) and B_1 from (58), the difference $\Delta B(\overline{q})$ is calculated by the following:

$$\begin{aligned}
\Delta B(\overline{q}) &= D_1(\overline{q}) - B_1 = \overline{q}F + \frac{\left(1-\overline{q}^2\right)}{2}F + \frac{(1-\overline{q})^2}{2}L - B_1 \\
&= \frac{(1-\overline{q})^2}{2}L + \left(1 - \frac{(1-\overline{q})^2}{2}\right)F - \frac{1}{2}L - \frac{1}{2}F \\
&= \left(\frac{(1-\overline{q})^2}{2} - \frac{1}{2}\right)L + \left(\frac{1}{2} - \frac{(1-\overline{q})^2}{2}\right)F \\
&= \left(\frac{1}{2} - \frac{(1-\overline{q})^2}{2}\right)(F-L) > 0
\end{aligned} \tag{A113}$$

For every $\overline{q} \in (0,1]$ the first term cannot be negative and with $F > L$ the second one cannot be negative either, so that $\Delta B(\overline{q}) > 0$.

A.38. Shareholders' payoff function without debtholder control

With $V(\overline{q})$ from (56), $D_1(\overline{q})$ from (57) and B_1 from (58), the shareholders' payoff function is calculated as follows:

$$\begin{aligned}
S_1(\overline{q}) &= V(\overline{q}) + B_1 - D_1(\overline{q}) - E[\mu] \\
&= -I + \overline{q}I + \frac{\left(1-\overline{q}^2\right)}{2}H + \frac{(1-\overline{q})^2}{2}L + B_1 - \frac{(1-\overline{q})^2}{2}L - \left(1 - \frac{(1-\overline{q})^2}{2}\right)F \\
&\quad - E[\mu] \\
&= -I + \overline{q}(I-F) + \frac{\left(1-\overline{q}^2\right)}{2}(H-F) + \frac{(1-\overline{q})^2}{2}\underbrace{(L-L)}_{=0} + B_1 - E[\mu]
\end{aligned} \tag{A114}$$

A.39. Calculation and analysis of $\overline{q}_1$

The first-order-condition of the shareholders' payoff function, given in (59) with $E[\mu] = w_0$, with respect to $\overline{q}$ is calculated as follows:

$$\begin{aligned}
\frac{\partial S_1(\overline{q})}{\partial \overline{q}} &= I - F - \overline{q}(H-F) = 0 \\
\leftrightarrow \quad \overline{q}(H-F) &= I - F \\
\leftrightarrow \quad \overline{q} &= \frac{I-F}{H-F} = \frac{I-L-(F-L)}{H-L-(F-L)} = \overline{q}_1
\end{aligned} \tag{A115}$$

With $\hat{q} = \frac{I-L}{H-L} < 1$, given by (41), subtracting the same value $(F - L)$ from the numerator and the denominator decreases the expression, which leads to $\overline{q}_1 < \hat{q}$. Following from the Appendix A.26, the policy $[\overline{q}_1]$ is riskier than the policy $[\hat{q}]$.

The derivative of $\overline{q}_1$ with respect to F is the same as the derivative of $\overline{q}$ with respect to F, given in (A99), which means that $\partial\overline{q}_1/\partial F < 0$.

A.40. Shareholders' payoff function with initial debtholder control

With $V(\overline{q})$ from (56) and $D_2(\overline{q}) = D_1(\overline{q})$, as in (57), the shareholders' payoff function is given by:

$$S_2(\overline{q}) = V(\overline{q}) + B_2 - D_2(\overline{q}) - E[\mu]$$
$$= -I + B_2 + \overline{q}(I - F) + \frac{(1-\overline{q}^2)}{2}(H - F) + \frac{(1-\overline{q})^2}{2}(L - L) - E[\mu] \tag{A116}$$

As $B_2 = D_2(\overline{q})$ and $E[\mu] = w_0$, this leads to:

$$S_2(\overline{q}) = V(\overline{q}) - E[\mu] = -I + \overline{q}I + \frac{(1-\overline{q}^2)}{2}H + \frac{(1-\overline{q})^2}{2}L - w_0 \tag{A117}$$

A.41. Calculation and analysis of investment policy $\overline{q}_2$

The first-order-condition of the objective function, given in (66), with respect to $\overline{q}$ is calculated as follows:

$$(1-\beta)\frac{\partial S_2(\overline{q})}{\partial \overline{q}} + \beta\frac{\partial D_2(\overline{q})}{\partial \overline{q}} = 0$$
$$\leftrightarrow \quad (1-\beta)[I - \overline{q}H - (1-\overline{q})L] + \beta[F - \overline{q}F - (1-\overline{q})L] = 0$$
$$\leftrightarrow \quad \overline{q}[-(1-\beta)H - \beta F + L] + (1-\beta)I - L + \beta F = 0$$
$$\leftrightarrow \quad \overline{q}[(1-\beta)H + \beta F - L] = (1-\beta)I - L + \beta F$$
$$\leftrightarrow \quad \overline{q} = \frac{(1-\beta)I - L + \beta F}{(1-\beta)H - L + \beta F}$$
$$\leftrightarrow \quad \overline{q} = \frac{(1-\beta)I - (1-\beta)L - \beta L + \beta F}{(1-\beta)H - (1-\beta)L - \beta L + \beta F}$$
$$\leftrightarrow \quad \overline{q} = \frac{I - L + \frac{\beta(F-L)}{(1-\beta)}}{H - L + \frac{\beta(F-L)}{(1-\beta)}} = \overline{q}_2 \tag{A118}$$

With $\hat{q} = \frac{I-L}{H-L} < 1$, given by (41), adding the same value $\frac{\beta(F-L)}{(1-\beta)}$ to the numerator and the denominator increases the expression, which leads to $\overline{q}_2 > \hat{q}$. Following from the Appendix A.26, the policy $[\overline{q}_2]$ is less risky than the policy $[\hat{q}]$.

For the determination of the derivative of $\overline{q}_2$ with respect to β, focus on the term $\frac{\beta(F-L)}{(1-\beta)}$ that is added to the numerator and denominator of $\frac{I-L}{H-L}$ in order to generate $\overline{q}_2$. If this term increases, so does the whole fraction. The derivative of the term with respect to β is given by:

$$\frac{\partial[\frac{\beta(F-L)}{(1-\beta)}]}{\partial\beta} = \frac{(F-L)(1-\beta)-(-1)(\beta(F-L))}{(1-\beta)^2} = \frac{(F-L)(1-\beta)+\beta(F-L)}{(1-\beta)^2}$$

$$= \frac{F-L}{(1-\beta)^2} > 0 \qquad \text{(A119)}$$

As the term added to the numerator and denominator increases in β, so does the fraction and it follows that $\partial\overline{q}_2/\partial\beta > 0$.

A.42. Calculation of $\hat{\alpha}_2$ with initial debtholder control

With (61) and (67), the shareholders calculate $\hat{\alpha}_2$ via the equation $\overline{q}_m = \overline{q}_2$:

$$\frac{I-F+\frac{\vartheta}{\alpha}}{H-F+\frac{\vartheta}{\alpha}} = \frac{I-L+\frac{\beta(F-L)}{(1-\beta)}}{H-L+\frac{\beta(F-L)}{(1-\beta)}}$$

$$\leftrightarrow \quad -F+\frac{\vartheta}{\alpha} = -L+\frac{\beta(F-L)}{(1-\beta)}$$

$$\leftrightarrow \quad \frac{\vartheta}{\alpha} = \frac{(1-\beta)(F-L)}{(1-\beta)} + \frac{\beta(F-L)}{(1-\beta)}$$

$$\leftrightarrow \quad \frac{\vartheta}{\alpha} = \frac{(F-L)}{(1-\beta)}$$

$$\leftrightarrow \quad \alpha = \frac{\vartheta(1-\beta)}{(F-L)} = \hat{\alpha}_2 \qquad \text{(A120)}$$

A.43. Expanded graphical presentation of $S_2(\overline{q}_2)$

Figure III expands the interval of β to $[0; 1]$ for figure 9.

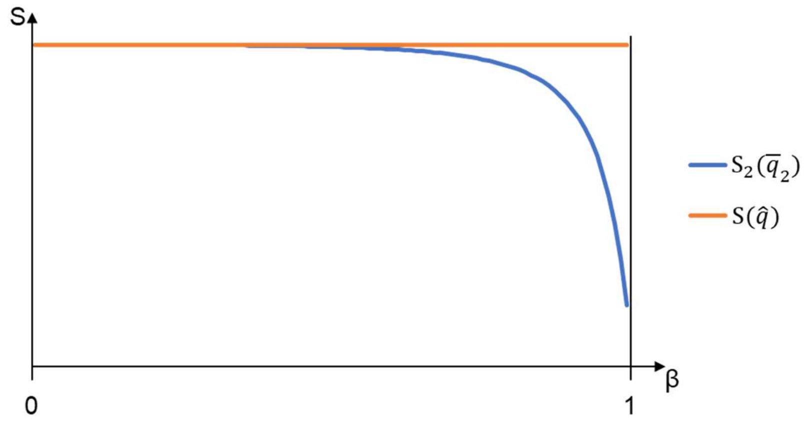

Figure III: Expanded illustration of shareholders' payoff $S_2(\overline{q}_2)$ vs. optimal expected payoff $S(\hat{q})$. Own graphic, $S(\hat{q})$ calculated according to John and John (1993), pp.957-958.[286]

A.44. Obtaining the optimal solution with initial debtholder control

With $\beta = 0$, the cutoff level $\overline{q}_2$, given in (67), becomes:

$$\overline{q}_2 = \frac{I - L + \dfrac{0 * (F - L)}{(1 - \beta)}}{H - L + \dfrac{0 * (F - L)}{(1 - \beta)}} = \frac{I - L}{H - L} = \hat{q} \qquad \text{(A121)}$$

Thus, the optimal policy, as in (41), is induced and the shareholders' expected payoff, as in (65), becomes:

$$S_2(\hat{q}) = -I + \hat{q}I + \frac{(1 - \hat{q}^2)}{2}H + \frac{(1 - \hat{q})^2}{2}L - w_0 = S(\hat{q}) \qquad \text{(A122)}$$

Note that above equation holds for $S(\hat{q})$, given by (44).

As shown in Appendix A.41, $\overline{q}_2 > \hat{q}$ for $\beta > 0$, so that the optimal solution is only obtained for $\beta = 0$.

[286] The values used to calculate the functions are: $L = 0;\ I = 5;\ H = 11;\ w_0 = 0;\ F = 1$.

A.45. Shareholders' payoff function with state-contingent debtholder control

With $V(\overline{q})$ from (56) and $D_3(\overline{q})$ from (69), the shareholders' payoff function is given by:

$$S_3(\overline{q}) = V(\overline{q}) + B_3 - D_3(\overline{q}) + \frac{(1-\overline{q})^2}{2}(F-L) - E[\mu]$$

$$= -I + B_3 + \overline{q}(I-F) + \frac{\left(1-\overline{q}^2\right)}{2}(H-F) + \frac{(1-\overline{q})^2}{2}(L+(F-L)-F) - E[\mu] \quad \text{(A123)}$$

The debtholders calculate the bond value $B_3 = D_3(\overline{q})$, which leads to:

$$S_3(\overline{q}) = V(\overline{q}) + \frac{(1-\overline{q})^2}{2}(F-L) - E[\mu] = -I + \overline{q}I + \frac{\left(1-\overline{q}^2\right)}{2}H + \frac{(1-\overline{q})^2}{2}F - E[\mu] \quad \text{(A124)}$$

A.46. Calculation and analysis of investment policy $\overline{q}_3$

With the shareholders' payoff function given in (70) and the condition (72), the first-order-condition with respect to $\overline{q}$ is calculated as follows:

$$\frac{\partial S_3(\overline{q})}{\partial \overline{q}} = I - \overline{q}H - (1-\overline{q})F + (1-\overline{q})C = 0$$

$$\leftrightarrow \quad \overline{q}[H-F+C] = I-F+C$$

$$\leftrightarrow \quad \overline{q} = \frac{I-F+C}{H-F+C} = \frac{I-L+(C-(F-L))}{H-L+(C-(F-L))} = \overline{q}_3 \quad \text{(A125)}$$

As per assumption $C > F-L$, so that a positive term $(C-(F-L))$ is added to the numerator and denominator of $\hat{q} = \frac{I-L}{H-L} < 1$, as given in (41). This leads to an increase of the whole fraction, which means that $\overline{q}_3 > \hat{q}$. Following from the Appendix A.26, the policy $[\overline{q}_3]$ is less risky than the policy $[\hat{q}]$.

The derivative of $\overline{q}_3$ with respect to C is given by:

$$\frac{\partial \overline{q}_3}{\partial C} = \frac{H-F+C-(I-F+C)}{(H-F+C)^2} = \frac{H-I}{(H-F+C)^2} > 0 \quad \text{(A126)}$$

A.47. Calculation of $\hat{\alpha}_3$ with state-contingent debtholder control

With (61) and (73) the shareholders calculate $\hat{\alpha}_3$ via the equation $\overline{q}_m = \overline{q}_3$:

$$\frac{I - F + \frac{\vartheta}{\alpha}}{H - F + \frac{\vartheta}{\alpha}} = \frac{I - F + C}{H - F + C}$$

$$\leftrightarrow \quad \frac{\vartheta}{\alpha} = \mathrm{C}$$

$$\leftrightarrow \quad \alpha = \frac{\vartheta}{C} = \hat{\alpha}_3 \tag{A127}$$

A.48. Obtaining the optimal solution with state-contingent debtholder control

In the following, the condition $C = F - L$ holds, so that $\overline{q}_3$, given by (73), becomes:

$$\overline{q}_3 = \frac{I - L + \big(F - L - (F - L)\big)}{H - L + \big(F - L - (F - L)\big)} = \frac{I - L}{H - L} = \hat{q} \tag{A128}$$

This holds for $\hat{q}$, given by (41), and leads to the following changes in the shareholders' expected payoff, given by (76):

$$\begin{aligned} S_3(\hat{q}) &= -I + \hat{q}I + \frac{(1 - \hat{q}^2)}{2}H + \frac{(1 - \hat{q})^2}{2}F - w_0 - \frac{(1 - \hat{q})^2}{2}(F - L) \\ &= -I + \hat{q}I + \frac{(1 - \hat{q}^2)}{2}H + \frac{(1 - \hat{q})^2}{2}L - w_0 = S(\hat{q}) \end{aligned} \tag{A129}$$

Note that above equation holds for $S(\hat{q})$, as given in (44).

As shown Appendix A.47, $\overline{q}_3 > \hat{q}$ for $C > F - L$. Moreover, $C > F - L$ means that the costs of the restructuring exceed its additional value, so that the terms do not eliminate each other, as they do in (A129). Thus, the optimum can only be obtained for $C = F - L$.

A.49. Comparison of policies: No control vs. initial control

Both of the critical values $\overline{q}_1$, given by (60), and $\overline{q}_2$, given by (67), are of the form $\overline{q}_l = \frac{(I-L)+n_l}{(H-L)+n_l}$, whereby the first- and second-order derivative with respect to n_l are calculated as follows:

$$\frac{\partial \overline{q}_l}{\partial n_l} = \frac{H - L + n_l - (I - L + n_l)}{(H - L + n_l)^2} = \frac{H - I}{(H - L + n_l)^2} > 0 \tag{A130}$$

$$\frac{\partial^2 \overline{q}_l}{\partial {n_l}^2} = \frac{-2(H - L + n_l)(H - I)}{(H - L + n_l)^4} \tag{A131}$$

For $n_l > -(H - L)$ it follows that $\partial^2 \overline{q}_l / \partial {n_l}^2 < 0$.

As $\partial \overline{q}_l / \partial n_l > 0$ and $n_1 = -(F-L) < \frac{\beta(F-L)}{(1-\beta)} = n_2$, it follows that $\overline{q}_1 < \overline{q}_2$.

According to the Appendix A.26, this means that the policy $[\overline{q}_1]$ is riskier than $[\overline{q}_2]$.

Given the cutoff levels in (41), (60) and (67), the following differences are calculated for further investigation:

$$\left|\hat{q} - \overline{q}_1\right| = \frac{I-L}{H-L} - \frac{I-L-(F-L)}{H-L-(F-L)} \tag{A132}$$

$$\left|\hat{q} - \overline{q}_2\right| = \frac{I-L+\frac{\beta(F-L)}{(1-\beta)}}{H-L+\frac{\beta(F-L)}{(1-\beta)}} - \frac{I-L}{H-L} \tag{A133}$$

Assume that $\beta = 0{,}5$, so that $n_1 = -(F-L)$ and $n_2 = (F-L)$. This means that $|n_1| = |n_2|$ and $n_2 > n_1 > -(H-L)$. As $\partial \overline{q}_l / \partial n_l > 0$ and $\partial^2 \overline{q}_l / \partial {n_l}^2 < 0$, the difference $\left|\hat{q} - \overline{q}_2\right|$ must be smaller than $\left|\hat{q} - \overline{q}_1\right|$. If $\beta < 0{,}5$, the values of n_2 and $\overline{q}_2$ decrease, so that the difference $\left|\hat{q} - \overline{q}_2\right|$ becomes even smaller. Hence, $\left|\hat{q} - \overline{q}_1\right| > \left|\hat{q} - \overline{q}_2\right|$ holds for $\beta \leq 0{,}5$.

A.50. Analytical and graphical presentation of $\boldsymbol{\beta_{crit}}$

In order to determine β_{crit}, $S_1(\overline{q}_1)$, given by (64), is compared with $S_2(\overline{q}_2)$, given by (65) with $\overline{q} = \overline{q}_2$:

$$\begin{aligned} -I + \overline{q}_1 I + \frac{\left(1-{\overline{q}_1}^2\right)}{2} H + \frac{\left(1-\overline{q}_1\right)^2}{2} L - w_o - \Delta B(\overline{q}_1) \\ = -I + \overline{q}_2 I + \frac{\left(1-{\overline{q}_2}^2\right)}{2} H + \frac{\left(1-\overline{q}_2\right)^2}{2} L - w_o \end{aligned} \tag{A134}$$

This equation holds for the cutoff levels $\overline{q}_1$and $\overline{q}_2$, given by (60) and (67), and for $\beta = \beta_{crit}$. Solving (A134) for β_{crit} leads to the following:

$$\beta_{crit} = \frac{\pm(I-H)\sqrt{\Phi} + \Psi}{\Omega} \tag{A135}$$

With:

$$\begin{aligned} \Phi = (H-F)^2L^2 + [(H-F)I^2 + (2FH-2H^2)I + F^2H - F^3]L + (F^2-FH)I^2 \\ + (2FH^2 - 2F^2H)I - F^2H^2 + F^3H \end{aligned} \tag{A136}$$

$$\Psi = (H-F)^2L + (H-F)I^2 + (2FH-2H^2)I + FH^2 - F^2H \tag{A137}$$

$$\Omega = (H-F)I^2 + (2FH-2H^2)I + 2FH^2 - 3F^2H + F^3 \tag{A138}$$

Note that the algebraic sign before $(I - H)\sqrt{\Phi}$ needs to be a minus in order to receive a plausible result. β_{crit} can be plotted as a function of F, with $L < F < I$, leading to figure IV. For illustration purposes, $\beta = 0{,}5$ and $\beta = 1$ have been inserted in the figure, too.

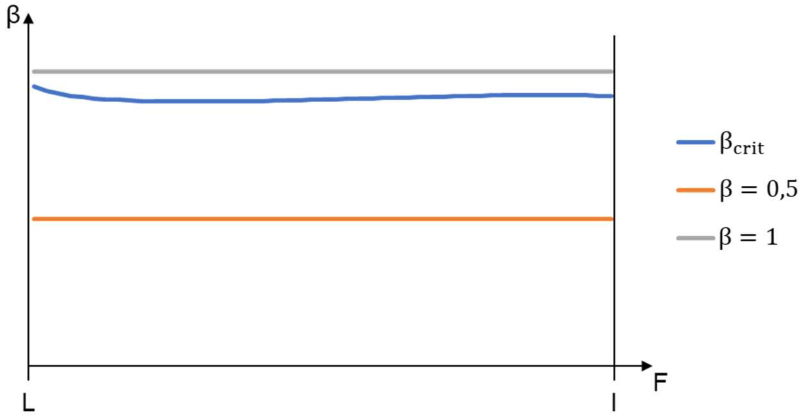

Figure IV: Critical value β_{crit} for comparison of options of no debtholder control and of initial debtholder control.[287]

A.51. Comparison of policies: No control vs. state-contingent control

The critical values $\overline{q}_1$ and $\overline{q}_3$, given by (60) and (73), have the form $\overline{q}_l = \frac{(I-L)+n_l}{(H-L)+n_l}$. With $\partial\overline{q}_l/\partial n_l > 0$, as shown in equation (A130) and $n_1 = -(F-L) < (C-(F-L)) = n_3$, it follows that $\overline{q}_1 < \overline{q}_3$.

According to the Appendix A.26, this means that the policy $[\overline{q}_1]$ is riskier than $[\overline{q}_3]$.

Given the cutoff levels in (41), (60) and (73), the following differences are calculated for further investigation:

$$\left|\hat{q} - \overline{q}_1\right| = \frac{I-L}{H-L} - \frac{I-L-(F-L)}{H-L-(F-L)} \tag{A139}$$

$$\left|\hat{q} - \overline{q}_3\right| = \frac{I-L+(C-(F-L))}{H-L+(C-(F-L))} - \frac{I-L}{H-L} \tag{A140}$$

For $C = 2(F-L)$ it follows that $n_1 = -(F-L)$ and $n_3 = (F-L)$. Hence, with $|n_1| = |n_3|$ and $n_3 > n_1 > -(H-L)$, this case is similar to the one in Appendix A.49, so that for $C = 2(F-L)$ the difference $\left|\hat{q} - \overline{q}_3\right|$ is smaller than the difference $\left|\hat{q} - \overline{q}_1\right|$. In case that $C < 2(F-L)$, the value of n_3 becomes smaller, leading to a further decrease of $\overline{q}_3$ and thus of the difference $\left|\hat{q} - \overline{q}_3\right|$. Overall, this means that $\left|\hat{q} - \overline{q}_1\right| > \left|\hat{q} - \overline{q}_3\right|$ for $C \leq 2(F-L)$.

[287] The values used to calculate the function of β_{crit} are: $L = 0;\ I = 5;\ H = 11;\ w_0 = 0;\ F = 1$.

A.52. Calculation of $C_{crit,1}$

For the determination of $C_{crit,1}$, $S_1(\overline{q}_1)$, given in (64), is compared with $S_3(\overline{q}_3)$, given in (76):

$$-I+\overline{q}_1 I+\frac{\left(1-\overline{q}_1{}^2\right)}{2}H+\frac{\left(1-\overline{q}_1\right)^2}{2}L-w_o-\Delta B(\overline{q}_1)$$
$$=-I+\overline{q}_3 I+\frac{\left(1-\overline{q}_3{}^2\right)}{2}H+\frac{\left(1-\overline{q}_3\right)^2}{2}F-\left(w_0+\frac{\left(1-\overline{q}_3\right)^2}{2}C_{crit,1}\right) \tag{A141}$$

The above holds for the cutoff levels $\overline{q}_1$ and $\overline{q}_3$, given by (60) and (73), and for $C = C_{crit,1}$. Solving (A141) for $C_{crit,1}$ leads to:

$$C_{crit,1}=-\frac{(H-F)^2L-FH^2+2F^2H-F^3}{(H-F)L+I^2-2HI+H^2-FH+F^2} \tag{A142}$$

This is shown graphically as a function of F in figure 11.

A.53. Comparison of policies: Initial control vs. state-contingent control

Given the cutoff levels in (41), (67) and (73), the following differences are calculated:

$$\left|\hat{q}-\overline{q}_2\right|=\frac{I-L+\frac{\beta(F-L)}{(1-\beta)}}{H-L+\frac{\beta(F-L)}{(1-\beta)}}-\frac{I-L}{H-L} \tag{A143}$$

$$\left|\hat{q}-\overline{q}_3\right|=\frac{I-L+(C-(F-L))}{H-L+(C-(F-L))}-\frac{I-L}{H-L} \tag{A144}$$

Both of the cutoff levels $\overline{q}_2$ and $\overline{q}_3$, given by (67) and (73), have the form $\overline{q}_l=\frac{(I-L)+n_l}{(H-L)+n_l}$. Assuming that $C=2(F-L)$ and $\beta=0{,}5$ leads to $n_2=n_3=F-L$, so that $\left|\hat{q}-\overline{q}_2\right|=\left|\hat{q}-\overline{q}_3\right|$. In case of $C>2(F-L)$, it follows that $n_3>F-L$, whereas $\beta<0{,}5$ leads to $n_2<F-L$, which means that with $\partial\overline{q}_l/\partial n_l>0$, as in (A130), the condition $\left|\hat{q}-\overline{q}_2\right|<\left|\hat{q}-\overline{q}_3\right|$ holds.

A.54. Calculation of $C_{crit,2}$

In order to determine $C_{crit,2}$, $S_2(\overline{q}_2)$, as given in (65), is compared with $S_3(\overline{q}_3)$, given in (76):

$$-I + \overline{q}_2 I + \frac{(1-\overline{q}_2{}^2)}{2}H + \frac{(1-\overline{q}_2)^2}{2}L - w_o = -I + \overline{q}_3 I + \frac{(1-\overline{q}_3{}^2)}{2}H + \frac{(1-\overline{q}_3)^2}{2}F - \left(w_0 + \frac{(1-\overline{q}_3)^2}{2}C_{crit,2}\right) \quad \text{(A145)}$$

The equation holds for $\overline{q}_2$ and $\overline{q}_3$, given by (67) and (73), and for $C = C_{crit,2}$. Solving (A145) for $C_{crit,2}$ leads to:

$$C_{crit,2} = -\frac{(\overline{q}_2 - 1)^2(H-F)L - I^2 + (\overline{q}_2 H + (1-\overline{q}_2)F)2I - \overline{q}_2{}^2 H^2 + (\overline{q}_2{}^2 - 1)FH}{(\overline{q}_2 - 1)^2 L + (\overline{q}_2 - 1)2I + (1-\overline{q}_2{}^2)H} \quad \text{(A146)}$$

As $\overline{q}_2$ is a function of β, $C_{crit,2}$ is also a function of β and can be graphically displayed, as shown in figure 12 as well as figure V.

A.55. Expanded graphical presentation of $C_{crit,2}$

Figure V expands the interval of β to $[0;1]$ for figure 12.

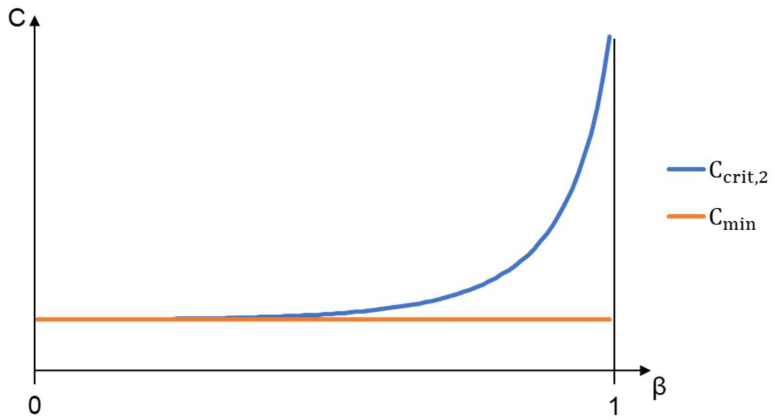

Figure V: Expanded illustration of the critical value $C_{crit,2}$ for comparison of options of initial debtholder control and of state-contingent debtholder control.[288]

[288] The values used to calculate the functions are: $L = 0;\ I = 5;\ H = 11;\ w_0 = 0;\ F = 1.$

A.56. Situations with negative expected payoffs of the shareholders

At first, consider the shareholders' expected payoff $S_1(\overline{q}_1)$, given in (64), and examine when it becomes negative:

$$S_1(\overline{q}_1) = -I + \overline{q}_1 I + \frac{(1-\overline{q}_1{}^2)}{2}H + \frac{(1-\overline{q}_1)^2}{2}L - w_o - \Delta B(\overline{q}_1) < 0$$

$$\leftrightarrow \quad -I + \frac{1}{2}F + \frac{1}{2}L + \overline{q}_1(I-F) + \frac{(1-\overline{q}_1{}^2)}{2}(H-F) + \frac{(1-\overline{q}_1)^2}{2}(L-L) - w_o < 0$$

$$\leftrightarrow \quad -I + \frac{1}{2}(L+H) + \overline{q}_1(I-F) - \frac{\overline{q}_1{}^2}{2}(H-F) - w_o < 0$$

$$\leftrightarrow \quad \frac{(-2I+L+H-2w_o)(H-F) + 2(I-F)^2 - (I-F)^2}{2(H-F)} < 0$$

$$\leftrightarrow \quad (-2I+L+H-2w_o)\left(\underbrace{H-F}_{>0}\right) + \left(\underbrace{I-F}_{>0}\right)^2 < 0 \qquad \text{(A147)}$$

Note that even for $w_0 = 0$ this inequation can be true if the first term is negative and its modulus is high enough. Hence, all combinations of L, I, H, F, for which (A147) holds with $w_0 = 0$, lead to $S_1(\overline{q}_1) < 0$, regardless of the value of w_0. Setting $w_0 > 0$ would even further decrease $S_1(\overline{q}_1)$.

Now, consider the shareholders' expected payoff $S_2(\overline{q}_2)$, given in (65) with $\overline{q} = \overline{q}_2$:

$$S_2(\overline{q}_2) = -I + \overline{q}_2 I + \frac{(1-\overline{q}_2{}^2)}{2}H + \frac{(1-\overline{q}_2)^2}{2}L - w_0 < 0$$

$$\leftrightarrow \quad V(\overline{q}_2) - w_0 < 0 \qquad \text{(A148)}$$

As $V(\overline{q})$ is a downward facing parabola with the maximum value at $\hat{q}$, as shown in Appendix A.26, and $\overline{q}_2 > \hat{q}$, as shown in Appendix A.41, $V(\overline{q}_2)$ becomes minimal for the maximum value of $\overline{q}_2$. Consider $\overline{q}_2 = 1$, so that:

$$V(1) = -I + 1 * I + 0 + 0 = 0 \qquad \text{(A149)}$$

Thus, even with the minimal value of V, inequation (A148) can only hold if $w_0 > 0$, so that $S_2(\overline{q}_2) \geq 0$, as long as $w_0 = 0$.

Finally, consider the shareholders' expected payoff $S_3(\overline{q}_3)$, given by (76):

$$S_3(\overline{q}_3) = -I + \overline{q}_3 I + \frac{(1-\overline{q}_3{}^2)}{2}H + \frac{(1-\overline{q}_3)^2}{2}(F-C) - w_0 < 0 \qquad \text{(A150)}$$

As $\overline{q}_3$ is set by the board, acting on behalf of the shareholders, it maximizes $S_3(\overline{q})$, so that the following condition holds:

$$S_3(\overline{q}_3) \geq S_3(\overline{q} = 1) = -I + 1 * I + 0 + 0 - w_0 = -w_0 \quad \text{(A151)}$$

Thus with $w_0 = 0$ it follows that $S_3(\overline{q}_3) \geq 0$.

For the comparison of the three different options, w_0 can be set equal to zero without the loss of generality, as it has exactly the same impact on each payoff function of the shareholders. It follows that for all combinations of I, L, H, F, for which inequation (A147) holds, the options with debtholder control dominate, as they cannot yield negative expected payoffs.

A.57. Endogenizing F in the case of initial debtholder control

If F is determined endogenously, this is done via the equation $B_2 = D_2(\overline{q}_2) = D_1(\overline{q}_2)$, with D_1 as in (57). This leads to:

$$B_2 = \overline{q}_2 F + \frac{\left(1 - \overline{q}_2^{\;2}\right)}{2} F + \frac{\left(1 - \overline{q}_2\right)^2}{2} L \quad \text{(A152)}$$

As $\overline{q}_2$, given by (67), is a function of β, solving (A152) for F leads to a solution, which is also a function of β.

With $\partial \overline{q}_2 / \partial \beta > 0$, as shown in Appendix A.42, and B_2 being constant, it follows from (A152) that F decreases in β. A decrease in F also decreases the term $\frac{\beta(F-L)}{(1-\beta)}$, which is added to the numerator and denominator of $\frac{I-L}{H-L}$ in order to create $\overline{q}_2$. As $\frac{I-L}{H-L} < 1$, a decrease in this term also decreases the whole fraction and therefore $\overline{q}_2$. While $\overline{q}_2$ decreases, it approaches the ideal cutoff level $\hat{q}$, which leads to an increase in $S_2(\overline{q}_2)$. This is in contrast to the direct effect of β on $S_2(\overline{q}_2)$, as $\overline{q}_2$ increases in β, so that the overall function decreases, as in figure 9.

A.58. Agency costs without debtholder control when debtholders assume $L = 0$

With the debtholders assuming that $L = 0$, B_1, given by (58), becomes:

$$B_{1,alt} = \frac{1}{2} F \quad \text{(A153)}$$

Consider the shareholders' expected payoff $S_1(\overline{q})$, given by (59). As $B_{1,alt}$ represents a constant, the first-order-condition and therefore the value of $\overline{q}_1$ do not change. $B_{1,alt}$ only affects $\Delta B(\overline{q}) = D_1(\overline{q}) - B_1$. With $\partial \Delta B(\overline{q}) / \partial B_1 = -1 < 0$ and $B_{1,alt} < B_1$, $B_{1,alt}$ leads to a higher $\Delta B(\overline{q})$ and therefore increases the agency costs.

A.59. Shareholders' expected payoff with long-term aspects

The debtholders price the debt equal to their expected payoff, which means $B = D$. The future savings Z can only be expected after the safe or the high return, as the relationship only continues in these cases. With (56), this leads to:

$$\begin{aligned} S_{LT}(\overline{q}) &= V(\overline{q}) + B - D - w_0 + E[Z] \\ &= -I + \overline{q}I + \frac{\left(1-\overline{q}^2\right)}{2}H + \frac{(1-\overline{q})^2}{2}L - w_0 + \overline{q}Z + \frac{\left(1-\overline{q}^2\right)}{2}Z \\ &= -I + \overline{q}(I+Z) + \frac{\left(1-\overline{q}^2\right)}{2}(H+Z) + \frac{(1-\overline{q})^2}{2}L - w_0 \end{aligned} \tag{A154}$$

A.60. Calculation of $\overline{q}_{LT}$

The first-order-condition of $S_{LT}(\overline{q})$, given by (79), with respect to $\overline{q}$ leads to:

$$\begin{aligned} \frac{\partial S(\overline{q})}{\partial \overline{q}} &= I + Z - \overline{q}(H+Z) - (1-\overline{q})L = 0 \\ \leftrightarrow \quad & \overline{q}(H+Z-L) = I + Z - L \\ \leftrightarrow \quad & \overline{q} = \frac{I-L+Z}{H-L+Z} = \overline{q}_{LT} \end{aligned} \tag{A155}$$

With $\hat{q} = \frac{I-L}{H-L} < 1$, as given in (41), adding the same value Z to the numerator and the denominator increases the expression, which leads to $\overline{q}_{LT} > \hat{q}$. As shown in Appendix A.26, this means that $[\overline{q}_{LT}]$ leads to a final cash flow that is less risky than the one generated by $[\hat{q}]$. It also follows that increasing the value of Z increases the fraction for the same reasons as stated above, so that $\partial\overline{q}_{LT}/\partial Z > 0$.