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Multi-Period Optimization of the Refuelling Infrastructure for Alternative Fuel Vehicles

Alexander Böhle

Karlsruher Institut für Technologie

Abstract

Alternative fuel vehicles (AFV) are gaining increasing attention as a mean to reduce greenhouse gas (GHG) emissions. One of the most critical barriers to the widespread adoption of AFVs is the lack of sufficient refuelling infrastructure. Although it is expected, that an adequate number of alternative fuel stations (AFS) will eventually be constructed, due to the high resource intensity of infrastructure development, an optimal step-by-step construction plan is needed. For such a plan to be actionable, it is necessary, that the underlying model considers realistic station sizes and budgetary limitations.

This bachelor thesis addresses this issue by introducing a new formulation of the flow-refuelling location model, that combines multi-periodicity and node capacity restrictions (MP-NC FRLM). For this purpose, the models of Capar and Kluschke have been extended, and the pre-generation process of sets and variables has been improved. The thesis furthermore adapts and applies the two evaluation concepts Value of the Multi-Period Solution (VMPS) and Value of Multi-Period Planning (VMPP) to assess the model's relative additional benefit over static counterparts. Besides, several hypotheses about potential drivers of the two evaluation concepts VMPS and VMPP have been made within the scope of a numerical experiment, to help central planners identify situations, where the additional complexity of a dynamic model would be worthwhile.

While the MP-NC FRLM has proven to provide additional benefit over static counterparts, it comes at the cost of a higher solving time. The main contributor to the higher solving is hereby the incorporation of a time module.

Keywords: Alternative fuel vehicle; refuelling infrastructure; optimal location; multi-period; fuel station.

1. Introduction and Problem Formulation

Over the last decade, the awareness of environmental problems and climate change has grown significantly. Via the internet and social media, it is now easier than ever for non-governmental organizations (NGO), scientists and activists to reach millions of people with their message: Climate Change is real, and if humanity in its entirety does not act with all necessary vehemence, the effects of global warming will be devastating. Even if the goals of the Paris Climate Agreement are accomplished, and global warming is held below 2°C compared to pre-industrial times, the consequences will still be grave. Risks to livelihoods, food security, water supply and impacts on biodiversity and ecosystems, including species loss and extinction, are the most commonly mentioned consequences. Nonetheless, global greenhouse gas (GHG) emissions continue to rise and path the way to a significant climate crisis. In contrast to the increasing production of greenhouse gases, GHG emissions in 2030 need to be approximately 25% respectively 55% lower than in 2017 to put the world on a pathway to limiting global warming to 2°C respectively 1,5°C (UN Environment (2018)).

Alongside the rising awareness for climate change and its consequences, alternative fuel vehicles (AFV) are gaining increasing attention. According to the IPCC, the transportation sector accounts for 14% of global greenhouse gas emissions, Edenhofer et al. (2014), in Europe for even 20% and rising (Rosca, Costescu, Rusca, and Burciu (2014)) (see figure 1). The step-wise replacement of combustion engines with, for example, battery-electric (BEV) or fuel cell electric vehicles (FCEV) is, hence, seen as an essential cornerstone for reducing greenhouse gases and other emissions. To become more popular, AFVs have to overcome several barriers. The most commonly discussed barriers are hereby the limited range of BEVs ,Capar and Kuby (2012); Lim and Kuby (2010), and the high cost for FCEVs (James, Huya-Kouadio, Houchins, and Desantis (2017)). While each AFV type has its respective barriers, one common problem is the lack of alternative refuelling infrastructure (Zhang, Kang, and Kwon (2017)).

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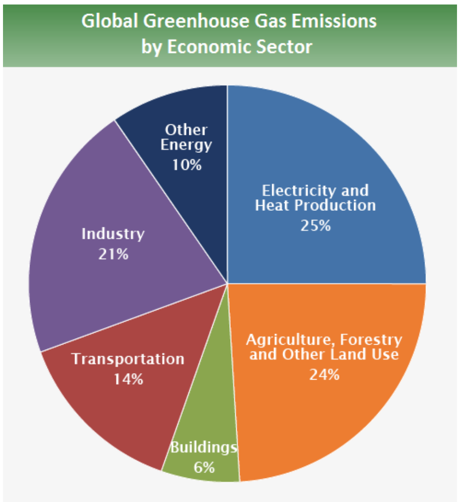


Figure 1: Global GHG Emissions by Economic Sector: With a share of 14% transportation contributes significantly to global emissions. (Edenhofer et al. (2014))

Although the popularity of AFVs is rising, many potential customers hesitate to buy BEVs and FCEVs, because the current level of refuelling infrastructure is not as mature as that of conventional gas stations and is not widely distributed (Zhang et al. (2017)). To facilitate the use of alternative drive technologies, it is, therefore, essential to plan and establish a refuelling infrastructure that is in line with the rising demands (Melendez (2006)). The decision where AFS should be placed is serious, because it has an influence on the allocation of further gas stations and might be decisive for the market success of alternative drive technologies. It becomes more important, as establishing a refuelling infrastructure is expensive, and decisions for a location are most probably final due to the high costs of changing the location. Hence, optimal allocation seems to be inevitable (Jochem, Brendel, Reuter-Oppermann, Fichtner, and Nickel (2016)).

In consequence numerous efforts have been made to determine the optimal siting of AFS alongside a road network by the means of mathematical optimization (Kuby and Lim (2005); MirHassani and Ebrazi (2013); Capar, Kuby, Leon, and Tsai (2013)). The road network in these optimization problems is represented as a Graph. Potential fuel station locations like cities, route intersections, road service areas are represented as the nodes. The roads are described as edges, that link the nodes. Traffic is depicted as flow that passes nodes and edges on a trip from an origin node to a destination node. The main goal of the optimization models is, to determine the optimal siting of a pre-specified number of p fuel stations so that the amount of refuelled origin-destination (OD) trips respectively the amount of refuelled traffic is maximal. Figure 2 shows how the trip from Cologne to Karlsruhe via Frankfurt could be modelled as a graph. Applied to this OD trip, the AFS siting models would determine which of the nodes would be the optimal location for a fuel station so that traffic on the way from Cologne to Karlsruhe is refuelled.

Over the last couple of years, models have become increasingly sophisticated, and scientists have begun to consider important real-world restrictions for fuel station siting in their optimization. Some of the more recent models, for example, consider that the possible storage amount of fuel at gas stations is not infinite. Restraining factors are, for example, limitations of the building land or laws that constrain the maximum amount of fuel stored at a single location. Hence, several authors included capacity restrictions in their model (Hosseini and MirHassani (2017);Kluschke et al. (2020)).

Alternative fuel stations are not only capacitated, the construction of such stations is also resource-intensive. Therefore it is unrealistic to assume the construction of a larger number of fuel stations within a short amount of time due to, for example, limitations of budget or labour. Hence, besides determining the optimal AFS placement, it is essential to provide an efficient step-by-step construction plan for the refuelling infrastructure. Thus, some authors have started to extend models by a temporal dimension. Time is hereby discretized into several planning respectively construction periods of equal length.

In consequence, these multi-period models have the objective of providing an optimal period-by-period construction plan, that respects periodic budget limitations (Chung and Kwon (2015); Zhang et al. (2017)).

Although there exist some multi-period AFS location models, a multi-period model that also respects budgetary constraints and capacity restrictions of the building sites have yet to be developed.

The main objective of this thesis is to address this issue by providing an optimization model that delivers an efficient construction plan for building up an alternative fuel station network based on empirical flow data. Therefore the node-capacitated flow-refuelling location model (NC-FRLM)

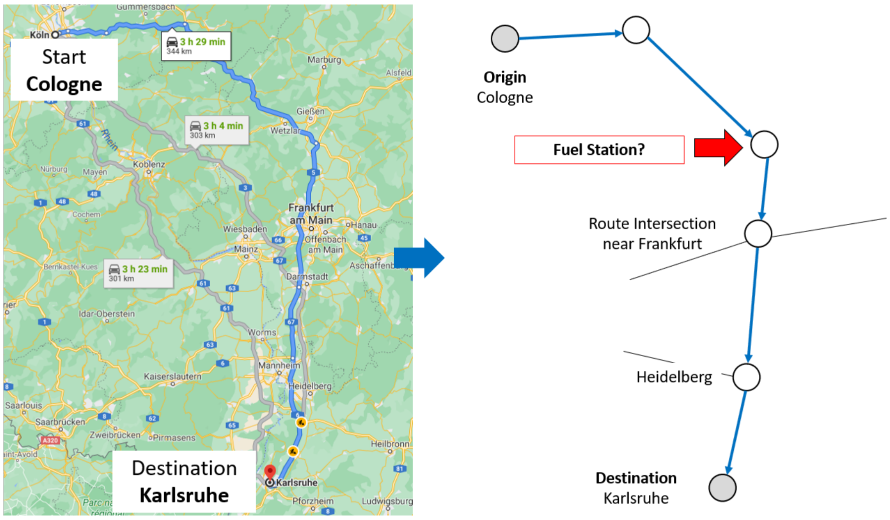


Figure 2: Possible transformation of a real-world road trip from Cologne to Karlsruhe into a graph.

by Kluschke et al. (2020) was extended by adding a period module alongside a periodic budget to more realistically represent changing demands and construction capacity. The thesis aims at answering the following research questions:

- How can the node-capacitated FRLM be extended to provide a multi-period construction plan for an alternative fuel station network while respecting changing demands and construction capacity?
- Does a multi-period model provide benefits compared to static modelling approaches and how can this benefit be quantified?

The thesis is structured as follows: First, a short overview of the existing literature for flow-refuelling location models (FRLM) is given in section 2.1. Section 2.2 introduces the basic FRLM, as well as the node-capacitated FRLM extension by Kluschke et al. (2020), which are explained before the new multi-period node-capacitated FRLM modelling approach is presented in section 2.3. To examine the benefits and the computational complexity of the model, a numerical experiment is conducted in section 2.4, before completing the thesis with a conclusion and suggestions for further research in section 3.

2. Literature Review

The following Literature Review is subdivided into two parts. The General Literature Review gives the reader a comprehensive overview of the current literature and thematizes mainly the flow-refuelling location model and its expansions. Apart from the basic FRLM and its origin, various approaches towards capacitated and multi-period extensions will also be discussed. The last subsection introduces the reader to the current state of the art FRLM modelling. More specifically, the two models on which the new FRLM extension is based, namely the arc-cover path-cover FRLM by Capar et al. (2013) and the node-capacitated FRLM by Kluschke et al. (2020), are described in detail.

2.1. General Literature Review

Of all facility location problems, the FRLM is the most commonly used model in AFV infrastructure planning. It is based on the idea by Hodgson (1990) to model traffic as flow, that is passing nodes along an origin-destination trip on a graph. The nodes of the network are considered candidate locations for fuel stations, that serve the refuelling demand. The FRLM can either be formulated as a set covering or a maximal covering problem. While the set covering formulation determines the minimal amount of fuel stations necessary to cover all OD trips, the maximal covering formulation maximizes the path/flow coverage with a given amount of p fuel stations. Current FRLM extensions include the consideration of station and node capacity limits and the inclusion of multiple construction periods. The two most common concepts for evaluating multi-period models are the Value of the Multi-Period Solution and the Value of Multi-Period Planning. Both concepts quantify the relative value difference between a multi-period model and pre-specified counterparts.

2.1.1. The Flow-Refueling Location Model

Capar et al. (2013) identify seven different models for solving a facility location problem: p-median problem, set covering problem, maximal covering location problem, flow interception location problem, flow-refuelling location problem, network interdiction problem and network sensor problem. In the context of infrastructure planning for alternative drive technologies, the flow-refuelling location model (FRLM) is the most commonly used (Kluschke et al. (2020)). Figure 3 classifies the new MP-NC FRLM within the different research streams for facility location models.

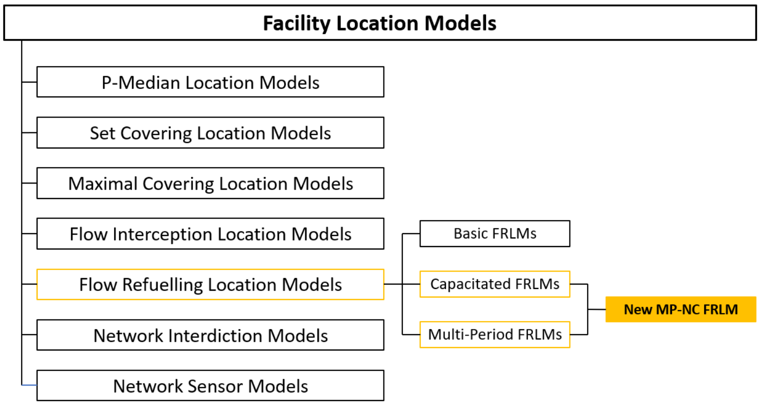


Figure 3: Overview over facility location models and classification of the MP-NC FRLM

The FRLMs and their expansions are based on the idea of Hodgson (1990) to not express demand as a stationary node attribute but to model it as a flow, that is passing nodes along an origin-destination (OD) trip on a graph. Within the application of alternative fuel station placement, the demand flow represents the vehicle traffic with its need for refuelling on the way from origin to destination.

The nodes of the network are candidate locations for the construction of gas stations to capture the flow and serve the demand. On a highway network, for example, nodes refer to highway entries, intersections or exits.

In their first formulation of the FRLM, Kuby and Lim (2005) extended Hodgson's model to consider the limited range of vehicles. Contrary to prior models, a trip was no longer refuelled, if the OD flow passes one single facility along its path. For refuelling, the entire path had to be covered, which might include more than one refuelling stop at a gas station, depending on the vehicle range, the path length and the node spacing. The possible need to refuel at several facilities required the pre-generation of valid facility combinations on each OD path, which made the model potentially difficult to solve in large networks. To address this problem, Lim and Kuby (2010) proposed heuristic algorithms to solve larger problems. Moreover, MirHassani and Ebrazi (2013) both developed FRLM formulations, that did not require the pre-generation of facility combinations and solved the model faster than the heuristics of Lim and Kuby (2010).

The arc-cover path-cover model, Capar et al. (2013) provided a new formulation of the FRLM, that was computationally more efficient than previous formulations and heuristics. The main idea of this new formulation was to refuel each OD path arc-wise. If all arcs on a path can be refuelled at one of the open facilities, the whole path is seen as refuelled and travelable.

Due to the efficiency of the formulation, Capar et al. (2013)'s model is the base for many of today's FRLM extensions like Hosseini and MirHassani (2017); Zhang et al. (2017); Kluschke et al. (2020) and will therefore be further discussed in section 2.2.

Although the FRLM initially followed a maximal coverage approach, intending to cover the maximal possible amount of flow through the allocation of p facilities, it can be reformulated into a set-covering problem. The set covering formulation aims at minimizing the number of stations necessary to cover a given share of flow respectively demand (Jochem et al. (2016)). Furthermore, Wang and Wang (2010) were the first ones to reformulate the FRLM into a set covering problem. Contrary to most FRLM formulations, their model only used origin-destination trip data as input without including information about the demand of the OD flows. Capar et al. (2013) also provide a set covering formulation of their arc-cover path-cover model, that, like their maximal covering formulation, considers the OD demand.

2.1.2. Capacitated FRLMs

Most articles on the FRLM do not consider capacity limits for facilities and assume, that all flows passing through a station can be served, regardless of its dimension. As AFS do have capacity limits and are expensive to set up, considering existing refuelling limitations is vital to improving the informative value of the models (Hosseini and MirHassani (2017)).

Upchurch, Kuby, and Lim (2009) were the first ones to address this issue with their capacitated FRLM. Their model defines the capacity of a station through the number of its interchangeable modular refuelling units, which can serve a certain amount of vehicles. In consequence, the main objective is not the optimal placement of p facilities, but p modular units on nodes of the network. As Upchurch et al. (2009) do not limit the number of modular units per node, the amount of refuelling capacity that could be built at each node is potentially infinite.

Wang and Lin (2013) provided a capacitated extension of Wang and Lin (2009)'s model that is designed explicitly for BEVs and considers multiple types of charging stations as well as a constrained facility budget. Like Upchurch et al. (2009), they model the capacity of stations through the number of vehicles, that they can serve. The capacity of each station type is calculated through the recharging efficiency of the used technology, given a pre-specified refuelling time. Contrary to Upchurch et al. (2009), the maximum number of facilities at the nodes is limited, so that node-specific restrictions, like local limitations of the power supply or the building land, can be included in the model. Therefore Wang and Lin (2013) can be considered as the first ones to apply node capacity restrictions to the FRLM.

Hosseini and MirHassani (2017) present a capacitated expansion of MirHassani and Ebrazi (2013)'s and Capar et al. (2013)'s models and solved them with a heuristic method based on Lagrangian relaxation. Hosseini and MirHassani (2017) assumed the stations to be fast-refuelling and determined the degree of capacity utilization through the actual amount refuelled. This approach differs from previous ones by Upchurch et al. (2009) and Wang and Lin (2013), who base facility capacity on the number of refuelable vehicles and not on the total amount of refuelling, the station can provide that. Following up on their article, the authors have published two further expansions of their capacitated FRLM.

Hosseini and MirHassani (2015) developed a stochastic version of their capacitated FRLM formulation, to consider the uncertainty of the traffic flow, based on a finite number of scenarios. As the solution time drastically increased with the network size and the number of considered scenarios, a solution heuristic for the stochastic model was presented and successfully tested on an intercity network for Arizona. [1]

The second expansion by Hosseini, MirHassani, and Hooshmand (2017) adds the drivers' willingness to deviate from their pre-defined shortest path to visit an AFS to the model. To be able to obtain a solution in a reasonable time for larger instances of the problem, an iterative-based heuristic algorithm was presented.

Most recently Kluschke et al. (2020) present a node capacitated formulation of the arc-cover path-cover formulation by Capar et al. (2013). Like Wang and Lin (2013) they base their model on the idea, that a potentially unlimited amount of refuelling at a single node is unrealistic. Reasons for that are, for example, technical limitations (e.g. the amount of electricity provided at a single location) or legal limitations (e.g. the quantity of hydrogen stored at a single location). The capacity of a station is modelled in units of the alternative fuel (e.g. kg of hydrogen), and its use to capacity is determined by the actual amount refuelled to serve the captured flows. Kluschke et al. (2020) successfully applied their model to the siting of hydrogen refuelling infrastructure for heavy-duty vehicles on the German highway network. Furthermore, they can be considered the first ones to combine node capacity restrictions, and OD demand flows in a model. As their model serves as the base for the FRLM extension presented in this thesis, it will be further discussed in section 2.3.

2.1.3. Multi-Period FRLMs

As pointed out by Hosseini and MirHassani (2017), AFS is not only capacitated, the construction of such stations is also resource-intensive. Therefore it might not be useful to assume the construction of a larger number of fuel stations within time due to, for example, limitations of budget or labour Chung and Kwon (2015)).

Furthermore, the development of an alternative fuel infrastructure constitutes a so-called "chicken-egg problem", Kuby and Lim (2005) and Wang and Wang (2010), that might only be solved through strategic multi-period planning controlled by a central authority (Chung and Kwon (2015)). On the one hand, companies are unlikely to invest in alternative fuel stations until there is sufficient demand for profitable operations. On the other hand, potential customers are less incentivized to buy alternative fuel vehicles unless there is an agreeable level of refuelling infrastructure (Bento (2008)).

Even though multi-periodicity seems to be an essential aspect of AFS infrastructure planning, the existing literature has rarely considered it.

Chung and Kwon (2015) first addressed the issue of multi-periodicity by extending the maximal covering FRLM formulation of MirHassani and Ebrazi (2013). They present three different methods for multi-period planning of flow-refuelling locations: a multi-period optimization method (M-opt), a forward myopic method (F-Myopic) and a backwards myopic method (B-Myopic). All three methods were applied for the siting of BEV charging stations along the Korean expressways.

The M-opt method solves a multi-period optimization model over T discrete time periods and sites $n_t,\ t \in T$ facilities per period to maximize the total amount of flow covered over all periods. Once a facility is sited, it must remain open until the final period. n_t depicts the total number of stations that are operational in period t of which $n_t - n_{t-1}$ are newly constructed in period t.

The F-Myopic method solves T single-period optimization models successively starting in period one. Like in the M-opt method, in each optimization problem (= time period) n_t—n_{t-1} facilities are allocated, given the siting of the stations in the prior period $t-1$. That means, for example, that all stations sited in period one are automatically allocated to the same spot in period two. Given the allocated stations from period one, n_2 facilities are sited in period two to maximize the amount of flow covered.

The B-Myopic method follows an approach similar to the F-Myopic method but begins the series of single-period optimization problems in the last period, T. The n_T nodes where facilities have been located in period T serve as candidate nodes for the siting of n_{T-1} facilities in period $T-1$. The same procedure is repeated until period one.

Chung and Kwon (2015) stated that the M-Opt method produces the best result in all cases, but also requires the

[1] Even though (Hosseini and MirHassani,2017) appeared in the November issue of *International Transactions in Operational Research*, it was first published in October 2015. Therefore the publishing order of the capacitated FRLM and its stochastic expansion by the authors still follows the logical timeline

most computational resources. Although the myopic methods produce significantly worse results on some demand profiles, the B-Myopic method is recommended for larger problems as the B-Myopic solutions are nearly as good as the M-Opt solutions in most cases.

Li, Huang, and Mason (2016) present a multi-period multi-path refuelling location model, that seeks to minimize the roll-out costs for refuelling infrastructure that serves all origin-destination trips. The model takes the drivers' willingness to deviate from their shortest OD path for refuelling into account. An OD pair is considered served, if at least one path, either the shortest path or a path within a reasonable deviation, is refuelled. In their model, (Li et al. (2016)) allow the costly relocation of facilities, but do not include traffic flows between OD pairs.

In a case study for the development of a fast-charging network in South Carolina, Li et al. (2016) applied both, a multi-period optimization method as well as F-Myopic and B-Myopic methods and compared the outcomes. Their findings are consistent with the results of Chung and Kwon (2015) as both of their myopic methods performed worse than the multi-period optimization approach in terms of the objective function value.

Miralinaghi, Keskin, Lou, and Roshandeh (2017)'s model takes a different approach and aims at minimizing the total system cost. The model includes facility construction costs, facility operating costs and the total travel costs experienced by the users of the network. Although Miralinaghi et al. (2017) work with OD pairs, they pre-calculate neither the shortest path nor a path with a reasonable deviation that a driver would take. They implicitly assume that drivers are willing to take any detour necessary to refuel on their trip. They applied the model to an intra-city transportation network and solved it via branch-and-bound and Lagrangian relaxation algorithms.

Zhang et al. (2017) base their multi-period capacitated FRLM on the maximal covering arc-cover path-cover formulation of Capar et al. (2013) and are the first ones to combine multi-periodicity and capacity restrictions in an FRLM formulation. They furthermore model demand as an endogenous variable that depends on demand dynamics and depicts the interaction of network users and network planners.

In their model, demand is displayed as the AFV market share of an OD flow for path q in period t. The market share per path and per period depends on several factors: the market share of the prior period, the natural growth of the market share and the path-specific flow coverage compared to the average flow coverage in the network.

Zhang et al. (2017) model the capacity restrictions of fuel stations according to Upchurch et al. (2009) as the number of vehicles that can charge at a refuelling module per period. Like Upchurch et al. (2009) the number of refuelling modules per node was not limited, which proved to be problematic when the model was applied to a case study about the siting of AFS in the Washington DC - New York - Boston area. The results suggest the construction of up to 70 refuelling modules per single node, which seems to be unrealistic when considering technical and legal limitations to the total refuelling capacity per single node (Kluschke et al. (2020)).

2.1.4. Assessment of Multi-Period Models

For assessing the additional benefit of multi-period models, the two most frequently found concepts in the literature are the "Value of the Multi-Period Solution "(VMPS) and the "Value of Multi-Period Planning" (VMPP).

The Value of the Multi-Period Solution is a concept first introduced by Alumur, Nickel, Saldanha-da Gama, and Verter (2012), that aims at quantifying the additional benefit of a multi-period model compared to a static counterpart. The static counterpart is a model, that looks for a time-invariant solution of the multi-period problem and scales the outcome adequately to compensate for only solving the problem for one single period (Laporte, Nickel, and Saldanha da Gama (2015)).

As there are several possibilities to define the static counterpart to a multi-period problem, the Value of the Multi-Period Solution can vary along with the definition of the counterpart. Laporte et al. (2015), for example, name several possibilities on how to consider time-varying demands in a static counterpart. While it is one possibility to average all demands over the planning horizon, it is also possible to determine a reference value, e.g. the maximum value observed throughout the planning horizon, for calculating the counterpart's solution. The VMPS is finally calculated as the relative difference between the multi-period model's solution and the one of its counterpart.

The Value of Multi-Period Planning is an evaluation concept first mentioned by Ballou (1968). Although it has yet to be precisely defined, the concept aims at quantifying the additional benefit from considering multiple periods while planning, contrary to continuously solving static problems for each period, given the results of the prior calculations. Two possible comparison models are the F-Myopic and the B-Myopic solution approaches, that was, for example, utilized by Chung and Kwon (2015).

For retrieving comparable results as well as for decision-makers to consider multi-period over step-wise optimizing models, it is essential to assume, that demand and economic data can be accurately predicted for every considered period. The Value of Multi-Period Planning is obtained by subtracting the solution value of the myopic comparison model from the value of the multi-period model and dividing it by the myopic model's value. Ballou (1968) postulate, that given the assumption of predictive accuracy the Value of Multi-Period Planning should always be positive, which goes along with Chung and Kwon (2015)'s findings.

2.1.5. Contribution of this thesis

In summary, there are only four studies that address the application of the flow-refuelling location model over multiple periods. Furthermore, of those studies, Zhang et al. (2017) are the only ones also to incorporate capacity restrictions in their model. However, the results of their conducted case study indicate that their use of station capacity limits might be of limited practicability due to existing node-specific capacity limitations. To provide a plan for the construction of an AFS network over time with realistic stations sizes on nodes, the use of node capacity restrictions is necessary.

To the author's best knowledge, this thesis is the first piece of work to design and test a multi-period and also node-capacitated FRLM (MP-NC FRLM). In addition to proposing a general model, this thesis adopts the two assessment criteria, VMPS and VMPP, to fit the specific case of bench-marking the model's additional benefit. The thesis furthermore discusses several factors, that potentially influence the VMPS and VMPP and in this context, addresses the issue of computational complexity.

2.2. Introduction to Current State of the Art FRLM Modelling

The previous paragraph provided a general overview of the FRLM and its current extensions. The two most significant extensions are the consideration of station/node capacity restrictions and the optimization over multiple periods. In the following, the two models on which the MP-NC FRLM model extension is based, are explained in further detail.

Capar et al. (2013) take a different and more efficient way of formulating the FRLM than previous authors. Their main idea is to refuel each OD path arc-wise. If every arc on an OD round-trip can be refuelled at one of the open fuel stations, the whole path is seen as refuelled and travelable. Kluschke et al. (2020) later extend Capar et al. (2013)'s model with capacity restrictions, that limit the total amount of refueling at fuel stations. To better fit their case study of siting hydrogen fuel stations for trucks along the German highway, Kluschke et al. (2020) modify and add several model assumptions. Contrary to Capar et al. (2013), they use single OD trips instead of round-trips and make detailed presumptions about starting and ending fuel level of drivers as well as the total amount refuelled during the trip.

2.2.1. The Basic Arc-Cover Path-Cover FRLM | Capar et al. 2013

The following section describes the arc-cover path-cover FRLM (AC-PC-FRLM) of Capar et al. (2013) in further detail, starting with the model assumptions. After introducing the set covering formulation of the AC-PC FRLM, the calculation of the set $K^q_{j,k}$ and the functionality of the model are illustrated using a simple example. $K^q_{j,k}$ represents the set of facility locations, that could refuel the arc $a_{j,k}$on path q. For concluding, the maximal covering formulation of Capar et al. (2013)'s AC-PC FRLM is given.

Capar et al. (2013)'s arc-cover path-cover FRLM (AC-PC-FRLM) can be either formulated as a set covering or as a maximal covering problem. The main objective of the set covering problem is to determine the minimal amount of alternative fuel stations and their location on a Graph $G = (N,A)$ under the condition, that at least a pre-specified share of the total fuel demand S is satisfied. On the other hand, the maximal covering formulation aims at maximizing the served demand with p facilities. The vehicle traffic is depicted as flow, that passes from an origin O to a destination D on the graph. Traffic flow is considered refuelled or served if vehicles can travel from origin to destination and back to the origin without running out of fuel.

Model Assumptions and Mathematical Formulation

Capar et al. (2013) formulate their model under the following assumptions:

1. The traffic between an origin-destination pair flows on the shortest path through the network.
2. The traffic volume between OD pairs is known in advance.
3. Drivers have full knowledge about the location of the fuel stations along their path and refuel sufficiently to complete their round trips.
4. Only nodes of the network are considered as possible refuelling facility locations.
5. All vehicles are assumed to have similar driving ranges, a similar fuel tank capacity and similar fuel consumption.
6. The fuel consumption is directly proportional to the distance travelled.
7. Refuelling stations are incapacitated.

Assumptions 1-3 seem reasonable because every driver has access to a navigation system, either through car equipment or a smartphone, that can provide information about the shortest route, refuelling opportunities and traffic information. As Capar et al. (2013) specifically apply the model to private BEVs, the adoption of round trips rather than single trips is comprehensible considering the fact, that the passengers will want to return to their homes (=the origin) at some point after reaching the destination. In Assumption 4, Capar et al. (2013) limit potential station locations to the network nodes and by that prohibit the possibility of siting a station anywhere on an arc between two nodes. Restricting the siting to the nodes reduces the complexity of the model without significantly negatively impacting the results when applied to real transportation networks except in remote areas (Kuby and Lim (2007)). Assumptions 5-7 are further technical simplifications of reality.

Capar et al. (2013) define the set covering formulation of their arc-cover path-cover FRLM as follows:

$$\min \sum_{i \in N} z_i \tag{2.1}$$

$$\text{s.t.} \sum_{i \in K^q_{j,k}} z_i \geq y_q \forall q \in Q, a_{j,k} \in A_q \tag{2.2}$$

$$\sum_{q \in Q} f_q y_q \geq S \forall i \in N \tag{2.3}$$

$$z_i, y_q \in \{0,1\} \forall q \in Q, i \in N \tag{2.4}$$

Sets	
N	Set of all nodes on the Graph G
Q	Set of all OD pairs
A_q	Set of all directional arcs on the OD path $q \in Q$ from origin to destination and back
$K^q_{j,k}$	Set of all potential station locations, that can refuel the directional arc $a_{j,k} \in A_q$
Variables	
z_i	Binary Variable that equals to one, if a refuelling facility is constructed at node i
y_q	Binary Variable that equals to one, if the flow on path q is refuelled
Parameters	
f_q	Total vehicle flow on the OD path q
S	Proportion of the minimal amount of total flow refuelled

The objective function (2.1) of the model minimizes the total number of stations built on the nodes N of Graph G. Constraint (2.2) presents the core of the new arc-cover path-cover formulation by Capar et al. (2013), which allows them to formulate their FRLM without the pre-generation of all possible facility combinations for all paths. $K^q_{j,k}$ is hereby the set of all nodes, where a constructed facility could refill the arc $a_{j,k}$ on the OD-path q. (2.2) ensures, that a path is only counted as refuelled if the built stations z_i refuel all arcs on the OD-path q. Constraint (2.3) guarantees the refuelling of at least S * 100% of all OD-flows f_q of all OD trips q. (2.4) defines the two binary variables z_i and y_q. z_i equals to one if a facility is constructed at node i, whereas y_q equals to one if a path q is refuelled.

Pre-Calculation of the Set $K^q_{j,k}$

Like mentioned in the previous paragraph, the set $K^q_{j,k}$ is the core of Capar et al. (2013)'s new FRLM formulation. For each arc of each path, $K^q_{j,k}$ provides a list of candidate nodes for facilities, that could refuel the directional arc $a_{j,k}$. A path q can only be considered as covered, if every arc of this path is refuelled by a gas station from their candidate set. $K^q_{j,k}$ is calculated prior to the optimization of the model by applying the following logic, depicted in Code Listing 1:

```
for all q ∈ Q:
    for all a_{j,k} ∈ A_q:
        for all nodes i on the OD round trip q,
        that are not the destination node k of
        the arc a_{j,k}
            if the distance(node i, node k) ≤
            vehicle_range, following the round
            trip, then
                add node i to K^q_{j,k}
```

Code Listing 1: Algorithm for determining the set $K^q_{j,k}$ in the AC-PC FRLM

To determine the set of potential facility locations $K^q_{j,k}$, every node i, that lies between the origin node and the destination node of the arc $a_{j,k}$ on path q, will be inspected, to whether it qualifies for hosting a station, that can refuel the arc aj,k. Node I will be added as a potential location, if the destination node of the arc $a_{j,k}$ is reachable leaving node i with a full tank.

Contrary to the first FRLM formulation by Kuby and Lim (2005), vehicles here do not start from the origin with a prespecified fuel level, e.g. half of the tank. Capar et al. (2013) determine the initial tank filling based on the location of the AFS on the path, assuming that drivers only frequent the same OD trip. If there is a fuel station sited at the origin, the vehicle will start with a full tank; if there is no fuel station at the origin, the vehicle will start the trip with the remaining fuel from the last fill-up on the same OD round trip.

For better understanding, the calculation of the set $K^q_{j,k}$ and the model functionality illustrated in a simple example below.

Figure 4 shows a four-node sized network with the origin-destination pair (1,2). The vehicle range is assumed to be 120 km. For satisfying the demand flow, each arc on the round trip from the origin to the destination and back has to be refuelled. Therefore, the candidate node set $K^q_{j,k}$ has to be determined before the optimization process, starting with the arc $a_{1,2}$. The origin node one is the only node on the trip from the origin to the destination node of arc two.

As z_1 lies within the vehicle range (120 km) of node z_2 coming from the origin, it counts as a potential station location for refuelling the arc $a_{j,k}$. Hence, z_1 is added to the set $K^{(z_1,z_4)}_{1,2}$. Node two that is visited during the way back from the destination to the origin is on the path $a_{2,1} + a_{1,2} = 80$ km $\leq$ 120 km away from node two and counts as well a potential station location. As the distance from node 3 is $a_{3,2} + a_{2,1} + a_{1,2} = 110$ km $\leq$ 120 km, z_3 is added as well as a potential station location. Following the flow direction of the OD path, node 4 is 140 km away from node 2 is, therefore, no station locations. Thus, the set of potential facility sites, that could refuel the arc $a_{j,k}$ is $K^{(z_1,z_4)}_{1,2} = \{z_1, z_2, z_3\}$. The potential station locations for the other arcs on the round trip are listed in the table 1.

For refuelling the whole round trip (1,4), a facility must be built at least one of the candidate locations of each set $K^{(1,4)}_{j,k}$. Although several combinations of fuel stations could serve the vehicle flow, placing a station at node 2, with the variable $z_2 = 1$, is the only option, that serves the whole demand with the construction of just one facility and is, therefore, the optimal solution of the minimization problem.

The maximal covering formulation of the arc-cover path-cover FRLM is created by switching the objective function of the set covering approach (2.1) with constraint (2.3) and modify them accordingly.

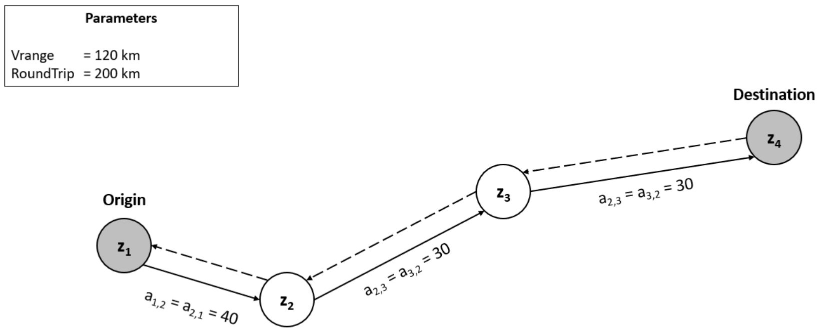


Figure 4: Exemplary graph network for illustrating the calculation of the set $K^q_{j,k}$

Sets	Potential Station Locations		
$K^{(1,4)}_{1,2}$	z_1	z_2	z_3
$K^{(1,4)}_{2,3}$	z_1	z_2	
$K^{(1,4)}_{3,4}$	z_1	z_2	z_3
$K^{(1,4)}_{4,3}$	z_2	z_3	z_4
$K^{(1,4)}_{3,2}$	z_2	z_3	z_4
$K^{(1,4)}_{2,1}$	z_2	z_3	z_4

Table 1: Set $K^q_{j,k}$ for the graph in Figure 4.

$$\max \sum_{q \in Q} f_q \; y_q \tag{2.5}$$

$$\text{s.t.} \sum_{i \in K^q_{j,k}} z_i \geq y_q \forall q \in Q, a_{j,k} \in A_q \tag{2.6}$$

$$\sum_{i \in N} z_i = p \tag{2.7}$$

$$z_i, \; y_q \in \{0,1\} \forall q \in Q, i \in N \tag{2.8}$$

p displays the number of stations that will be allocated to maximize the total flow covered on all OD paths.

2.2.2. FRLM Extension: Node Capacity Restrictions | Kluschke et al. 2020

In the previous paragraph, the reader was familiarised with AC-PC FRLM by Capar et al. (2013), which is the base model for Kluschke et al. (2020)'s extension. The AC-PC FRLM follows the idea of seeing each path as a sequence of arcs, that have to be refuelled for the path to be covered in its entirety. Kluschke et al. (2020) adopt this principle for their node-capacitated extension of the AC-PC FRLM's set covering formulation. The following sections begin by discussing the new FRLM assumptions, that were added by Kluschke et al. (2020). After presenting the mathematical formulation, a closer look is taken at the calculation of sets and parameters in Kluschke et al. (2020)'s model. Apart from adapting the $K^q_{j,k}$ generation algorithm, Kluschke et al. (2020) introduce the new parameter r_{iq}, that as well needs to be calculated before the optimization. The parameter r_{iq} is highly important to the model because it depicts the amount of refuelling of vehicles at the fuel station. If the total amount of refuelling of all vehicles at a station reaches Kluschke et al. (2020)'s capacity limit, it is no longer possible to fill up there.

To consider station location capacity limits, for example, local limitations of the power supply or the building land, Kluschke et al. (2020) added node capacity restrictions to the set covering formulation of Capar et al. (2013)'s arc-cover path-cover FRLM. Contrary to Capar et al. (2013), they do not apply their node-capacitated FRLM to refuelling BEV vehicles, but to fuel cell-powered heavy-duty vehicles and adjusted Capar et al. (2013)'s assumptions to fit their specific case.

Model Assumptions and Mathematical Formulation

The assumptions made by Kluschke et al. (2020) are listed below. Modified and additional assumptions are highlighted in italics:

1. The traffic between an origin-destination pair flows on the shortest path through the network.
2. The traffic volume between OD pairs is known in advance.
3. Drivers have full knowledge about the location of the fuel stations along their path and refuel efficiently *to complete their one-way origin-destination trips*.
4. Only nodes of the network are considered as possible AFS locations.
5. All vehicles are assumed to be homogeneous. The maximum driving range that can be achieved in a single refuelling is similar for each vehicle.

6. The fuel consumption is directly proportional to the distance travelled.
7. *Nodes and fuel stations are capacitated.*
8. *refuelling is only required on trips longer than 50 km.*
9. *Each vehicle starts and ends its trips with the same fuel level, which is sufficient for a specific range. There are no refuelling stations at the origins and destinations.*

Kluschke et al. (2020) use the first six assumptions of Capar et al. (2013) with one small adjustment to better fit the model to their case study that thematizes the siting of AFS along with the German highway network. As trucks usually receive a delivery order to another location, once it reaches the destination (tramp traffic), they model the OD routes as one-way trips instead of round trips

Assumption 7 is the first general difference between the two models, as Kluschke et al. (2020) formally introduce the node capacity restrictions to their model. In Assumption 8, a lower bound for OD trip lengths is introduced to reduce the total number of considered OD trips and thus reduce the computational complexity of the model. In this context, Kluschke et al. (2020) speculate, that short trips of less than 50 km might not require public refuelling infrastructure. Although it is not clear how likely this speculation is, it shifts the model focus to refuelling mainly long haul transportation. It can easily be relaxed for an application in different contexts.

Assumption 9 simultaneously incorporates two suppositions. As Kluschke et al. (2020) want to focus on public refuelling infrastructure, they assume, that there are no private AFS. In consequence, they prohibit the siting of facilities at the origin and the destination nodes of the paths, as trucks start and end their trips at the private cargo bays of the transportation companies. Assumption 9 furthermore implies, that truck drivers refuel efficiently and by that do not make unnecessary refuelling stops. In consequence, they only refuel the exact amount needed to travel their route. As vehicles end with the same fuel level as they started with, they have to refuel at least once per trip.

Kluschke et al. (2020) extend the arc-cover path-cover FRLM as follows:

$$\min \sum_{i \in N} z_i \tag{2.9}$$

$$\text{s.t.} \sum_{i \in K^q_{j,k}} z_i \geq y_q \forall q \in Q, a_{j,k} \in A_q \tag{2.10}$$

$$\sum_{q \in Q} f_q \, p \, r_{iq} \, y_q \, g_{iq} \, x_{iq} \leq c \, z_i \forall i \in N \tag{2.11}$$

$$\sum_{i \in K^q_{j,k}} x_{iq} = y_q \forall q \in Q, a_{j,k} \in A_q \tag{2.12}$$

$$\sum_{i \in N} x_{iq} = y_q \, l_q \forall q \in Q \tag{2.13}$$

$$x_{iq} \leq z_i \forall i \in N, q \in Q \tag{2.14}$$

$$z_i \in \{0,1\} \forall i \in N \tag{2.15}$$

$$0 \leq x_{iq} \leq 1 \forall i \in N, q \in Q \tag{2.16}$$

Sets	
N	Set of all nodes on the Graph G
Q	Set of all OD pairs
A_q	Set of all directional arcs on the OD path $q \in Q$ from origin to destination
$K^q_{j,k}$	Set of all potential station locations, that can refuel the directional arc $a_{j,k} \in A_q$
Variables	
z_i	Binary Variable that equals to one, if a refuelling facility is constructed at node i
x_{iq}	Semi-Continuous Variable that indicates the proportion of vehicles on path q that are refuelled at node i
Parameters	
p	Fuel efficiency / fuel consumption per vehicle range
c	refuelling capacity per node
f_q	Total vehicle flow on the OD path q
y_q	Proportion of vehicles that are to be refuelled on path q
l_q	Number of refuelling occasions on path q depending on the total path distance, l_q = ceil {total trip distance / vehicle range}
g_{iq}	Binary indicator, that is set to one, if node i is a potential station location on path q
r_{iq}	refuelled driving distance at node i on path q

For the consideration of node capacity limits, Kluschke et al. (2020) added constraints (2.11) - (2.13) to the FRLM. Constraint (2.11) limits the total amount refuelled at node i to the maximum capacity of the station built there. The demand served at node i is computed as the flow of trucks (f_q) multiplied with the fuel consumption per vehicle range (p) and the amount of refuelled km (r_{iq}). Given three exemplary values, the total refuelling amount would be calculated like this: $2 * 0{,}5 \frac{\text{l}}{\text{km}} * 100 \text{ km} = 100 \text{ l}$
Two trucks refuel enough fuel to travel 100 km. As they consume 0,5 litres per kilometre, they in total fill up 100 l of fuel.

The total amount refuelled is further influenced by the proportion of vehicles that shall be refuelled on path q (y_q), the proportion of vehicles on path q that refuel at node i (giq) and whether node i is a potential station location at all (g_{iq}). Note, that unlike in the original AC-PC FRLM formulation by Capar et al. (2013), y_q is not a variable, but a parameter. As Kluschke et al. (2020)'s main objective is to determine the minimal amount of AFS that serve the total demand, y_q is set to 1 for all $q \in Q$.

Constraint (2.12) defines, that all vehicles on path q can refuel the arc $a_{j,k}$ at any of the possible locations given by the set $K^q_{j,k}$. Constraint (2.13) ensures that all vehicles of a flow refuel at the ensured number of refuelling occasions on path q. (2.14) states that vehicles can only refuel at node i if there is an open facility. (2.15) and (2.16) define the decision variables.

Calculation of Sets and Parameters

The previous paragraph discussed Kluschke et al. (2020)'s model assumptions and introduced the reader to their node-capacitated FRLM formulation. As their model is an extension of Capar et al. (2013), the core of the AC-PC FRLM, the set of fuel station candidate locations $K^q_{j,k}$, is present as well. Kluschke et al. (2020) modified $K^q_{j,k}$ due to their utilisation of single trips instead of round trips.

Hence they use a different pre-generation algorithm, which is explained in the following.

To compare the total amount refuelled at a station with its capacity limit, Kluschke et al. (2020) introduce the parameter r_{iq} that displays the amount of refuelling of each vehicle at gas stations. As r_{iq} needs to be computed before the optimization, its pre-generation process is as well illustrated in the upcoming section.

Contrary to Capar et al. (2013), Kluschke et al. (2020) use single trips instead of round trips. They furthermore assume that vehicles end their trips with the initial fuel level. Thus, they take a different approach in the pre-generation of the set $K^q_{j,k}$. The main adjustments are:

- $K^q_{j,k}$ is generated while iterating node-wise and not arc-wise over each path q.
- All vehicles start with a similar, predetermined initial fuel range (IFR). The IFR of vehicles in the AC-PC FRLM, on the other hand, is endogenously determined by the optimal location of fuel stations on each path.
- Arcs are divided into *critical* and *non-critical* arcs. An arc is considered as *critical*, if the travelability or refuelling of the path is not automatically guaranteed, e.g. through the initial fuel level.
- As vehicles start and end the trips with the same fuel level, drivers have to refuel at their last stop enough to not only reach the destination but to reach it with the initial fuel level. To respect the maximum capacity of the tank, an adjusted trip distance is used to calculate the set $K^q_{j,k}$, every time the iteration reaches a destination node. The adjusted distance formula is calculated as follows: $\text{AD}_q = \text{TD}_q + \text{IFR} - \text{DO}_q$

  The adjusted distance of path q (AD_q) equals the sum of total trip distance (TD_q) and initial fuel range (IFR) minus the network access distance from the origin node (DO_q). Note, that DO_q only has a value greater than zero, if the origin node does not lie within the considered network. Else wise $\text{DO}_q = 0$.

A pseudo-code for the generation of $K^q_{j,k}$ according to Kluschke et al. (2020) is given in the Code Listings 2 and 3. For better clarity, the process of identifying potential station locations was black-boxed in the code below and explained in a separate listing. A flowchart of the algorithm can be found in Appendix A for further illustration.

To generate the set $K^q_{j,k}$, the algorithm determines the set of potential facility locations, in case that reaching the end of an arc $a_{j,k}$ requires refuelling. Therefore, only arcs are considered, whose destination node either lies outside the initial fuel range from the origin or is as well the destination node of the OD trip.

```
for all q ∈ Q:
    for all nodes i on path q:
        if distance(origin, node i) ≤ initial
        fuel range:
            """if arc with destination node i
            lies within IFR, it is
            non-critical and already refuelled"""
            if node i is destination node of
            path q:
                identify potential station
                locations** using the
                adjusted distance
                """before reaching the
                destination, vehicles need
                to refuel to end with IFR"""
            else:
                continue with the next node in
                the loop
        else:
            if node i is destination node of
            path q:
                identify potential station
                locations** using the
                adjusted distance
            else:
                identify potential station
                locations*
```

Code Listing 2: Algorithm for determining the set $K^q_{j,k}$ in the NC-FRLM

Apart from the set $K^q_{j,k}$ it is also necessary to pre-calculate the values of the parameter r_{iq} in the NC-FRLM. r_{iq} displays the driveable distance, that shall be added to the current vehicle range by refuelling at node i on path q, in case a gas station is built there. Within the model, r_{iq} is used to both represent the drivers' efficient refuelling strategy and determine the total amount of refuelling at a gas station z_i. Although it is an essential part of Kluschke et al. (2020)'s model extension, the calculation of r_{iq} and its role as a parameter are only partially described. The following approaches this issue by providing a comprehensive overview of the parameter r_{iq} and its pre-generation process.

For the NC-FRLM, Kluschke et al. (2020) assume, that vehicles start and end their trips with the same fuel level and refuel efficiently on their way. That implies that drivers only take as many refuelling stops on the route as needed. Hence, Kluschke et al. (2020) define the following, underlying refuelling strategy for their model: A driver will always fill up the maximal possible amount until the last stop. There, the driver refuels the difference between the total fuel needed to complete the trip and the total fuel filled up during the previous gas station stops. In the end, the driver has refilled the exact amount of fuel that he had consumed during the trip. Hence, he ends the route with the initial fuel level in the tank.

```
# * if i is not the destination node of path q
for all nodes k, that lie on the path from
    origininto node i:
    if distance(origin, node i) -distance(origin,
    node k) ≤ vehicle range:
        if node k is a potential station location
        (parameter g_kq = 1):
            add node k to K^q_{i-1,i}

# ** if i is the destination node of path q
for all nodes k, that lie on the path from origin
to destination node i:
    if adjusted distance(origin, node i) -
    distance(origin, node k) ≤ vehicle range:
        if node k is a potential station location
        (parameter g_kq = 1):
            add node k to K^q_{i-1,i}
```

Code Listing 3: Identification of potential station locations in the $K^q_{j,k}$ algorithm in the NC-FRLM

Calculating the difference between the current and the maximum fuel level, to determine the amount of possible refuelling, might seem relatively easy. Especially, as fuel consumption is assumed to be directly proportional to the distance travelled. The necessary consideration of refuelling along the route, however, adds a variable component to the calculation.

The exact fuel level, and therefore the current vehicle range, depends on the following three factors:

- The initial fuel range at the origin node,
- The total distance travelled from the origin node to node i,
- Possible refuelling and the corresponding refuelling amounts at nodes along the way from the origin node to node i, which necessitates the modelling as a decision variable.

For the benefit of the model complexity, Kluschke et al. (2020) desist from precisely calculating the current fuel level and modelling the refuelling through additional decision variables and constraints. Instead, they estimate the values of the refuelling amount r_{iq}.

Kluschke et al. (2020) therefore subdivide each OD path into l_q route sections. l_q is the model parameter that indicates the number of refuelling occasions on path q. l_q is calculated as l_q = ceil {total trip distance / vehicle range}. As l_q depicts the number of refuelling occasions on path q as well as the number of route sections, each vehicle refuels only once per route section. Although the refuelling capacity of the tank, the difference between the current and the maximal fuel level, varies between nodes, the values of r_{iq} are identical for each node within a route section. Following the refuelling strategy, drivers will refuel the maximal tank capacity in every route section, but the last one. Depending on the location of the fuel stations along the route, it is likely to happen, that the vehicle's tank is not empty when refuelling the maximal tank capacity. Refuelling then leads to exceeding the maximal tank capacity and therefore to overflowing.

Kluschke et al. (2020) assume, that due to their definition of the set $K^q_{j,k}$ and the model's objective of minimizing the total amount refuelling stations, facilities will be built preferably at the end of the subdivided route sections so that the minimal amount of facilities can refuel every arc of the OD path. In consequence, vehicles would fill up instead at the end of the sections and with only a small amount of rest fuel left in the tank, so that the overflowing would not be substantial. In the model, the overflowing is not considered, and the refuelled amount is added to the tank.

For the calculation of r_{iq}, Kluschke et al. (2020) differentiate between three cases:

- if $l_q = 1$ (one refuelling stop, one route sections) vehicles shall fill up the amount of fuel necessary to cover the total trip distance at any of the potential station locations.
- if $l_q = 2$ (two refuelling stops, two route sections) in the first section, defined by the initial fuel range, vehicles shall refuel the maximal vehicle range. In the second section, vehicles shall refuel the difference between the total trip distance and the already refuelled amount.
- if $l_q \geq 2$ (multiple refuelling stops, multiple route sections) in every route section, apart from the last one, vehicles shall refuel the maximal vehicle range. On the last stop, vehicles refuel the difference between the total trip distance and the already refuelled distance. The first $l_q - 1$ route sections are defined by the vehicle range.

The pseudo-code for the algorithm, that determines the parameter r_{iq} is displayed in Code Listing 4:

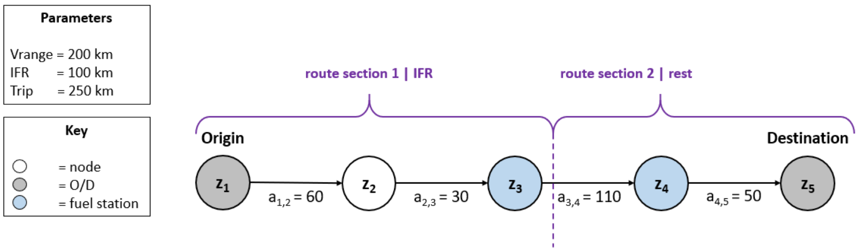
According to the algorithm, vehicles will refuel the maximum tank capacity in every route section, but the last one. In the last gas station stop, they will fill up the difference between the total fuel needed to travel the OD path and the refuelled amount in the previous gas station stops.

In case the trip has only one route section, the amount of fuel needed to travel the whole distance will be refuelled in the only fill-up. As vehicles refuel once per route section, drivers refuel precisely the amount needed to travel the OD path and therefore end the trip with the initial fuel level.

For a better understanding, the parameter r_{iq} and its role in the model is illustrated in a simple example below:

Figure 5 shows a five node network with the origin-destination pair (1,5). The total trip distance is 250 km. The maximal vehicle range amounts to 200 km, and the initial fuel range is 100 km. Therefore the number of necessary refuelling stops l_q is two.

The Parameter r_{iq} is calculated to constitute the driving distance a vehicle would refuel the nodes of the trip, in case there was an operating fuel station. According to the above-presented algorithm, the OD trip is subdivided into two sections, of which the first one has the length of the initial fuel



	z_1	z_2	z_3	z_4	z_5
Current fuel range	100	40	210	150	100
r_{iq}	200	200	200	50	50
Amount refueled	0	0	200	50	0
Total refueling	0	0	200	250	250

Figure 5: Exemplary calculation of the parameter r_{iq} for a five node graph network

```
for all paths q ∈ Q:
    for all nodes i on the path q:
        if the total number of refuelling stops
        l_q = 1:
            r_iq = total path distance
        else:
            if the total number of refuelling
            stops l_q = 2:
                if the distance(origin, node i) ≤
                initial fuel range:
                    r_iq = maximal vehicle range
                 else:
                    r_iq = (total path distance -
                    vehicle range)
            else:
                if the distance(origin, node i) ≤
                (vehicle range * (l_q - 1)):
                    r_iq = maximal vehicle range
                 else:
                    r_iq = (total path distance -
                    (vehicle range * (l_q - 1)))
```

Code Listing 4: Algorithm for determining r_{iq} in the NC-FRLM

range. The length of the second and final section accounts for the difference between the total trip distance and the distance of the prior section. As described, the value of r_{iq} is similar for each node of the corresponding section and amounts to the vehicle range in the first one. The refuelled distance in the final section equals the difference between the total trip distance and the total amount refuelled in the prior sections. As drivers in total filled up precisely the amount needed to travel the trip distance, they end with the initial fuel level.

Given an optimal solution of the problem, z_3, z_4 = 1, fuel stations are constructed at nodes three and four. When a vehicle refuels at node three according to the model, it has fuel for 10 km left in the tank and would overflow while refuelling 200 km. Kluschke et al. (2020) hereby pretend that refuelling more than the maximal refuelling capacity is possible, and the excess fuel is not wasted.

Kluschke et al. (2020)'s general idea of estimating the refuelling amount rather than precisely calculating it through decision variables to benefit the model complexity is reasonable. Although the tank level and the refuelling amount is not accurate, the share of overflowing in the total amount refuelled, and therefore the inaccuracy, is small due to the model setting. Thus, the capacity utilization of the station can still be approximated rather well. Although the extent of the inaccuracy and its possible impact on the optimal solution have not been further examined by Kluschke et al. (2020), it seems like a fair trade-off for the reduced model complexity.

As the calculation of r_{iq}, the corresponding assumptions and their motivation seem comprehensible, the multi-period node-capacitated FRLM applies the same logic with minor corrections to the r_{iq} generation algorithm.

3. New FRLM Extension: The Multi-Period Node-Capacitated FRLM

The previous chapter provided the reader with a comprehensive overview of the current FRLM literature in the first part and subsequently introduced the reader to the current state of the art FRLM modelling. Most importantly, profound knowledge about the MP-NC FRLM predecessor models, the AC-PC FRLM by Capar et al. (2013) and the NC-FRLM by Kluschke et al. (2020), was conveyed.

Capar et al. (2013)'s model refuels the OD trips arc-wise. If every arc on a trip can be refuelled at one of the operating fuel stations, the whole path is considered refuelled and travelable. Kluschke et al. (2020) extend Capar et al. (2013)'s AC-PC FRLM with capacity restrictions, that limit the total amount of refueling at fuel stations.

The multi-period node-capacitated FRLM is formulated as a maximal covering problem and seeks to maximize the number of refuelled OD trips given fuel station construction costs and a periodic budget. The model considers the possible value change of parameters over time and in turn provides a period-by-period plan for the step-wise development of an AFS refuelling infrastructure.

In the following sections, the model assumptions, the mathematical formulation, possible problems and the calculation of the sets and parameters are discussed and explained. The chapter is concluded by adapting the two multi-period model evaluation concepts, the VMPS and the VMPP, to the MP-NC FRLM and discussing further situational model assumptions and their impact on the model.

3.1. Model Assumptions

The following paragraph explains and discusses the assumptions made in the MP-NC FRLM. Apart from adapting previous assumptions by Capar et al. (2013) and Kluschke et al. (2020), assumptions that have been used by Kluschke et al. (2020) but are not explicitly stated in their assumption list, have also been added. Furthermore, additional MP-NC FRLM modelling presumptions, have been appended alongside further suppositions, that provide a better understanding of the model circumstances and possible use cases. The assumptions are thus subdivided into *General Modeling Assumptions*, which define the general model setting and are needed for obtaining a feasible solution, and *Case Specific Assumptions*. The below-listed assumptions describe a more general modelling framework than Kluschke et al. (2020), as they have specifically tailored their model assumptions to their case study.

The *General Modeling Assumptions* are listed below, modified and additional assumptions are highlighted in italics. It is important to note that Assumptions 9 and 10 have already been used by Kluschke et al. (2020).

Kluschke et al. (2020) do not address these assumptions in their assumption listing, but chose to introduce them later within their application context. In the interests of completeness, they are added to the *General Modeling Assumptions* and also highlighted in italics, as they have not yet formally appeared in assumption form.

1. The traffic between an origin-destination pair flows on the shortest path through the network.
2. The traffic volume between OD pairs is known in advance for all periods.
3. Drivers have full knowledge about the location of the fuel stations along their path and refuel efficiently to complete their trips. To minimize the number of refuelling stops on the road, drivers will always refuel the maximum tank level until their last stop.
4. Only nodes of the network are considered as possible refuelling facility locations.
5. All vehicles *on the same OD path* are assumed to be homogeneous in terms of maximal driving range, initial fuel level and fuel consumption.
6. The fuel consumption is directly proportional to the distance travelled.
7. Nodes and fuel stations are capacitated.
8. The initial fuel level and the ending fuel level have to be known in advance for every path.
9. *A station constructed at a node i will always have the maximal possible size, and the station capacity, therefore, equals the node-specific capacity limit.*
10. *The OD path is subdivided into* $\mathrm{l_q} = ceil\{\mathrm{d_q} \,/\, \theta_\mathrm{q}\}$ *route sections, with* $\mathrm{d_q}$ *being the total distance of path* q *and* θ_q *displaying the vehicle range. The amount of refuelling per vehicle is similar for each node of the corresponding route section if a station is built there. Each vehicle refuels once per route section.*
11. *The distances between two connected nodes are sufficiently small.*
12. *The number of periods is predetermined and each period has an equal length.*
13. *Once a facility is built at a node i, it has to remain open until the final period.*
14. *A periodic budget limits the number of fuel stations constructed per period.*
15. *The situation is modelled from a central planner's perspective.*

The first eight assumptions were taken from Kluschke et al. (2020) with two minor adjustments so that further parametric specification is possible. Assumption 5 relaxes the presumption that all vehicles on all paths have to be homogeneous and represents the minimal vehicle requirements for obtaining a feasible solution. While it is not possible to differentiate between vehicles on one path, path specific vehicle characteristics, like the maximal range or the fuel consumption, can be respected. In the context of transportation, an example of different vehicle characteristics on different OD paths would be the use of different truck types for long-haul transportation and local good distribution.

With a similar relaxation, Assumption 6 allows a pathwise specification of the vehicles' initial and ending fuel level. As there are no conditions on the choice of the initial and ending fuel levels, the siting of fuel stations at the origin and destination nodes of paths is contrary to Kluschke et al. (2020) allowed. Even though it is not necessary for obtaining a feasible solution, it is still reasonable to assume that drivers refuel only the total trip distance and therefore end the trip with the initial fuel level.

Assumption 9 defines that the capacity of a station will always equal the maximum capacity of the corresponding node. Although the MP-NC FRLM hereby applies the same logic as Kluschke et al. (2020), the maximal node capacity in the MP-NC FRLM is not unitary and can be defined nodewise. Although the partial utilization of the maximal node capacity is not possible in this case, it could theoretically be modelled through different station sizes going along with additional variables and constraints.

Assumptions 10 and 11 provide the foundation for the heuristic calculation of the refuelling amount at a gas station represented through the previously discussed parameter r_{iq}. For the benefit of the model complexity, the refuelling amount at node i on path q is estimated and not precisely calculated through decision variables similarly to Kluschke et al.

(2020). When the network edges are relatively long (compared to the vehicle range) or the total trip distance of an OD path is close to an integral multiple of the vehicle range it can occur, that the number of refuelling occasions l_q is not sufficient for refuelling the path. In that case, it is advised to incrementally increase the path specific l_q value to be able to cover the trip entirely. This peculiarity is further discussed in the next section.

For formulating a multi-period optimization problem, the number of considered periods has to be known in advance. As dividing time into periods of equal length is common in optimization models and general planning, Assumption 12 can be considered as a relatively standard assumption.

It is further assumed that a gas station, once opened, cannot be closed or relocated due to the high cost involved in the process (Assumption 13).

Assumption 14 limits the number of stations constructed per period. Limiting the station numbers seems reasonable because the construction of fuel stations is resource-intensive and resources, like budget or labour, are limited. Finally, it is essential to, once again, emphasize, that the model aims at providing a plan for developing a refuelling infrastructure over time from a central planner's perspective. The MP-NC FRLM in its current form is not suited for profit maximization for, e.g., a gas station operating firm.

The *Case Specific Assumptions* are a non-definite set of assumptions, that are made in the context of the application case. They, for example, contain information about the presumed development of parameters over time, like the vehicle flow on a path, or the definition of the sets. In the below-presented base formulation of the MP-NC FRLM, only three *Case Specific Assumptions* are considered. A more comprehensive list of possible *Case Specific Assumptions* and their impact on the model can be found in section 2.3.4.

1. The vehicle flow is expected to rise periodically due to an increase in the AFV market share.
2. The model mainly considers construction costs. Operating costs are not respected.
3. Different node capacities do not impact the facility construction cost.

Considering the fact, that AFVs currently stand at the beginning of their product life cycle and are mainly bought by "Early Adopters", it seems reasonable to assume an increase in market share and therefore a rise of the AFV vehicle flow (Assumption 1). The MP-NC FRLM models the development of alternative fuel refuelling structures from the perspective of a central planner who constructs fuel stations respectively subsidizes their construction. As the central planner is assumed not to be the operator of the fuel stations, Assumption 2 limits the budgetary expense to the siting of the facilities.

Even though drastic differences in node capacity limits, and thus large differences in the station sizes, have an impact on the facility construction costs, it is not considered in this model to reduce the estimation effort for parameters. The further *Case Specific Assumptions* and the extended model in section 2.3.4, on the other hand, do include the impact of station capacity on construction costs.

3.2. Mathematical Formulation and Possible Problems

In the upcoming section, the mathematical formulation of the MP-NC FRLM is presented as further explained. The main differences to the node-capacitated FRLM by Kluschke et al. (2020) are as follows:

- A maximal covering objective has been selected instead of a set covering.
- A time module has been added to consider multiple construction periods.
- A periodic budget was introduced, that limits the number of fuel stations constructed per period.

Furthermore, two problems of the formulation are discussed. To maximize the path coverage it can occur, that even though a majority of the paths is covered, only a small part of the total flow is covered. Two possible solutions are the introduction of a lower bound for periodic flow coverage in the constraints and the maximization of the flow instead of the path coverage. The second part of the problem discussion examines specific parametric constellations, where the prespecified number of l_q fuel stations are insufficient to cover the OD trip.

$$\max \sum_{t\in T}\sum_{q\in Q} y_q^t \tag{3.1}$$

$$\text{s.t.} \sum_{i\in K_{j,k}^q} z_i^t \geq y_q^t \,\forall\, q\in Q,\ a_{j,k}\in A_q,\ t\in T \tag{3.2}$$

$$\sum_{q\in Q} f_q^t\, p\, r_{iq}\, g_{iq}\, x_{iq}^t \leq c_i\, z_i^t \,\forall\, i\in N,\ t\in T \tag{3.3}$$

$$\sum_{i\in K_{j,k}^q} x_{iq}^t = y_q^t \,\forall\, q\in Q,\ a_{j,k}\in A_q,\ t\in T \tag{3.4}$$

$$\sum_{i\in N} x_{iq}^t = y_q^t\, l_q \,\forall\, q\in Q,\ t\in T \tag{3.5}$$

$$x_{iq}^t \leq z_i^t \,\forall\, i\in N,\ q\in Q,\ t\ in\ T \tag{3.6}$$

$$z_i^t \leq z_i^{t+1} \,\forall\, i\in N,\ t\in T\backslash\{n\} \tag{3.7}$$

$$z_i^t - z_i^{t-1} \leq k_i^t \,\forall\, i\in N,\ t\in T\backslash\{1\} \tag{3.8}$$

$$z_i^1 \leq k_i^1 \,\forall\, i\in N \tag{3.9}$$

$$\sum_{i\in N} o\, k_i^t \leq b_t \,\forall\, t\in T \tag{3.10}$$

$$\sum_{t\in T} k_i^t \leq 1 \,\forall\, i\in N \tag{3.11}$$

$$z_i^t,\ k_i^t \in \{0,1\} \,\forall\, i\in N,\ t\in T \tag{3.12}$$

$$0 \leq x_{iq}^t \leq 1 \,\forall\, i\in N,\ q\in Q,\ t\in T \tag{3.13}$$

$$0 \leq y_q^t \leq 1 \,\forall\, q\in Q,\ t\in T \tag{3.14}$$

Sets	
N	Set of all nodes on the Graph G
Q	Set of all OD pairs
T	Set of all time periods
A_q	Extended set of all critical arcs on the path $q \in Q$ from origin to destination
$K^q_{j,k}$	Set of all potential station locations, that can refuel the directional arc $a_{j,k} \in A_q$
Variables	
z^t_i	Binary Variable that equals to one, if a refuelling facility is open at node i in time period t
k^t_i	Binary Variable that equals to one, if a refuelling facility is constructed at node i in time period t
x^t_{iq}	Semi-Continuous Variable that indicates the proportion of vehicles on path q that are refuelled at node i in time period t
y^t_q	Semi-Continuous Variable that indicates the proportion of flow served on path q in time period t
Parameters	
p	Fuel efficiency / fuel consumption per vehicle range
o	Facility opening costs / construction costs
c_i	refuelling capacity at node i
d_q	total distance of path q
θ_q	vehicle range of vehicles on path q
l_q	Number of refuelling occasions on path q depending on the total path distance, $l_q = \text{ceil}\left\{d_q \ / \ \theta_q\right\}$
b_t	Available budget in period t
$f^{\,t}_q$	Total vehicle flow on the OD path q in time period t
g_{iq}	Binary indicator, that is set to one, if node i is a potential station location on path q
r_{iq}	refuelled driving distance at node i on path q

Contrary to Kluschke et al. (2020), the MP-NC FRLM does not seek to minimize the total number of stations necessary to cover 100 % of the flow. The objective function (3.1) aims at maximizing the total number of refuelled paths over all periods. Thus, the early coverage of paths is rewarded, and a refuelling network is planned, that covers as many OD trips as possible as early as possible.

Constraints (3.2) - (3.6) are similar to Kluschke et al. (2020) except that it is now possible in (3.3) to set the node capacity limits node-wise to be more responsive to the local capacity restrictions.

Constraint (3.7) ensures, that once a facility is opened at a node i, it has to remain open until the final period. Constraints (3.8) and (3.9) define, that a facility is constructed, if it is either open in a period t, but has not been open in the previous period or if it is open in the first period. Constraint (3.10) states that the total amount spent on the construction of fuel stations in a period t must be within the scope of the budget of the corresponding period. According to (3.11), a station can be constructed only once at a node over all periods. (3.12) - (3.14) conclude the model by defining the decision variables.

Objective Function and Model Purpose

Following the logic of the objective function, the MP-NC FRLM attempts to site fuel stations in a way that, at best, facilities contribute to refuelling multiple OD paths. In consequence, the model tends to construct a network of connected refuelled OD paths. As only the number of covered OD paths and not the covered flow volume is taken into account, it is possible, that even though a majority of the paths in a final period T is covered, the share of refuelled flow might be relatively low. This can be problematic, as a central planner, on the one hand, aims at covering a wide area, but on the other hand, wants as many drivers as possible to profit from the constructed fuel stations. An exemplary situation demonstrating this conflict is illustrated below.

Figure 6 shows a ten node network with the OD pairs (1,5), (1,6), (7,5) and (8,10). The periodic budget is sufficient to build one fuel station per period, and two periods are considered in this case. While the flow on OD paths (1,5), (1,6) and (7,5) amounts to two in every period, the flow on (8,10) is considerably greater with a value of 100.

As can be seen in table 2, constructing stations at nodes 3 and 4 is the optimal solution of the problem, as it covers the maximal possible amount of three OD paths while respecting the budgetary constraints of constructing only one station per period. For the optimal solution, it is irrelevant whether station 3 or 4 is constructed first. In the final period, even though 75 % of all OD trips are covered, only 5.67 % of the total flow is served.

Two possible approaches to address this issue are:

- **Setting a lower bound to the minimum flow covered per period**

  A possible approach to respecting the flow volume while maximizing the number of paths covered is the introduction of a new constraint, that sets a lower bound for the fraction of total flow covered in period t. v^t represents this new lower bound, with $v^t \in [0,1]$.

$$\text{s.t.} \ \frac{\sum_{q \in Q} y^t_q * f^{\,t}_q}{\sum_{q \in Q} f^{\,t}_q} \geq v^t, \ \ \forall \ t \in T \qquad (3.15)$$

  When applying constraint (3.15) to the MP-NC FRLM, the proper selection of v^t is essential. In case the available budget is not sufficient to cover the predetermined minimal fraction of flow, the model becomes infeasible. While the dimensioning of v^t falls to the preferences of the central planner, the maximal lower bound is determined through solving the maximal flow covering formulation of the MP-NC FRLM (c.p.). The formulation

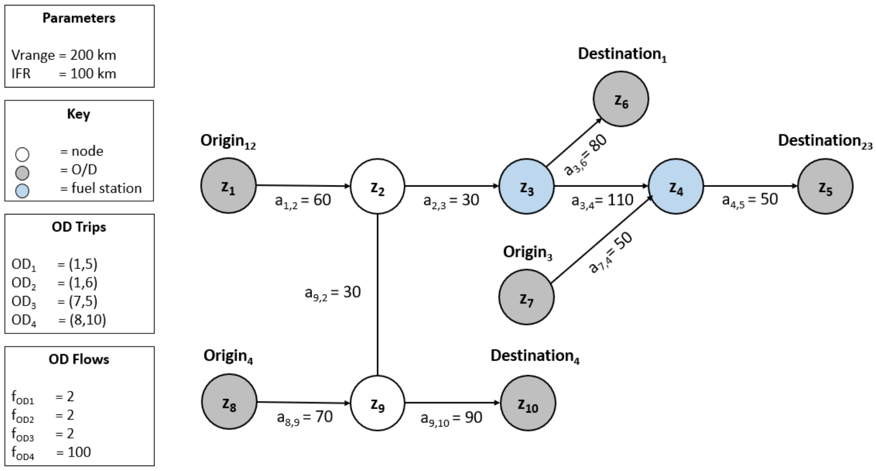


Figure 6: Exemplary display of potential problems with the max. path coverage objective function

Sets	Potential Station Locations				
$K_{3,4}^{(1,5)}$	z_1	z_2	z_3		
$K_{4,5}^{(1,5)}$				z_4	
$K_{3,6}^{(1,6)}$			z_3		
$K_{4,5}^{(7,5)}$				z_4	
$K_{9,10}^{(8,10)}$					z_9

OD Trip	Possible station combinations
(1,5)	(z_1,z_4), (z_2,z_4), (z_3,z_4)
(1,6)	(z_3)
(7,5)	(z_4)
(8,10)	(z_9)

Table 2: Set $K_{j,k}^{q}$ and potential facility combinations, that could cover the OD trips in the problem in figure 6

of the MP-NC FRLM including constraint (3.15) can be found in Appendix A.

- **Maximizing the flow coverage instead of maximizing the path coverage in the objective function**

  Another possible approach to considering the flow volume in the MP-NC FRLM is to weigh the OD paths with the corresponding flow in the objective function. Weighting the OD paths leads to a maximization of the flow coverage instead of the number of paths covered.

$$\max \sum_{t \in T} \sum_{q \in Q} f_q^t \; y_q^t \tag{3.16}$$

  While the base model has the natural tendency to create a coherent network of refuelled OD paths, the connectivity of refuelled OD paths in the maximal flow covering MP-NC FRLM solely depends on whether the OD paths with a high flow volume are linked. As refuelling paths with a higher flow volume are preferred over covering OD trips, that share the same route for most of their trip, the served OD paths can be scattered throughout the network. Thus, the refuelled routes are less likely to be interconnected in early construction stages, which, in turn, restrains the possibilities of free-roaming travel within the underlying road network. The formulation of the maximal flow covering MP-NC FRLM can be found in Appendix A.

Parameter l_q and the Coverability of Routes

Another potential problem for the functionality of the model can be posed by the current definition of the parameter l_q. As mentioned above, it can occur, that in some cases l_q refuelling locations are not enough to cover an OD trip. Although an insufficient number of ensured refuelling locations does not lead to an infeasible model, the optimal allocation of refuelling stations along the way becomes trivial, and the path will not be respected during the optimization. An exemplary situation demonstrating this problem is shown below.

Figure 7 shows a five node network with a single OD pair (1,5). The vehicle range is 200 km, the initial fuel range 100 km and the total trip length accumulates to 380 km. According to the above-defined formula, $l_q = \text{ceil}\left\{d_q \,/\, \theta_q\right\} = 2$. Although two stations are assigned to this path, three would be necessary to cover the whole path. Due to the topological structure of the underlying network, drivers would have to refuel before every, but the first arc. In case the IFR was lower than the length of the first arc, 100 km, even four stations would be needed to refuel the OD path.

In consequence it is not possible to cover the path with $l_q = 2$ stations and the optimal solution is therefore $y_q = 0$. It is important to note that this result is a feasible solution and

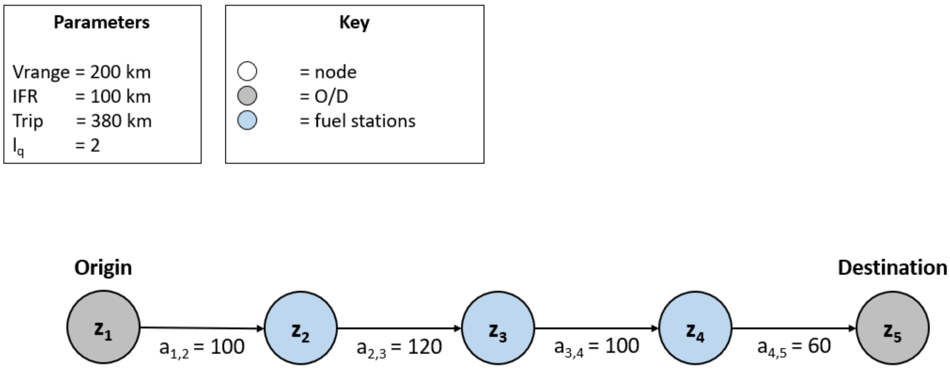


Figure 7: Illustration of the problems with the definition of l_q in the MP-NC FRLM.

does not violate the model constraints.

While the 380 km long OD trip can not be covered under this parametric constellation, it is possible to refuel a similar path with two fuel stations, after changing the arc lengths of $a_{3,4}$ and $a_{4,5}$ (see figure 8).

While it has not been possible to identify non-coverable paths before the calculation reliably, one can say, that in general, l_q fuel stations are insufficient to cover an OD path, if the character respectively the length of the path edges make extra refuelling stops necessary. Additional refuelling stops become necessary, if the total amount of wasted but refilled fuel, exceeds a certain critical level. The amount of fuel wasted equals the total overflowing amount at the refuelling stops (fuel overflowing is further explained within the context of the definition of the parameter r_{iq} in section 2.2.).

The critical level respectively, the tolerable refuelling error margin declines, the closer the total trip distance gets to the next greater integral multiple of the vehicle range. In the extreme case, that the path length is only infinitesimally shorter than an integral multiple of the vehicle range, the error margin becomes zero. Then, every bit of fuel is needed to reach the destination with the IFR left in the tank. Therefore, the path edges would need to be entirely in line with the refuelling, meaning that every vehicle has to arrive at the gas stations with an entirely empty tank.

This issue becomes more evident when taking a closer look at the borderline example in figure 9.

Figure 9 shows two similar OD paths. The length of the first one is infinitesimally shorter than the doubled vehicle range, whereas the second path length is infinitesimally longer. In consequence, l_q equals two in the first case and three in the second case.

As can be seen in the figure, path one is not coverable, as three gas stations are needed, while only two are allowed by l_q. Two refuelling stops are insufficient because drivers need to fill up at z_3, even though they still have enough fuel in the tank to travel another 100 km. If the arcs were perfectly in line with the refuelling, meaning $a_{3,4} = 100$ and $a_{4,5} = 100 - \xi$, the OD trip would be fully refuelable. On the other hand, path two is fully coverable and has the maximal tolerable refuelling error, as its trip distance is as far away from the next greater integral multiple of the vehicle range as possible.

With the concept of non-coverable paths in mind, it is possible to find an upper bound for the number of necessary gas stations on a path. Therefore a generic path setting can be constructed, that maximizes the amount of overflowing by forcing the vehicle to refuel before every arc of the path, but one. Within this setting, every pair of adjacent arcs has to have a combined length that is greater than the vehicle range, except for the last pair of arcs. This way, every path length is representable in such a form.

In the exemplary path setting in figure 10, the length of every pair of arcs, except for the last one, is subdivided into $\frac{\text{vrange}}{2}$ and $\frac{\text{vrange}}{2} + \xi$. The IFR is insufficient, to travel the first arc, so vehicles have to refuel right at the beginning. Although the value of the IFR does not influence the upper bound, it does influence the location of the stations. In case the IFR is sufficient to travel the first arc of the path, vehicles have to refuel at every node but the first one. If the IFR is smaller than the length of the first arc, vehicles fill up at every node except for the destination node.

For the case, that the IFR equals the length of the first arc, it depends on the length of the two final arcs, whether vehicles refuel at the destination node.

The upper bound of l_q is calculated as following with θ_q representing die vehicle range on path q and d_q representing the total path distance of path q:

$$l_q^{\max} = \text{ceil}\left\{\frac{d_q}{0,5\ \theta_q}\right\} \tag{3.17}$$

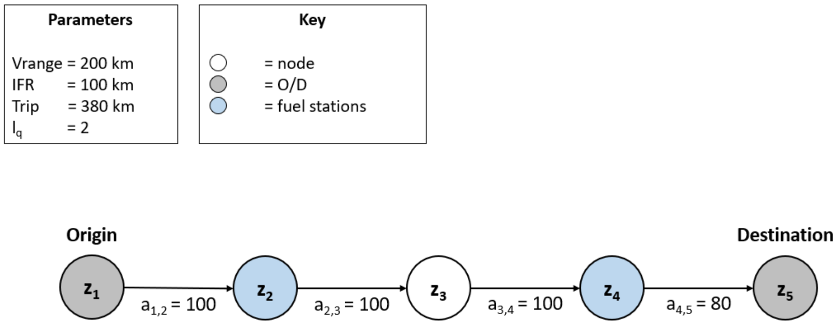


Figure 8: Illustration of the problems with the definition of l_q in the MP-NC FRLM 2

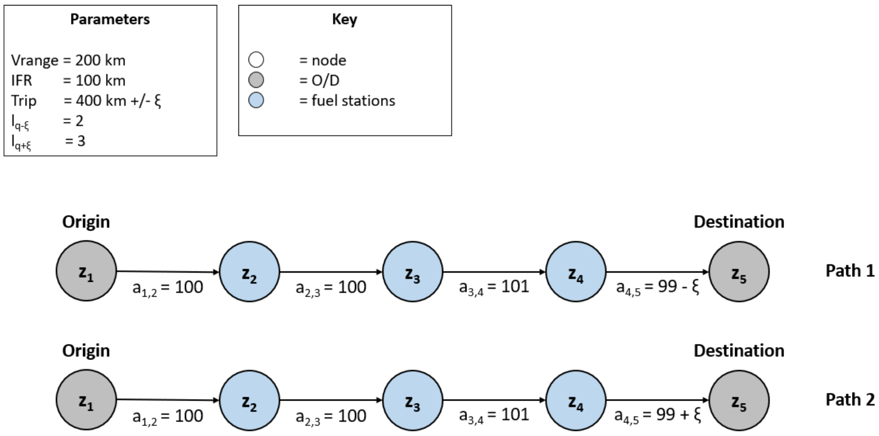


Figure 9: Borderline Case for the tolerable refuelling error margin in the MP-NC FRLM.

Hence, the difference between l_q and its maximal value is:

$$\begin{aligned} \Delta_{\max} &= l_q^{\max} - l_q && (3.18)\\ &= \operatorname{ceil}\left\{\frac{d_q}{0{,}5\ \theta_q}\right\} - \operatorname{ceil}\left\{\frac{d_q}{\theta_q}\right\} && (3.19)\\ &= \operatorname{ceil}\left\{\frac{d_q}{\theta_q}\right\} && (3.20)\end{aligned}$$

In consequence, the maximal possible discrepancy between l_q and its maximal value grows, the greater the total path distance and the smaller the vehicle range is.

The search for a better path-specific calculation method for l_q has not yet been successful, but would be of great benefit for the model. As l_q constitutes the number of refuelling occasions on a path q, it is crucial to find a method, so that l_q matches precisely the number of necessary fuel stations, because every excess fuel station built unnecessarily stresses the periodic budget.

Although the current calculation method does not achieve fitting results for every OD path, it will work for most, under the assumption of sufficiently short edges. Until there is a better method, it is advised to keep a closer look at the OD paths with a length close to an integral multiple of the vehicle range, check for their coverability and manually increase l_q as necessary, even though this might be laborious on larger problems.

3.3. Calculation of Sets and Parameters

In the previous section, the reader was familiarised with the mathematical formulation of the MP-NC FRLM and its main differences to Kluschke et al. (2020)'s formulation. The second part discusses two problems that go along with the model formulation: For once the difficult choice between a maximal path and maximal flow covering problem and for second the illustration of topological circumstances, where l_q facilities are insufficient to cover an OD trip.

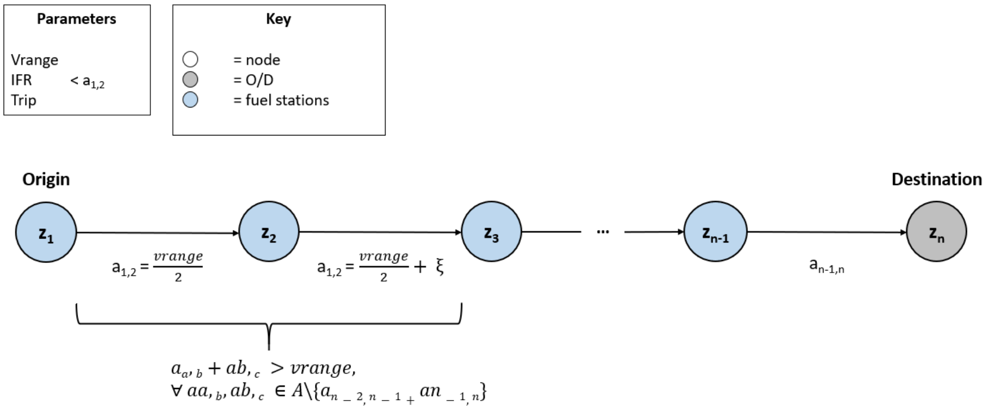


Figure 10: Construction of the l_q upper bound in the MP-NC FRLM.

The following section thematizes the calculation of sets and parameters in the multi-period node-capacitated FRLM. Although the model components, that require a pre-calculation, namely the set $K^q_{j,k}$ and the parameter r_{iq}, have been discussed before, their computation differs from Capar et al. (2013) and Kluschke et al. (2020).

For the calculation of $K^q_{j,k}$, the MP-NC FRLM follows Jochem, Szimba, and Reuter-Oppermann (2019)'s idea of splitting the pre-generation of the set into two parts. In the first part, the set of critical arcs A_q is derived by removing all non-critical arcs from the set of all arcs on path q. Besides, a new virtual destination arc technique is applied to A_q to address the shortcomings of Kluschke et al. (2020)'s Adjusted Distance method in guaranteeing the equality of starting and ending fuel levels. In the second part, $K^q_{j,k}$ is calculated by determining all possible refuelling locations for the critical arcs in A_q.

Furthermore, an improved algorithm for the calculation of the parameter r_{iq} is presented. The new algorithm is more compact and solves a shortcoming in Kluschke et al. (2020)'s route sectioning, which can lead to a violation of the initial equals ending fuel level assumption.

Extended Set of all Critical Arcs A_q

The calculation of the extended set of critical arcs A_q is subdivided into two parts. In the first step, a new virtual arc technique is applied to A_q to ensure that the initial and ending fuel levels can be identical. The technique adds a virtual arc with the length of the initial fuel range (IFR) to each path that has to be fully travelable by the vehicles. In the second step, the set of critical arcs is determined by removing all non-critical arcs from A_q, according to Jochem et al. (2019).

In the first part of the A_q calculation, a virtual arc with the length of the initial fuel range is added at the end of each path q and thus appended to A_q. Following the addition, the new virtual arc has to be treated as a real arc, although vehicles will end their OD at the OD path's destination node. In the end, the remaining fuel in the tank must equal the exact amount necessary to fully travel the virtual arc. Adding this virtual arc and respecting it in the $K^q_{j,k}$ serves as a measure to ensure that the initial fuel level equals the ending fuel. This method is used instead of the Adjusted Distance method by Kluschke et al. (2020), as its use can lead to an infeasible model in some cases, which is illustrated below.

Problems of the Adjusted Distance method

As demanded in Assumption 8, drivers have to refuel in such a way, that they do not only reach the destination node but reach it with the initial fuel level left in the tank. Kluschke et al. (2020) solve this difficulty, through artificially prolonging the last arc of the OD paths.

While vehicles refuel sufficiently to exactly reach the destination node via the adjusted distance, drivers end the trip with the initial fuel level left in the tank, as the actual distance of the last arc is shorter than the Adjusted Distance. The Adjusted Distance is calculated as follows:

$$\mathrm{AD}_q = d_{\mathrm{origin},i} + d_{i,\mathrm{destination}} + \mathrm{IFR}_q - \mathrm{DO}_q$$

The Adjusted Distance consists of the path distance from the origin to node i, ($d_{\mathrm{origin},i}$), the length from node i to the destination ($d_{i,\mathrm{destination}}$), the initial fuel range (IFR_q) and the access distance from the origin node to the network (DO_q). DO_q is a case-specific parameter, that only has a value > 0, if the origin node does not lie within the examined network.

From the pseudo-code in figure 11 follows, that a node i counts as a potential station location, if the Adjusted Distance from i to the destination node is smaller than the vehicle range θ_q:

$$d_{i,\mathrm{destination}} + \mathrm{IFR}_q - \mathrm{DO}_q \le \theta_q$$

In cases, where i is the starting node of the last arc of a path q and $d_{i,\mathrm{destination}} + \mathrm{IFR}_q - \mathrm{DO}_q > \theta_q$, no node satisfies the condition for being a potential station. In consequence, the final arc, and therefore the whole path, is not refuelable and the model is hence infeasible. As Kluschke et al. (2020) applied their model to cases with heavy-duty vehicles, that have a range of 800 km and an initial fuel level around 50

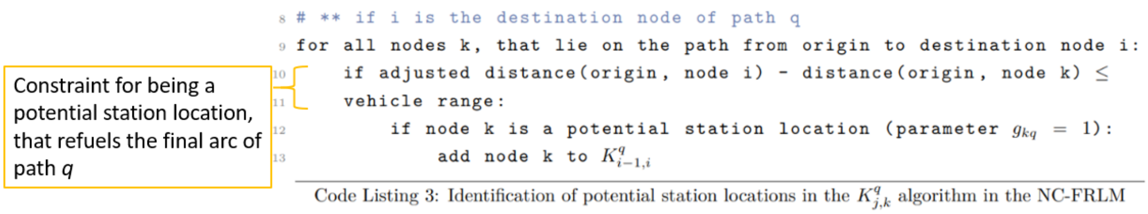


Figure 11: Implementation of the Adjusted Distance method by Kluschke et al. (2020).

%, a situation where no node satisfies the condition for refuelling the last arc of a path never occurred. A possible situation, where the last arc is not refuelable, is illustrated in figure 12.

Figure 12 shows a five node network similar to the one in figure 14, except, that in this example, the initial fuel range is 150 km. Both origin and destination node, lie in the considered network. DO_q is, therefore, 0.

The constraint for node z_4 to be a potential station location, that can refuel the final arc $a_{4,5}$, is as follows:

$$60 \text{ km } (d_{i,\text{destination}}) + 150 \text{ km } (\text{IFR}_q) \leq 200 \text{ km}$$

As this inequality is not right for the case-specific values, neither z_4 nor other nodes ($d_{i,\text{destination}}$ is greater for other nodes) can refuel the final arc. Therefore the whole path q is not refuelable.

Extended Set of all Critical Arcs A_q - Step 1

The shortcoming of the Adjusted Distance method is solved in the MP-NC FRLM by adding a virtual arc with the length of the initial fuel range at the end of each trip. Contrary to Kluschke et al. (2020), the ending fuel range is not seen as a part of the last arc. It is rather seen as an own edge at the end of the trip, that has to be travelled. In consequence, an own set $K^q_{j,k}$ is created for the virtual last arc. As the destination node of the trip is then the starting node of the virtual edge, it is a potential station location for refuelling this virtual arc. That way, all initial fuel levels, including 100 %, can be modelled. Contrary to Kluschke et al. (2020), it is, therefore, possible to model cases with private refuelling infrastructure and high IFRs with the MP-NC FRLM. One example would be BEV passenger car cases, where vehicles start fully loaded, as they can be charged at home.

Figure 13 shows the addition of the virtual arc a_{VD} to the OD trip from the previous example in figure 12.

Extended Set of all Critical Arcs A_q - Step 2

When removing all non-critical arcs in the second part of the A_q calculation, Jochem et al. (2019), unlike other authors, not only consider nodes within the initial fuel range as non-critical. For the case, that there is only one valid site for a fuel station z_i, that can refuel a critical arc $a_{j,k}$ on a path q, it is certain, that this fuel station z_i will be built. If $a_{j,k}$ is not the last arc of the path and z_i therefore not the last refuelling stop on the route, it is also known, that drivers will refuel the maximal tank level according to the refuelling strategy (Assumption 3). In consequence, the travelability of all $a_{j,k}$'s subsequent arcs $a_{l,m}$, that lie within the vehicle range of z_i, is guaranteed. Hence, these subsequent arcs can be considered as non-critical and can as well be deleted from the set A_q.

The pseudo-code for the removal of all non-critical arcs from the set A_q is displayed in code Listing 5. For further illustration, a flowchart of the algorithm is added to Appendix A.

```
for all OD trips q ∈ Q:
    create an empty delete-items} list for all non-
    critical arcs on A_q
    for all arcs a_{j,k} ∈ A_q:
        # arcs within the vehicle range are
        deleted from A_q
        if arc a_{j,k} is travelable with the initial
        fuel level:
            add a_{j,k} to the delete-items list
        else:
            if a facility at node i is the only
            valid possibility to refuel a_{j,k}:
                add all arcs a_{l,m} ,with m > l ≥ k,
                to the delete-items list that
                are as well refuelled by the fuel
                station at node i and are not the
                last arc on path q
    remove all arcs from the delete-items list
    from A_q
```

Code Listing 5: Algorithm for determining the set of necessary arcs A_q in the MP-NC FRLM

As can be seen in the pseudo-code, for every new path q, that the algorithm iterates over, an empty *delete-items* list is created. Every time the algorithm identifies an arc on path q as non-critical, the arc is added to this list. After iterating over all arcs of the extended path q, the arcs on the *delete-items* list

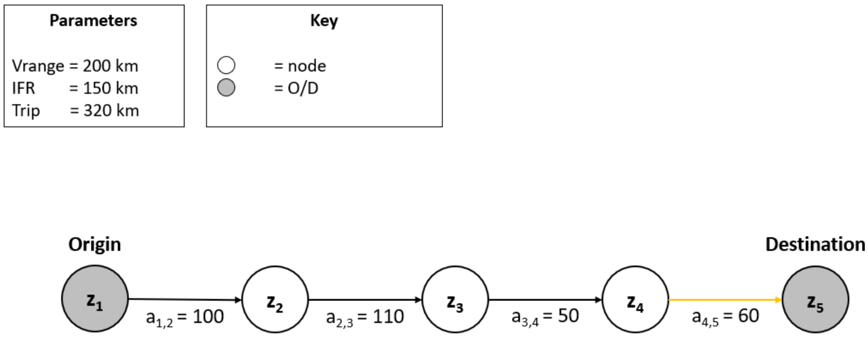


Figure 12: Permissibility problem of the Adjusted Distance method by Kluschke et al. (2020).

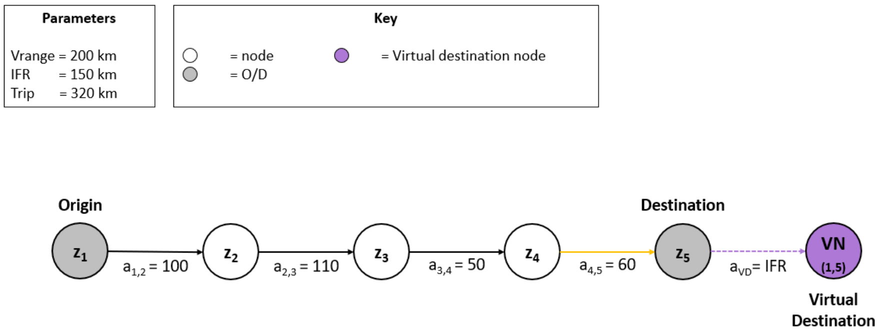


Figure 13: Addition of a virtual arc in the new A_q calculation method.

are removed from the set A_q, before the algorithm continues with the next path. It is important to note, that although it is possible to delete the non-critical arcs directly after their identification, removing items from a list, that is currently being iterated over will most likely produce non-desired results. Therefore the *delete-items* list functions as temporary storage so that the non-critical arcs can be deleted from the set after iterating over it. Thus results are not negatively impacted.

For a better understanding of the algorithm, that removes the non-critical arcs from A_q, its functionality is illustrated in a simple example.

Figure 14 shows a 320 km long, five node network with a single OD pair (1,5) and the newly added virtual destination node from the first part of the A_q calculation. The maximal vehicle range is 200 km, and the initial fuel range 150 km %. To convert A_q from the set of all arcs on path q to the set of critical arcs, all non-critical arcs have to be removed. As z_2 is the only station location, that can refuel the arc $a_{2,3}$, it is marked as a certain station location. Hence, all arcs within vrange of the certain station are travelable and therefore non-critical. Table 3 shows the arc-wise iteration of the algorithm along with the changes in the *delete-items* list.

Set of all Potential Station Locations $\mathbf{K^q_{j,k}}$

In the previous paragraph, the set of critical arcs A_q was obtained through appending a virtual arc with the length of the IFR at the end of each trip and afterwards removing all non-critical arcs from A_q. In the following, the A_q is used to calculate $K^q_{j,k}$.

The MP-NC FRLM algorithm for determining $K^q_{j,k}$ equals the *Identifying potential station locations* sequence of Kluschke et al. (2020)'s $K^q_{j,k}$ generation algorithm, which is marked blue in the flowchart in figure 22 (Appendix A). While Kluschke et al. (2020) iterate over every node on the way from origin to the destination node of the critical arc, to see whether it qualifies as a potential station location, the below-presented algorithm takes a different approach.

Instead of iterating forwards from the origin, the algorithm goes backwards on the path from the destination node of the examined critical arc $a_{j,k}$. If a node i is

- within the vehicle range of the destination node of the critical arc **and**,
- if the node qualifies as a potential station location (modelled through the parameter g_{iq}),

it is added to the set $K^q_{j,k}$.

When the algorithm reaches the first node, that is outside the vehicle range, it *breaks* the iteration and moves to the next critical arc, as all succeeding nodes would also be outside the vehicle range.

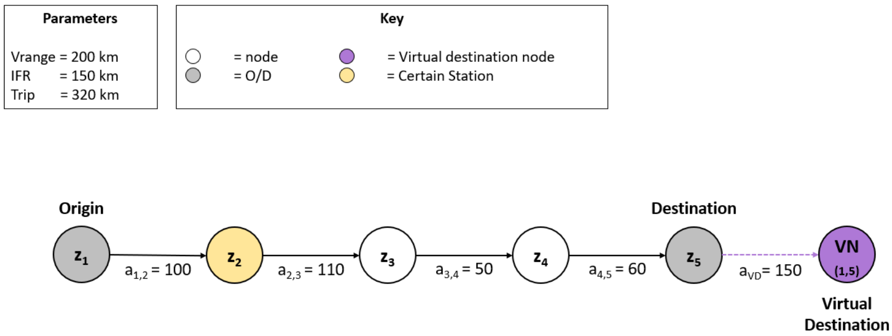


Figure 14: Usage of certain station placements on OD paths to identify further non-critical arcs

Step	Arc	*delete-items* list	Explanation
1	$a_{1,2}$	$\{a_{1,2}\}$	$a_{1,2}$ lies within vehicle range and is not the last arc of path (1,5) ->add to *delete-items*
2	$a_{2,3}$	$\{a_{1,2},\ a_{3,4}\ \}$	As a station at z_2 is the only possibility to refuel the arc $a_{2,3}$, subsequent arcs of $a_{2,3}$ are checked for their criticalness 1) $a_{3,4}$ is within vrange of z_2 and not the last arc of path (1,5) ->add to *delete-items* 2) $a_{4,5}$ is not within vrange of z_2
3	$a_{3,4}$	$\{a_{1,2},\ a_{3,4}\ \}$	$a_{3,4}$ is already in the *delete-items* list
4	$a_{4,5}$	$\{a_{1,2},\ a_{3,4}\ \}$	$a_{4,5}$ is not within vrange of z_2 and therefore critical
5	a_{VD}	$\{a_{1,2},\ a_{3,4}\ \}$	a_{VD} is not within vrange of z_2 and therefore critical
6	$A_q\ =\ \{a_{2,3},\ a_{4,5},\ a_{VD}\}$		All arcs from the *delete-items* list are removed from A_q

Table 3: Set of all critical arcs A_q of the example in figure 14

The pseudo-code for the generation of the set $K^q_{j,k}$ is given below. A flowchart of the algorithm can be found in Appendix A.

```
for all OD trips q ∈ Q:
    for all arcs a_{j,k} ∈ A_q:
        for all nodes i on the reversed path q
        with i ≤k:
            if distance (i,j) ≤ vrange and
            a potential station location:
                add node i to K^q_{j,k}
            else:
                break
```

Code Listing 6: Algorithm for determining the set $K^q_{j,k}$ in the MP-NC FRLM

Refuelled Driving Distance r^q_i

The last part illustrated the pre-generation process of the set $K^q_{j,k}$, which is split into the calculation of the set of critical arcs A_q and the computation of $K^q_{j,k}$. While the calculation of A_q and $K^q_{j,k}$ in the MP-NC FRLM follows a different approach than Kluschke et al. (2020), the algorithm for the generation of r_{iq} adapts and improves Kluschke et al. (2020) pre-calculation process. The new and improved algorithm is more compact and solves existing problems by harmonizing the route sectioning.

As described in section 2.2.2, Kluschke et al. (2020) subdivide the OD paths for the estimation of the refuelling amounts, r_{iq}, in l_q route sections. A vehicle is supposed to refuel once per section and fill up the maximal tank capacity, respectively enough fuel, to reach the end of the last section with the initial fuel level. When refuelling once per route section, the vehicles fill up in total the exact amount of fuel consumed during the OD trip.

Although this works well for most cases with $l_q = 1$ and $l_q = 2$, the definition of the route sections for $l_q > 2$ can cause vehicles to refuel twice in the penultimate, but not at all in the last route section. In consequence, vehicles refuel more than the amount of fuel consumed on the trip. Therefore Kluschke et al. (2020)'s assumption, that each vehicle starts and ends its trip with the same fuel level is violated, and more than the necessary amount is refuelled. Depending on the path's flow value, this can lead to greater distortions in the degree of capacity utilization and hence the model outcome.

As can be seen in the commented pseudo-code in figure 15, Kluschke et al. (2020) define the first route sections for the cases with multiple refuelling stops, $l_q = 2$ and $l_q \geq 2$, differently. $l_q = 2$'s first route section has the length of the initial fuel range and is therefore fully travelable from the origin. Contrary to that, all of $l_q > 2$'s route sections, except the last one, are defined by the maximal vehicle range, which makes the first route section not fully travelable with the initial fuel level, unless the vehicle's tank is full at the beginning.

In case the total trip length is close to $l_q\ \theta_q$ km, $l_q \geq 2$, with the destination node being the only node in the last section, all refuelling opportunities for refuelling the last arc lie in the penultimate section, which leads to a double refuelling there. As the applied refuelling logic causes drivers to fill up maximal tank capacity in every section except for the last, the ending vehicle range is $l_q\ \theta_q - d_q$ higher than the initial fuel range. For better illustration, an example is given below.

Figure 16 shows a six node network with the OD trip (1,6) of length 450 km. The vehicle range is 200 km and the initial fuel range 100 km. Thus, l_q is 3, and the route is subdivided according to Kluschke et al. (2020)'s algorithm. As the vehicle is supposed to finish the trip with an ending fuel range of 100 km, possible refuelling locations can only be within 100 km of the destination node 6. Therefore z_5 is the only location for refuelling the final arc of trip q, but lies in the second, and not the last route section and will refill the maximal vehicle range, 200 km.

When optimizing this problem with the solver Gurobi, the optimal solution is the construction of stations $z_2,\ z_4\ and\ z_5$. Thus, drivers would refuel 600 km instead of the trip length of 450 km.

The here proposed algorithm for the generation of the r_{iq} in Code Listing 7 addresses this shortcoming by harmonizing the route sectioning for all paths with $l_q \geq 2$ ß and reformulating the algorithm compactly. The primary adjustment is that the first route section for $l_q > 1$ will always have the length of the initial fuel range and vehicles refuelling at any node within this first section will fill up the maximum tank capacity. Furthermore, all potential route sections between the first and the last one (case $l_q > 2$) are defined by the vehicle range, and drivers will refuel the maximum tank capacity as well. Like in Kluschke et al. (2020), drivers will fill up the difference between the total trip distance and the sum of the previous refuelling amounts on the trip.

Figure 17 shows the route sectioning of the OD trip from figure 16 according to the improved algorithm for determining the parameter r_{iq}. While $z_2,\ z_4\ and\ z_5$ remain the optimal station location in this problem, due to the new definition of the route sections, the refuelling amount is equal to the total trip distance. The initial and ending fuel range are therefore equal.

```
for all OD trips q ∈ Q:
    for all nodes i on path q:
        if the number of refuelling occasions
        l_q ≤ 1:
            r_iq = total path distance
        else:
            if the distance (origin,i) ≤ (initial
            fuel range + vehicle range *
            max{0, l_q - 2}:
                r_iq = vehicle range
            else:
                r_iq = total path distance -
                vehicle range * (l_q - 1)
```

Code Listing 7: Algorithm for determining the parameter r_{iq} in the MP-NC FRLM

3.4. Measuring the MP-NC FRLM's Additional Benefit

In the previous section, the calculation of sets and parameters for the MP-NC FRLM was illustrated. Although no additional set respectively parameter that requires pre-generation was added compared to Kluschke et al. (2020), their computation differs. The set $K^q_{j,k}$ is calculated in two steps. In the first step, a virtual arc is added to A_q before removing all non-critical arc from the set. Subsequently, $K^q_{j,k}$ is generated on bases of A_q. The new r_{iq} pre-generation algorithm is more compact than Kluschke et al. (2020)'s formulation and has harmonized the route sectioning. For $l_q \geq 1$ the first route section now has the length of the IFR. For assessing the additional benefit of the MP-NC FRLM over non-multi-period models, the following paragraph introduces two of the most frequently found evaluation concepts, the "Value of the Multi-period Solution" and the "Value of Multi-Period Planning" and discusses their calculation in the context of the MP-NC FRLM. VMPS and VMPP respectively display the relative value difference between the MP-NC FRLM and pre-specified comparison models.

Value of the Multi-Period Solution

As stated in section 2.1.4, "Assessment of multi-period models", the VMPS is defined as the relative improvement of a multi-period model compared to its static counterpart. Laporte et al. (2015) While there are several ways to define the counterpart, for the application in context with the MP-NC FRLM, it is calculated as follows:

In the first step, the static counterpart is defined as the optimal solution of a single period NC-FRLM, that considers only the last period of the planning horizon. While the time-invariant parameters remain constant and the model flow equals the flow of the last period f^n_q, the budget in the counterpart has to be altered so that the potential amount of constructed stations in the dynamic and the static problem are equal. The outcome is a set of facilities that maximizes the value of the objective function in the static counterpart.

For the static counterpart to be comparable to be the MP-NC FRLM solution, it is necessary to determine a step-wise construction plan for the set of optimal facilities, that respects the budgetary constraints b_t for each period. Therefore, the

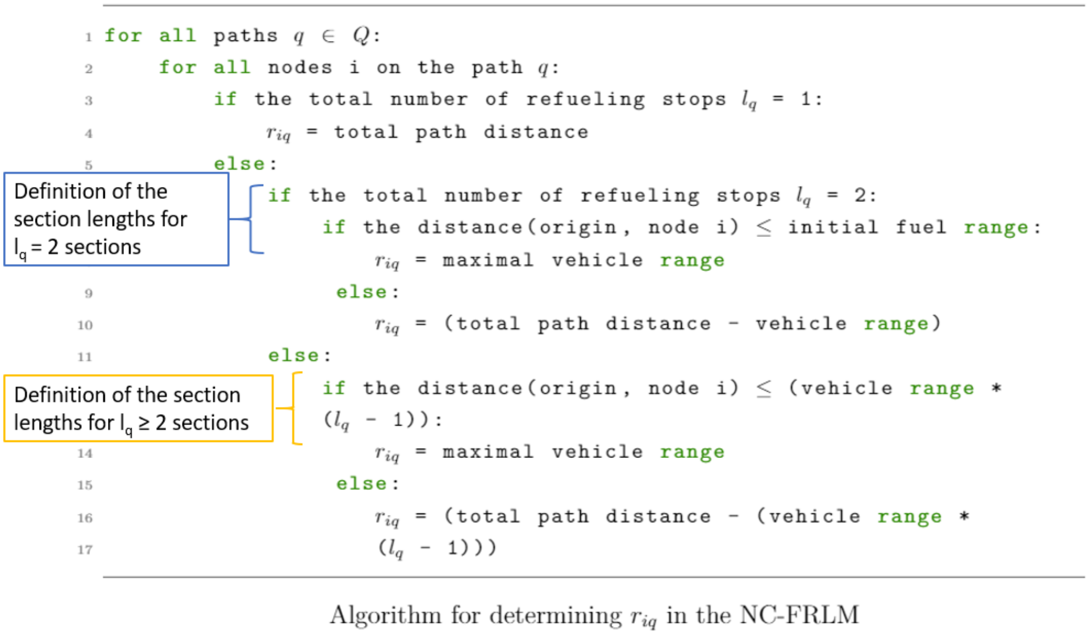


Figure 15: Definition of the l_q route sections in Kluschke et al. (2020).

Sets	
$T = n$	Only the last period n of the planning horizon is considered in the static counterpart
Parameters	
$b = \sum_{t=1}^{n} b_t$	The budget of the static counterpart is the sum of the periodic budgets from periods 1 to n
$f_q = f_q^n$	The vehicle flow in the static counterpart equals the vehicle flow in the last period $t = n$

MP-NC FRLM is optimized with the set of optimal facilities from the static counterpart being the only possible station locations. In case the set of optimal facilities in the multi-period model and the static counterpart are identical, the solution values will be identical as well.

In the last step, the Value of the Multi-Period Solution is obtained by subtracting the value of the objective function of the static counterpart MP-NC FRLM from the value of the objective function of the MP-NC FRLM and standardizing the difference with the counterpart's solution. For a VMPS greater than zero to occur, it is necessary, that the set of optimal facilities in the MP-NC FRLM and its counterpart differ. The variable V represents the value of the multi-period model, respectively of its counterpart.

Nonetheless, a difference in the set of optimal facilities does not necessarily mean that there is a VMPS greater than zero. On the other hand, if the two sets are identical, the VMPS is always zero. After all, the static counterpart can be seen as a building-site constrained version of the MP-NC FRLM. Its solution value will be lesser than or equal to the one of the original model with the full set of available building sites.

$$\text{VMPS} = \frac{V_{\text{MP-NC-FRLM}} - V_{\text{Counterpart}}}{V_{\text{Counterpart}}}, \ \text{VMPS} \geq 0$$

Value of Multi-Period Planning

The value of Multi-Period Planning quantifies the additional benefit of considering future periods while planning, contrary to continuously solving static problems for each period, given the results of the prior calculations. Although this definition would fit for the Forward-Myopic and the Backwards-Myopic method alike, it seems, from an economic point of view, more reasonable to optimize from the current period on forwards.

For retrieving comparable results and measuring the additional benefit, it is essential to assume that demand and economic data can be accurately predicted for every considered period. The Value of Multi-Period Planning is obtained by subtracting the solution value of the F-Myopic method from the value of the multi-period model and standardizing it with the F-Myopic value. The variable V represents the value of the multi-period model, respectively of its counterpart.

Compared to the multi-period model, its counterpart for the VMPP calculation can be considered as a greedy algorithm, as it makes the locally optimal choice for each period. Hence, the VMPP is zero, only if the locally optimal station placements in each period are as good as respectively identical to the siting choices made when considering other periods as well.

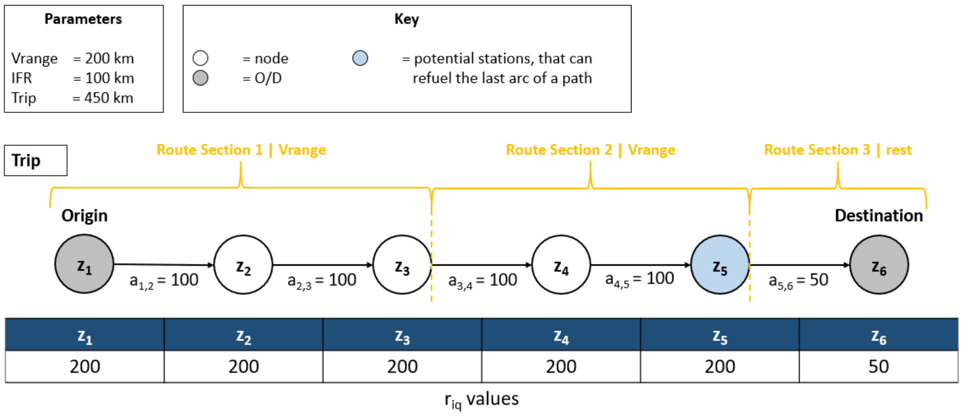


Figure 16: Illustration of the problem of the l_q definition by Kluschke et al. (2020)

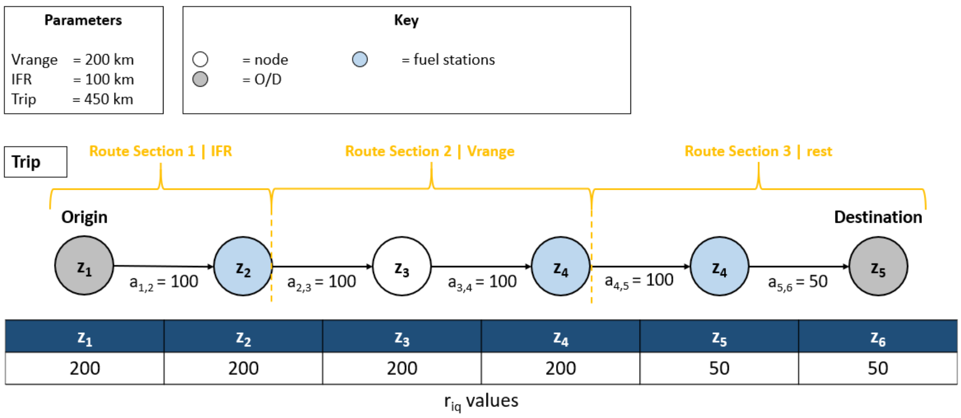


Figure 17: Implications of the adjusted l_q calculation method.

$$VMPP = \frac{V_{MP-NC-FRLM} - V_{F-Myopic}}{V_{F-Myopic}}, \; VMPP \geq 0$$

3.5. Further Case-Specific Assumptions and their Impact on the model

In the previous chapter, the two evaluation concepts for multi-period models, the VMPS and the VMPP have been introduced and adapted for assessing the MP-NC FRLM's additional benefit. The VMPS displays the relative value difference between the MP-NC FRLM and its static counterpart. On the other hand, the VMPP displays the difference between the MP-NC FRLM and a corresponding F-Myopic model.

As briefly discussed in section 2.3.1, assumptions for the multi-period node-capacitated FRLM are subdivided into two groups. While the *General Modeling Assumptions* define the general model setting and are needed for obtaining a feasible solution, the *Case Specific Assumptions* are a non-definite set of assumptions, that are made in the context of the application case. Although only three *Case Specific Assumptions* are made in the base case defined in section 2.3.2, a more comprehensive list of possible further situational assumptions and their impact on the model is presented below:

1. The vehicle flow is expected to rise periodically due to an increase in the AFV market share.
2. The driving range is assumed to increase due to technological advances.
3. Fuel station construction costs are assumed to be linear depending on the storage capacity.
4. Fuel station construction costs are influenced by local characteristics of the building site (e.g. topography).
5. Fuel station construction costs are expected to decline over time due to economies of scale and learning effects.
6. Certain nodes (e.g. origins, destinations, route intersections) are discarded as station locations.
7. refuelling is only required on trips longer/shorter than XX km.

The MP-NC FRLM considers the OD flows of alternative fuel vehicles, which currently stand at the beginning of their product life cycle. It is possible to include changes in the vehicle flow into the model by adding a temporal dimension to the vehicle flow on path q. The two main drivers of the AFV flow on path q in period t, f_q^t are

- the total vehicle flow and
- the AFV market share.

As both drivers, total vehicle flow and AFV market share, are expected to rise all around the world, Ahlswede (2020); FEV (2018), it is reasonable to assume an increase in AFV flow like in Assumption 1.

Assumption 2 thematizes the possible changes in vehicle range over time, which might be worth considering when applying the MP-NC FRLM to a BEV case. In the model, the vehicle range consists of

- the maximal tank/battery capacity and
- fuel/energy consumption per km.

An increase in the vehicle range has multiple implications for the model and causes the addition of a temporal dimension to A_q, $K_{j,k}^q$, r_{iq}, l_q and p. While adding an index t to the fuel consumption parameter p seems rather obvious, the necessity for adjusting the rest of the parameters mentioned above becomes clearer at a second glance.

When determining the set of critical arcs from A_q, all arcs, that lie within the vehicle range of the origin node or another certain fuel station, are considered uncritical and therefore removed. Hence, the set of critical arcs must be updated every time, the vehicle range changes.

The set $K_{j,k}^q$ is influenced by a changing vehicle range not only through an adjusted set of critical arcs A_q^t. Furthermore, a greater vehicle range can enlarge the set of potential facility locations, that can refuel an arc $a_{j,k}$ on path q. Therefore an updated set $K_{j,k}^q$ is needed for every period $t \in T$.

l_q is the number of necessary refuelling stops on a path q. It is calculated by rounding up the solution of dividing the total path distance through the vehicle range and thus needs a temporal dimension as well. Apart from ensuring the number of refuelling occasions on a path in a model constraint, l_q and the vehicle range influence the route sectioning when calculating the parameter r_{iq}. Among possible changes in the route sectioning, the values for r_{iq} change alongside the vehicle range.

Like in the base model, it is still assumed, that the constructed fuel station at a node i will always have the maximal possible capacity (= node capacity). While the base model does not consider the impact of station capacity on construction costs, it is possible to do so. Assumption 3 presumes a correlation between the capacity of a station and its construction costs and assumes it for simplification purposes to be linear. In consequence the construction cost of a fuel station is calculated as follows: $o_i = c_i$ cost per kg stored H_2.

To provide a more accurate estimation of the utilized budget, Assumption 4 gives the possibility to respect the impact of local characteristics of the building site on the station cost at a node i via a parameterα_i, $\alpha_i \geq 0$. An exemplary factor with an impact on the construction costs is the topography of the building land.

While it is more expensive to construct a station on hilly ground compared to the baseline cost o_i, with $\alpha > 1$, a plane construction site might be cost-wise more attractive, $\alpha < 1$. Possible other local factors influencing the construction costs could be the energy grid connection, forest vegetation or pre-existing stations.

Assumption 5 considers the possibility of economies of scale and learning effects, that might arise due to a large number of alternative fuel stations constructed within the project time. The parameter β^t, with $\beta^t \in \{0,1\}$, displays a time-dependent cost reduction factor for o_i. Like in Kluschke et al. (2020), it might be desired to discard specific nodes, e.g. origins, destinations or route intersections as a station location. (Assumption 6) Discarding can be achieved by setting the parameter g_{iq} for the discarded locations to zero. Assumption 7 restricts the set of all OD paths Q, considering only paths, whose lengths exceed respectively are below a certain threshold. Limiting the set of considered trips can be reasonable when focusing, e.g. on long-haul transportation or reducing Q to benefit the solving time of the model, like in Kluschke et al. (2020).

The following formulation is an extended version of the MP-NC FRLM that respects the above mentioned, possible *Case Specific Assumptions*.

$$\max \sum_{t \in T} \sum_{q \in Q} y_q^t \tag{3.21}$$

$$\text{s.t.} \sum_{i \in K_{j,k}^{q,t}} z_i^t \geq y_q^t \forall\, q \in Q,\, a_{j,k} \in A_q,\, t \in T \tag{3.22}$$

$$\sum_{q \in Q} f_q^t\, p^t\, r_{iq}^t\, g_{iq}\, x_{iq}^t \leq c_i\, z_i^t \forall\, i \in N,\, t \in T \tag{3.23}$$

$$\sum_{i \in K_{j,k}^{q,t}} x_{iq}^t = y_q^t \forall\, q \in Q,\, a_{j,k} \in A_q,\, t \in T \tag{3.24}$$

$$\sum_{i \in N} x_{iq}^t = y_q^t\, l_q^t \forall\, q \in Q,\, t \in T \tag{3.25}$$

$$x_{iq}^t \leq z_i^t \forall\, i \in N,\, q \in Q,\, t\ in\ T \tag{3.26}$$

$$z_i^t \leq z_i^{t+1} \forall\, i \in N,\, t \in T \backslash \{n\} \tag{3.27}$$

$$z_i^t - z_i^{t-1} \leq k_i^t \forall\, i \in N,\, t \in T \backslash \{1\} \tag{3.28}$$

$$z_i^1 \leq k_i^1 \forall\, i \in N \tag{3.29}$$

$$\sum_{i \in N} \alpha_i\, \beta^t\, o_i\, k_i^t \leq b_t \forall\, t \in T \tag{3.30}$$

$$\sum_{t \in T} k_i^t \leq 1 \forall\, i \in N \tag{3.31}$$

$$z_i^t,\, k_i^t \in \{0,1\} \forall\, i \in N,\, t \in T \tag{3.32}$$

$$0 \leq x_{iq}^t \leq 1 \forall\, i \in N,\, q \in Q,\, t \in T \tag{3.33}$$

$$0 \leq y_q^t \leq 1 \forall\, q \in Q,\, t \in T \tag{3.34}$$

Sets	
N	Set of all nodes on the Graph G
Q	Set of all OD pairs
T	Set of all time periods
A_q	Set of all directional arcs on the path $q \in Q$ from origin to destination
$K_{j,k}^{q,t}$	Set of all potential station locations, that can refuel the directional arc $a_{j,k} \in A_q$ in period t
Variables	
z_i^t	Binary Variable that equals to one, if a refuelling facility is open at node i in time period t
k_i^t	Binary Variable that equals to one, if a refuelling facility is constructed at node i in time period t
x_{iq}^t	Semi-Continuous Variable that indicates the proportion of vehicles on path q that are refuelled at node i in time period t
y_q^t	Semi-Continuous Variable that indicates the proportion of flow served on path q in time period t
Parameters	
p^t	Fuel efficiency / fuel consumption per vehicle range in period t
o_i	Facility opening costs / construction costs for a facility at node i, $o_i = c_i *$ cost per kg stored H_2
v^t	Fraction of the minimal amount of flow covered in period t
c_i	refuelling capacity at node i
d_q	total distance of path q
θ_q	vehicle range of vehicles on path q
l_q	Number of refuelling occasions on path q depending on the total path distance, $l_q = \text{ceil}\{d_q / \theta_q\}$
b_t	Available budget in period t
f_q^t	Total vehicle flow on the OD path q in time period t
g_{iq}	Binary indicator, that is set to one, if node i is a potential station location on path q
r_{iq}	refuelled driving distance at node i on path q
α_i	Semi-Continuous parameter, that depicts the impact of local factors (e.g. topography of the building land,...) on the construction cost of a station
β^t	Semi-Continuous parameter, that indicates the general cost reduction for construction fuel stations compared to $t = 0$ due to learning effects and possible economies of scale

4. Numerical Experiment: Additional Benefit and Computational Complexity

The previous chapter introduced the new FRLM extension of Capar et al. (2013)'s and Kluschke et al. (2020)'s models, the multi-period node-capacitated FRLM, to the reader. The section discussed the new MP-NC FRLM model assumptions and presented the model's mathematical formulation. After addressing two modelling problems, the calculation of the sets A_q and $K_{j,k}^q$ and the parameter $r_i q$ was illustrated. The chapter concludes by adapting the two multi-period model evaluation concepts, the VMPS and the VMPP, to the MP-NC FRLM and discussing further case-specific assumptions and their impact on the model.

To understand the implications of the MP-NC FRLM and to identify cases and parametric constellations, where the use of the MP-NC FRLM provides the most additional benefit, a numerical experiment is conducted. The two applied assessment criteria are the concepts of the *Value of the Multi-Period Solution* (VMPS) and the *Value of Multi-Period Planning* (VMPP), which are defined above in section 2.3.4. In the first step, the VMPS and the VMPP are illustrated through an exemplary network. After proving the existence of a VMPS, respectively a VMPP greater than zero, several hypotheses about parametric constellations driving the values of the assessment criteria are discussed. As the benefits of applying a multi-period model come at the cost of increased complexity and hence, a higher solving time, the final paragraph will assess the maximal complexity for the problem to be solved efficiently within ten minutes.

4.1. Illustrating the Benefits: VMPS and VMPP in the MP-NC FRLM

In the following, the VMPS and the VMPP are illustrated, each through an exemplary network. In a first step, the existence of a VMPS, respectively VMPP, greater than zero is proven, before taking a closer look at why and how VMPS and VMPP greater than zero occur and which factors might benefit these two assessment values. For a better understanding of the VMPS and VMPP, the chosen examples will only have a value greater than zero for the assessment criterion, that is examined in the particular paragraph. While it is likely, that larger problems have both, a VMPS and a VMPP greater than zero, this is also possible on smaller networks, as shown in Appendix A (Appendix A: network in figure 25, OD flows in table 12, solution values in table 13). All of the below-presented problems were solved with Gurobi Version 9.0.

Value of the Multi-Period Solution

Figure 18 shows the thirteen node network, which is used for further examining the VMPS. There are seven OD trips, of which the four trips, (1,5), (1,6), (1,7) and (1,13), share the same edges for their first 350 km. The maximal vehicle range is 400 km, and the initial fuel level 50 %. The considered time horizon is three periods, and the budget is sufficient for building one fuel station per period. Each OD trip has a flow amount of ten in the first period, fifteen in the second and twenty-five in the third and final period.

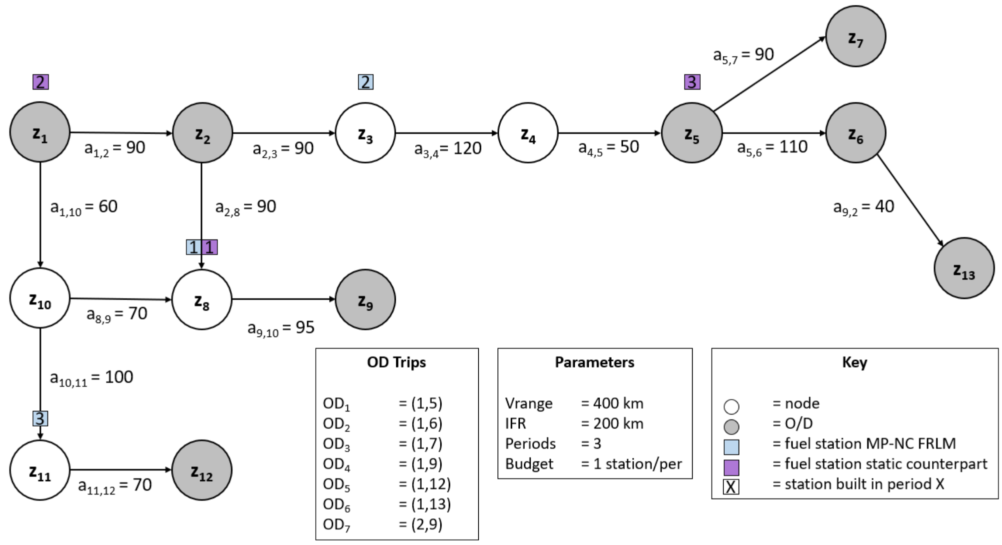


Figure 18: Exemplary problem in the numerical experiment with VMPS > 0 and VMPP $= 0$

While the MP-NC FRLM and the F-Myopic model have the same solution (leading to a VMPP of zero), the results of the multi-period model and its static counterpart differ. Following its definition, a VMPS greater zero could occur, as the set of optimal facilities in the multi-period model and its static counterpart differ.

As described in section 2.3.4, the set of optimal facilities in the counterpart is determined, by solving the static problem, given the OD flows from the last period and the possibility of building three facilities. Even though the flows of (1,6), (1,7) and (1,13) cannot be fully served due to the limited capacity of the fuel stations, it is optimal to serve them, as these three OD flows can be served with the two stations z_1 and z_5 (see table 4). Hence, the y_q value for these paths is below one.

After solving botch, the multi-period problem and the static counterpart, the total number paths covered over all periods, $\sum_{t \in T} y_q$ appears to be equal. The solution value of the static counterpart is smaller than the one of the multi-period model because the stations in the counterpart can not serve all flows on the refuelled paths as shown in table 4. The VMPS, therefore, amounts to 6.51 (see table 5).

While a disparity in the sets of optimal facilities is necessary for a VMPS greater than zero, the sole existence of this discrepancy does not make any predictions about the amount of the benefit. Although exact VMPS drivers have not yet been scientifically identified and confirmed, some parametric constellations and patterns frequently occurred while testing the model. This leads to the following hypotheses about the VMPS:

Although it is certain, that other factors, like the number of possible fuel station constructions per period or the length of the deviation paths, do influence the VMPS, it has not been easy to assess their exact effects. One of the primary reasons for that is that changing these factors also has consequences on other model parameters.

A good example is the possible influence factor "constructed fuel stations per period". The differences in the sets of optimal facilities mainly occur, because in the static model, station placements are optimal that cover multiple and mostly overlaying paths at the same time. In the multi-period model, these paths might be suboptimal, as it, for example, takes more time to cover them entirely. Hence, it seems logical, that the more time it takes to cover them, the longer it takes for the path to contribute to the solution value. In consequence, the solution value of the counterpart would decline and the VMPS rise.

However, this is difficult to prove because changing the number of constructed stations per period has consequences on at least either the total number of stations built or the number of considered periods. In case the number of periods remains constant, the total number of stations built changes, which affects the set of optimal facilities in both, the multi-period model and the static counterpart. When keeping the number of stations constant, while altering the number of periods, the solution values will differ significantly from the previous ones, as the objective function sums over all y_q variables over all periods.

Nonetheless, further analysis of VMPS drivers and influence factors might be useful for further research.

Value of Multi-Period Planning

For the illustration of the VMPP, the thirteen node network from above is extended by two nodes. OD trips, as well as their flows, have also been adapted (see figure 19. In this problem, there are six considered OD trips, (1,5), (1,6), (1,7), (1,12), (1,13) and (2,9). All of them have a flow amount of five in the first period, ten in the second and fifteen in the third period. The maximal vehicle range is 400 km, and the initial fuel level 50 %. The considered time hori-

Stations constructed	Paths Covered
z_8	$y_{1,6} = 0.817$
z_1	$y_{1,7} = 0.816$
z_5	$y_{1,13} = 0.816$
	$y_{1,9} = 1$
	$y_{2,9} = 1$

Table 4: Operating Stations and covered paths of the static counterpart of the numerical experiment in figure 18.

		Period t=1	Period t=2	Period t=3
MP-NC FRLM	**Operating Stations**	z_8	z_8, z_3	z_8, z_3, z_{11}
	Paths Covered	(1,9), (2,9)	(1,9), (2,9), (1,5)	(1,9), (2,9), (1,5), (1,12)
Static Counterpart	**Operating Stations**	z_8	z_8, z_1	z_8, z_1, z_5
	Paths Covered	(1,9), (2,9)	(1,9), (2,9)	(1,9), (2,9), (1,6), (1,7), (1,13)

Solution Value		**Assessment Criterion**	
MP-NC FRLM	9.0	VMPS	6.51 %
Static Counterpart	8.45	VMPP	0 %
F-Myopic	9.0		

Table 5: Solution value and assessment criteria of the numerical experiment in figure 18.

zon is three periods, and the budget is sufficient to build one fuel station per period.

While the MP-NC FRLM and the static counterpart model have the same solution (leading to a VMPS of zero), the results of the multi-period model and the F-Myopic differ. As described in section 2.3.4, the F-Myopic model can be considered a greedy algorithm, that makes the optimal decision for each period. Table 6 illustrates the differences in the decision making between the F-Myopic model and the MP-NC FRLM: From a holistic perspective it is optimal to first construct a station z_5, as z_5 contributes to refuelling multiple paths, that need more than one refuelling stop en route. With the construction of z_1 in the next period, three paths, (1,6), (1,7) and (1,13) can be refuelled at the same time. As the F-Myopic model, on the other hand, only aims at maximizing the payoff of the current period, it misses out on the opportunity of covering the paths (1,6), (1,7) and (1,13), because the paths take two periods to be covered.

Hence, the MP-NC FRLM achieves an overall better solution at the possible cost of a worse performance within the time horizon. This means that whenever the MP-NC FRLM and its F-Myopic correspondent do not have the same value, the solution of the F-Myopic model has a higher solution value in at least one time period.

Solving both, the MP-NC FRLM and the F-Myopic model leads to a VMPP of 16.67%.

Although there has not been an extensive scientific study examining potential VMPP drivers, frequently observed parametric constellations lead to the following hypotheses about the VMPP:

- **The VMPP strongly depends on the network and OD trip topology as well as on OD trip quantity**

  For a VMPP to exist, it is necessary, that the station placements for maximizing the benefit of the current period and for maximizing the overall benefit differ in at least one period. Contrary to the greedy F-Myopic model, the MP-NC FRLM accepts a short term worse solution value to invest into station combinations, that refuel several paths at the same time, take several periods to build and pay off in a later period (see table 6). As covering these paths requires "long-term planned" investments, these paths are in the following called "invest paths". Hence invest paths are necessary for a VMPP greater than zero.

  To be considered an invest path, OD trips have to fulfil the following two topological requirements:

  1. Invest paths can not be covered with the budget of one period.
  2. Covering an invest path refuels multiple other paths along the way.

  In the above-given example (figure 19) either (1,6), (1,7) or (1,13) would count as an invest path, as covering one of these paths automatically covers the other two paths along.

  The more invest paths there are in a problem, and the more OD trips are refuelled along with an invest path, the higher the VMPP gets. The impact on the VMPP is exemplified in table 7, as there are step-wise OD trips

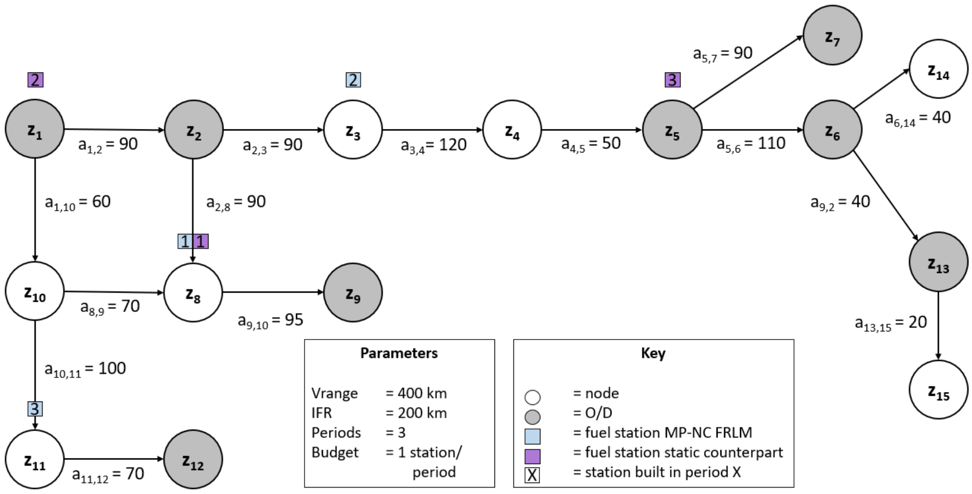


Figure 19: Exemplary problem in the numerical experiment with VMPP > 0 and VMPS = 0

		Period t=1	Period t=2	Period t=3
MP-NC FRLM	**Operating Stations**	z_5	z_5, z_1	z_5, z_1, z_8
	Paths Covered		(1,6), (1,7), (1,13)	(1,6), (1,7), (1,13), (2,9)
F-Myopic	**Operating Stations**	z_{10}	z_{10}, z_8	z_{10}, z_8, z_3
	Paths Covered	(1,12)	(1,12), (2,9)	(1,12), (2,9), (1,5),

Solution Value		**Assessment Criterion**	
MP-NC FRLM	7.0	VMPS	0 %
Static Counterpart	7.00	VMPP	16.67 %
F-Myopic	6.0		

Table 6: Operating stations and covered paths of the numerical experiment in figure 19.

added to the simultaneously coverable invest paths. With each addition of the two trips (1,14) and (1,15), the VMPP grows.

- **The VMPP is negatively correlated with the amount of non-coverable flow passing through the MP-NC FRLM's "invest paths"**

  Similar to the "deviation path effect" for the VMPS, the lesser flow is covered on the "invest paths", the smaller is the MP-NC FRLM's solution value. Hence, the VMPP declines.

  This effect is illustrated in table 8 by varying the flows of the invest paths (1,6), (1,7) and (1,13) in t = 3 in the previous problem in figure 19. As described above, one can see, that with the increase of the invest flows, the MP-NC FRLM solution value declines to the point, where the F-Myopic solution becomes optimal.

  As the invest paths are already covered in period two, it is contrary to the "deviation path" example in the VMPS, possible to alter the MP-NC FRLM solution by also varying the flow parameters in period two. Varying flows in prior periods has not been done for better clarity. Either way, the minimal MP-NC FRLM solution value hereby equals the F-Myopic value of six.

- **The VMPP is positively correlated with the number of considered periods as long as the sets of covered paths are not identical.**

  As described above, whenever there is a VMPP greater than zero, the MP-NC FRLM's solution value will, at least at one point in time, be lesser than the F-Myopic value as a tradeoff for future benefit. That means that the value of covering an invest path is greater than the value of paths that are covered in the meantime by the F-Myopic model. The additional value of covering an invest path can, for example, be seen in table 6, as the MP-NC FRLM achieves a surplus of one covered path compared to the F-Myopic model in period two, due to

OD trips on invest paths	Stations MP-NC FRLM	Value MP-NC FRLM	VMPP
(1,6), (1,7), (1,13)	5,1,8	7	16,67%
(1,6), (1,7), (1,13), (1,14)	5,1,8	9	50,00%
(1,6), (1,7), (1,13), (1,14), (1,15)	5,1,8	11	83,33%
Stations F-Myopic	3,8,10		
Value F-Myopic	6		

Table 7: Effects of network and OD trip topology on the VMPP in the problem in figure 19.

Invest Flows in t = 3	Stations MP-NC FRLM	Value MP-NC FRLM	VMPP
15	z_5, z_1, z_8	7	16,67%
16,75	z_5, z_1, z_8	6,99	16,0%
20	z_5, z_1, z_8	6,74	12,33%
30	z_5, z_1, z_8	6,24	4,00 %
37	z_5, z_1, z_8	6,01	0,17%
37,5	z_3, z_8, z_{10}	6	0%
40	z_3, z_8, z_{10}	6	0%
Stations F-Myopic	z_3, z_8, z_{10}		
Value F-Myopic	6		

Table 8: Effects of invest flow variations on the VMPP in the problem in figure 19.

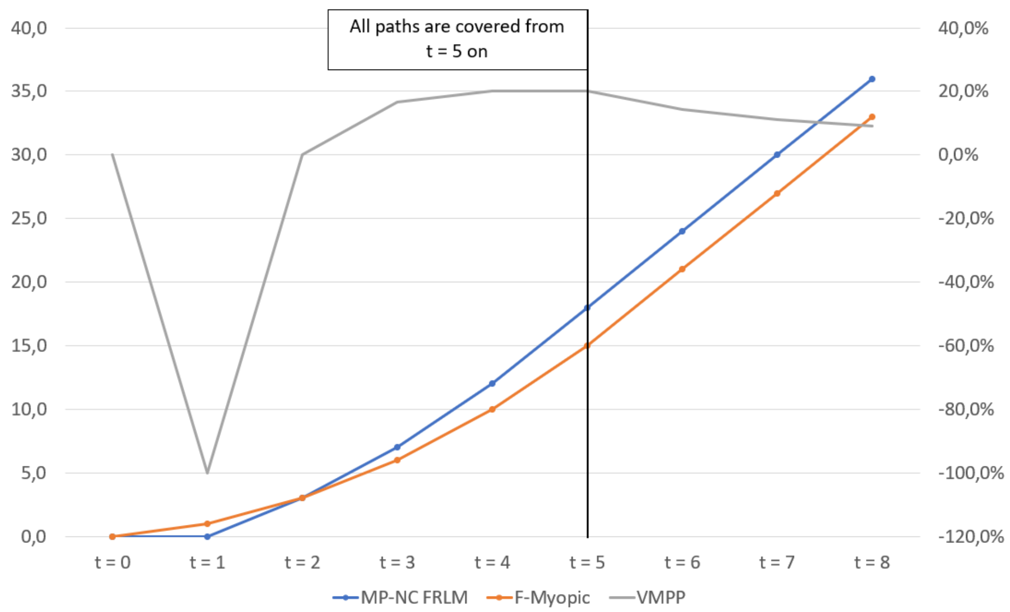


Figure 20: MP-NC-FRLM and F-Myopic solution values and VMPS development over time.

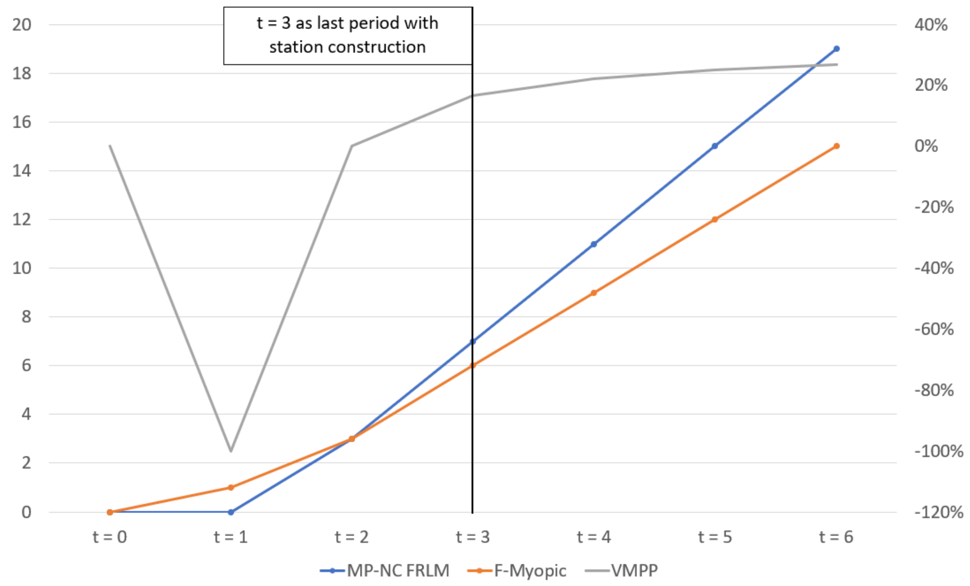


Figure 21: MP-NC-FRLM and F-Myopic solution values and VMPP development over time.

varied parameters	constant parameters	
number of nodes	x coordinate min/max	0/660
node connection probability	y coordinate min/max	0/880
number of OD paths	start value of the OD flows	5
number of periods	flow increment per period	5
	construction budget	5 facilities/period
	maximal vehicle range	400
	initial fuel range	200

Table 9: Parameters used in the randomised problem generation in the numerical experiment.

covering the invest paths. Until these invest paths are as well covered by the F-Myopic model, the surplus will contribute to an increasing VMPP in each period. Once the sets of covered paths concur, the VMPP declines with every further period.

Figure 20 exemplifies this effect by depicting the solution values of the MP-NC FRLM and the F-Myopic model as well as the VMPP for different periods in the problem in figure 19.

Until the two sets of covered paths are identical and the invest paths are also covered by the F-Myopic model (see period t = 5; all six paths are covered), the VMPP grows. While the absolute value difference remains constant, the relative value difference lessens with every period.

Figure 21 shows the graph of a situation, where it is no longer possible to build fuel stations respectively to cover paths after period three. Since the sets of covered paths never concur, the VMPP grows with every period.

4.2. Computational Complexity of the MP-NC FRLM

While the previous paragraph has shown, that the multi-period model can provide even significantly better results than its static, respectively its F-Myopic counterpart, the superior performance comes at the cost of higher computational complexity.

The assessment of the model complexity in the following section is approached from a practical standpoint. How complex can a problem in terms of network topology, OD path quantity and periods be so that it can be solved with the MP-NC FRLM in under ten minutes?

For testing the solution time, the optimization was performed on problems, where the network, as well as the OD paths, were randomly generated. The models were run with an Intel Core i5-6200 CPU with 2,40 GHz and 8 GB RAM.

The parameters used for the generation of the problems are displayed in table 9.

The network is generated using the parameters "number of nodes", "x/y coordinate maximum" and "node connection probability". In the first step, nodes are randomly placed on

varied parameters		Results	
number of nodes	1500	average solution time	10:01
node connection probability	10%	standard deviation	5:07
number of OD paths	150	minimal solution time	4:34
number of periods	10	maximal solution time	19:34
		% solution time under 10 minutes	70%

Table 10: Results of the computational complexity test for ten randomly generated optimization problems with 1500 node network with 150 OD paths, a node connection probability of 10% and ten considered periods.

varied parameters		Results	
number of nodes	1500	average solution time	25:32
node connection probability	10%	standard deviation	21:45
number of OD paths	175	minimal solution time	5:36
number of periods	10	maximal solution time	1:16:28
		% solution time under 10 minutes	40%

Table 11: Results of the computational complexity test for ten randomly generated optimization problems with 1500 node network with 175 OD paths, a node connection probability of 10% and ten considered periods.

an experiment plane, which is defined by the minimal maximal values of the x/y coordinates.

In this case, the plane has an expansion of 880 km on the y-axis and 660 km on the x-axis. These values correspond to the maximal extent of Germany in the north-south, respectively the east-west direction. In a second step, the edges of the graph are created, whereas each potential connection between two nodes, except for self-connections, has a certain likelihood of existence. Hence, the expected amount of edges amounts to $node\ connection\ probability * number\ of\ nodes * (number\ of\ nodes - 1)$. The maximal vehicle range is 400 km, the initial fuel range 200 km. Origin and destination of the OD trips are randomly chosen from the set of nodes on the graph.

As the computation varies along with the network topology and the length of the OD trips, each parametric constellation has been tested ten times of random problems.

The results of the experiment indicate that an optimization problem over ten periods and on a 1,500 node network with an edge probability of 10% and 150 OD paths seems to be the largest problem, that can regularly be solved in under ten minutes. Although the average solving time amounts to 10:01 minutes, a total of 70% of the tested problems could be solved within the designated time. The standard deviation for solving the problems is 5:07 (see table 10).

When increasing the number of OD paths c.p., problems with 175 OD trips already have a considerably higher solving time and standard deviation. Solving the randomly generated problems took an average time of 25:32 minutes with a standard deviation of 21:45 minutes. Only 40% of the problems with 175 OD trips could be solved in under ten minutes (see table 11. It is noteworthy that the maximal solving time for a problem with 175 OD trips is at 1:16:28 nearly one hour higher than for problems with 150 OD trips.

The results of the experiment indicate, that optimization problems over ten periods and on a 1,500 node network with an edge probability of 10% and 150 OD paths seems to be the largest problem, that can regularly be solved in under ten minutes with a computer with a 2.4 GHz processor and 8 GB RAM. Furthermore, it was shown that the more complex the problems are, the greater become average solving time and its standard deviation. Although the results of this complexity assessment are not statistically significant, they, nonetheless, provide general reference points for real-life application and further research.

5. Conclusion and Recommendations for Further Research

For alternative-fuel vehicles, like BEVs and FCEVs, to succeed, a comprehensive alternative fuel station network is vital. However the development of such a refuelling network constitutes a "chicken-egg problem": On the one hand, companies are unlikely to invest until AFS operations promise to be profitable, whereas on the other hand consumers hesitate to buy AFVs unless there is an agreeable level of refuelling infrastructure. One possible solution for this problem is strategic multi-period planning, which is incentivized, respectively led by a central authority.

This thesis introduces a new flow-refuelling location model, that aims at providing a multi-period construction plan for an alternative fuel station network. Based on the idea of covering each arc of a path, the MP-NC FRLM maximizes the number of paths covered. Depending on the problem at hand, it is as well possible to either maximize the total flow covered or introduce a lower bound of flow coverage while maximizing the refuelled paths.

Besides including nodal capacity restrictions for fuel stations, the model respects changing demand flows and limitations of the construction capacity and is an extension of Kluschke et al. (2020)'s node-capacitated FRLM. Apart from the model extension, the pre-generation process for the set of potential station locations $K_{j,k}^q$ and the heuristic estimating the refuelling amount r_i^q at the nodes has been improved.

To illustrate the benefits of the multi-period model over a static counterpart and a comparable F-Myopic model, the two measures "Value of the multi-period Solution" and "Value of multi-period Planning" were adapted to this context and applied in a numerical experiment. The VMPS and the VMPP quantify the relative additional value of the MP-NC FRLM's solution to the ones of the static counterpart respectively the F-Myopic model. The VMPS and VMPP have proven to be positive, and several hypotheses were made about parametric constellations and patterns, that drive VMPS and VMPP.

The additional benefit of the MP-NC FRLM, however, comes at the cost of higher computational complexity due to the incorporation of the time module. Another potential problem, which has to be borne in mind, is the calculation of the number of ensured refuelling locations alongside a route l_q. In some cases, l_q locations can be insufficient to cover an OD trip due to unfavourable topological characteristics. The existence of unfavourable paths does not cause any calculation errors but are not respected by the model during the optimization.

Following the findings of this thesis, there are several possible streams for further research:

- **Providing further analysis of VMPS/VMPP drivers**

  Providing a better understanding of the VMPS/VMPP and its value drivers might help to detect early, where the application of the MP-NC FRLM compared to static models provides sufficient benefit.

- **Reducing the MP-NC FRLM model complexity**

  Reducing the model complexity, e.g. by linearising the bi-linear constraint, leads to a decreasing calculation time, which can prove valuable in the application.

- **Finding a precise calculation method for l_q**

  Especially in greater problems, it can be challenging to identify whether a path was not covered, because it was suboptimal to refuel or because the covering solution was trivial. Finding a precise calculation method for l_q, where the number of built stations equals the minimum number of necessary stations will eventually provide better, cost-effective results.

- **Applying the MP-NC FRLM to a real-world case setting**

  An application of the model on a real-world case might provide further insights into the additional model benefit. It would also be useful to compare these outcomes to other FRLM models in various settings.

- **Integrating deviation paths into the MP-NC FRLM**

  To make the model more realistic, one option could be, extend the MP-NC FRLM in a way, that considers the drivers' willingness to deviate from an optimal path to refuel. Considering deviation paths might prove useful especially for simulating the early stages of the AFS network construction, where the network is significantly less mature than that of conventional gas stations.

- **Including the stochasticity of demand**

  For the comparison of the MP-NC FRLM with the F-Myopic model, it is assumed, that future demand can be precisely predicted, which is a substantial simplification of reality. The model's accuracy and significance could profit from considering the uncertainty of demand flows.

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