Online-Appendix zu

„Prospect Theory and Stock Returns During Bubbles“

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Appendix

Appendix A: Commented proof of the Barberis et al. (2016) model

This appendix is a commented version of the proof by Barberis et al. (2016). Given the fractions of the two types of investors in the population, the market portfolio w_m can be written as:

$$w_m = \pi w_t + (1-\pi)(w_t + k w_{TK})$$

$$= w_t + \eta w_{TK}, \qquad \text{(A1)}$$

where $\eta = (1-\pi)k$.

For the central proposition of what determines asset prices Barberis et al. (2016) use the matrix solution to Portfolio Theory by Markowitz (2000) for the case of multiple risky assets and a risk free asset as a starting point. Generally the solution for the asset weights in the tangency portfolio w_t can be written as:

$$w_t = \frac{\Sigma^{-1}(\mu - r_f \cdot 1)}{1'\Sigma^{-1}(\mu - r_f \cdot 1)}, \qquad \text{(A2)}$$

where $\sum$ is the matrix of return covariances. After rearranging this more general model we get:

$$\mu - r_f \cdot 1 = \gamma \Sigma w_t\ , \qquad \text{(A3)}$$

for some γ, where μ is the $J\ x\ 1$ vector of mean asset returns, $\sum$ is specifically defined as the $J\ x\ J$ matrix of return covariances and 1 is a $J\ x\ 1$ vector of ones. The asset weights w_t can be substituted for by using equation (A1) from above:

$$\mu - r_f \cdot 1 = \gamma(\Sigma w_m - \eta \Sigma w_{TK})\ . \qquad \text{(A4)}$$

Premultiplying both sides by w_m:

$$\mu_m - r_f = \gamma \sigma_m^2 (1 - \eta \beta w_{TK})\ . \qquad \text{(A5)}$$

In order to define the excess return over the market we divide equation (A4) by equation (A5):

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \frac{\beta_j - \eta\sigma_{j,TK}/\sigma_m^2}{1-\eta\beta_{TK}} \tag{A6}$$

for all $j \in \{1, \dots, J\}$. By introducing the new variable $s_{j,TK}$ the covariance of returns between j and TK stocks $\sigma_{j,TK}$ in (A6) can be substituted for by $\sigma_{j,TK} = \beta_j\beta_{TK}\sigma_m^2 + s_{j,TK}$ because of following decomposition. The movement of prices with the market for both stocks is $\beta_j = \frac{\sigma_{j,m}}{\sigma_m^2}$ and $\beta_{TK} = \frac{\sigma_{TK,m}}{\sigma_m^2}$, respectively. After rearranging for the covariance and summarizing both terms the covariance $\sigma_{j,TK}$ can be expressed as:

$$\sigma_{j,TK} = \beta_j\beta_{TK}\sigma_m^2 \tag{A7}$$

Because β_j and β_{TK} are estimated in a regression we need to add the covariance of the residuals $s_{j,TK}$ as a new variable. It is obtained from a CAPM-style regression of market excess returns on j's and TK's returns:

$$\tilde{r}_j = r_f + \beta_j(\tilde{r}_m - r_f) + \tilde{\varepsilon}_j \tag{A8}$$

$$\tilde{r}_{TK} = r_f + \beta_{TK}(\tilde{r}_m - r_f) + \tilde{\varepsilon}_{TK}\ . \tag{A9}$$

The covariance $\sigma_{j,TK}$ now is the product of the estimated β_j, β_{TK} and σ_m^2 plus the covariance of the residuals from the regressions in (A8) and (A9). This simplifies equation (A6):

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \beta_j - \frac{\eta s_{j,TK}}{\sigma_m^2(1-\eta\beta_{TK})}\ . \tag{A10}$$

Under the additional assumption that the covariance of j's returns with returns of some stock i is zero, $cov(\varepsilon_i, \varepsilon_j) = 0$, the covariance of the residuals $s_{j,TK}$ is the variance of the residuals in J adjusted for the weights w_{TK}^j. We therefore get:

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \beta_j - \frac{\eta w_{TK}^j s_j^2}{\sigma_m^2(1-\eta\beta_{TK})}\ . \tag{A11}$$

We obtain the final equation of the model in Barberis et al. (2016) by substituting $w_{TK}^j = TK_j - \overline{TK}$ into (A11):

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \beta_j - \frac{\eta(TK_j - \overline{TK})s_j^2}{\sigma_m^2(1-\eta\beta_{TK})}\ . \tag{A12}$$

Appendix B: Proof for the partial derivative of η

This Appendix presents the formal proof for the partial derivative presented in equation (13). The starting point is equation (B1), where EX is the abbreviation for the fraction on the left hand side:

$$\frac{\mu_j - r_f}{\mu_m - r_f} = \beta_j - \frac{\eta(TK_j - \overline{TK})s_j^2}{\sigma_m^2(1-\eta\beta_{TK})}. \tag{B1}$$

$$\frac{\partial EX}{\partial \eta} = -\frac{\sigma_m^2(1-\eta\beta_{TK})\left((TK_j - \overline{TK})s_j^2\right) - (\eta(TK_j - \overline{TK})s_j^2 * (-\sigma_m^2\beta_{TK})}{\sigma_m^2(1-\eta\beta_{TK})\sigma_m^2(1-\eta\beta_{TK})} \tag{B2}$$

$$\frac{\partial EX}{\partial \eta} = -\frac{\sigma_m^2(1-\eta\beta_{TK})\left((TK_j - \overline{TK})s_j^2\right) - (TK_j - \overline{TK})s_j^2 * (-\sigma_m^2\eta\beta_{TK})}{\sigma_m^2(1-\eta\beta_{TK})\sigma_m^2(1-\eta\beta_{TK})} \tag{B3}$$

$$\frac{\partial EX}{\partial \eta} = -\frac{(\sigma_m^2 - \sigma_m^2\eta\beta_{TK})(TK_j - \overline{TK})s_j^2 - (TK_j - \overline{TK})s_j^2 * (-\sigma_m^2\eta\beta_{TK})}{\sigma_m^2(1-\eta\beta_{TK})\sigma_m^2(1-\eta\beta_{TK})} \tag{B4}$$

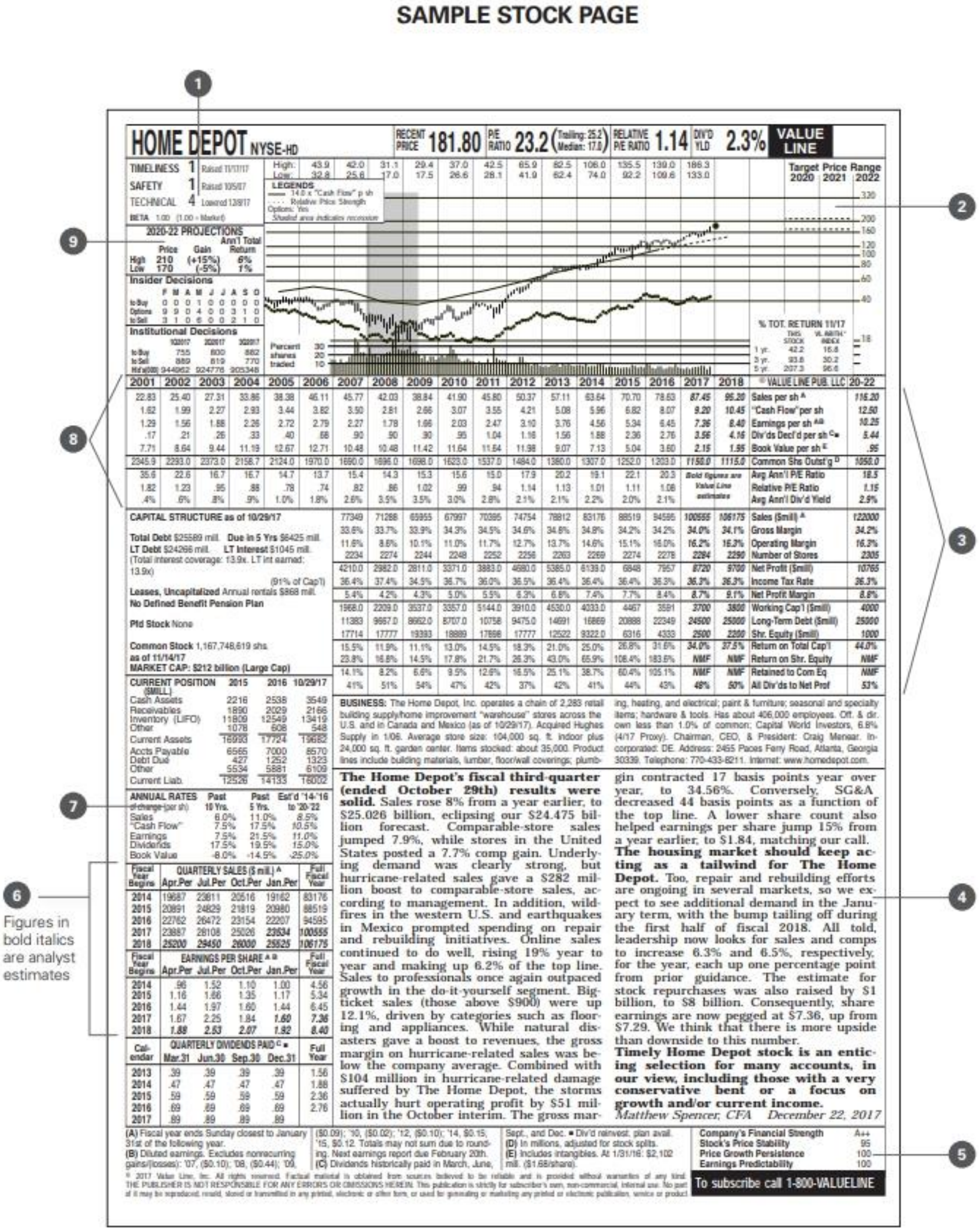
$$\frac{\partial EX}{\partial \eta} = -\frac{(TK_j - \overline{TK})s_j^2\left(\sigma_m^2 - \sigma_m^2\eta\beta_{TK} - (-\sigma_m^2\eta\beta_{TK})\right)}{\sigma_m^2(1-\eta\beta_{TK})\sigma_m^2(1-\eta\beta_{TK})} \tag{B5}$$

$$\frac{\partial EX}{\partial \eta} = -\frac{(TK_j - \overline{TK})s_j^2(\sigma_m^2 - \sigma_m^2\eta\beta_{TK} + \sigma_m^2\eta\beta_{TK})}{\sigma_m^2(1-\eta\beta_{TK})\sigma_m^2(1-\eta\beta_{TK})} \tag{B6}$$

$$\frac{\partial EX}{\partial \eta} = -\frac{\sigma_m^2(TK_j - \overline{TK})s_j^2}{\sigma_m^2(1-\eta\beta_{TK})\sigma_m^2(1-\eta\beta_{TK})} \tag{B7}$$

$$\frac{\partial EX}{\partial \eta} = -\frac{(TK_j - \overline{TK})s_j^2}{\sigma_m^2(1-\eta\beta_{TK})^2} \tag{B8}$$

Appendix C: Sample stock page



Appendix C: Value Line Investment Survey sample stock page (Taken from Product Guide – Value Line Investment Survey, 2018, p.14).
The figure presents the sample stock page from the Value Line Investment Survey. Barberis et al. (2016) assume that investors evaluate stocks based on the information presented in this prospectus.

Appendix D: Alternative specification of Table 7

Panel A	Non-Bubble				
	(1)	(2)	(3)	(4)	(5)
TK	**-0.078*****	**-0.067*****	**-0.039****	**-0.061****	**-0.077*****
	(-3.94)	(-4.42)	(-2.76)	(-3.23)	(-3.37)
Beta		0.162	0.219	0.325	0.410
		(0.63)	(0.86)	(1.43)	(1.67)
Size		**-0.122*****	**-0.121*****	**-0.133*****	**-0.133*****
		(-4.54)	(-4.56)	(-4.85)	(-4.87)
Bm		**0.203*****	**0.220*****	**0.202*****	**0.162*****
		(4.15)	(4.54)	(5.01)	(3.91)
Mom		**0.004****	0.002	**0.003***	0.003
		(2.67)	(1.51)	(2.16)	(1.59)
Illiq		**0.236****	**0.244****	**0.334*****	**0.392*****
		(3.00)	(3.12)	(4.21)	(4.46)
Rev			**-0.048*****	**-0.055*****	**-0.055*****
			(-13.80)	(-14.36)	(-14.20)
Ltrev				0.066	0.051
				(0.64)	(0.46)
Max				0.004	0.007
				(1.28)	(1.95)
Min				**-0.051*****	**-0.054*****
				(-8.82)	(-8.64)
Skew					**0.089***
					(2.16)
R-squared	0.015	0.044	0.049	0.056	0.060

t-statistics in parentheses *p<0.05 **p<0.01 ***p<0.001 (continued)

Appendix D: Fama-MacBeth regressions (Based on Barberis et al., 2016, p.3090).

Panel B	Bubble				
	(1)	(2)	(3)	(4)	(5)
TK	**-0.191*****	**-0.194*****	**-0.157*****	**-0.129***	**-0.147***
	(-4.64)	(-5.42)	(-4.46)	(-2.51)	(-2.11)
Beta		0.639	0.732	0.854	0.748
		(1.14)	(1.37)	(1.96)	(1.70)
Size		-0.145	-0.135	-0.148	-0.107
		(-1.40)	(-1.34)	(-1.54)	(-1.14)
Bm		0.175	0.195	0.104	0.130
		(0.97)	(1.10)	(0.80)	(1.03)
Mom		**0.012****	**0.010***	**0.009***	0.007
		(2.92)	(2.50)	(2.08)	(1.38)
Illiq		0.234	0.276	0.299	0.284
		(1.64)	(1.93)	(1.90)	(1.45)
Rev			**-0.051*****	**-0.062*****	**-0.066*****
			(-5.52)	(-6.11)	(-6.38)
Ltrev				-0.335	-0.092
				(-1.12)	(-0.27)
Max				0.017	0.007
				(1.64)	(0.69)
Min				**-0.040****	**-0.042*****
				(-3.16)	(-3.48)
Skew					0.154
					(1.28)
R-squared	0.012	0.050	0.057	0.066	0.069

t-statistics in parentheses *p<0.05 **p<0.01 ***p<0.001

Appendix D: Fama-MacBeth regressions (Based on Barberis et al., 2016, p.3090-3091).

The Table shows the results of the Fama-MacBeth approach. In each cross-sectional regression, percentage return is the dependent variable. *TK* is a stock's prospect theory value, measured at the beginning of the month using a distribution consisting of 36 monthly returns (see equation (7)). *Beta* is a stock's market beta, computed with the returns of 100 portfolios formed on size and pre-beta in June of each year, following Fama and French (1992). *Size* is the log equity market value at month t-1; the product of common shares outstanding in December of year t-1 and monthly stock price. *Bm* is the log book-to-market ratio calculated as the difference of the log book value and the log equity market value in December of year t-1. Book value is computed following Daniel and Titman (2006). *Mom* is a stock's cumulative return from month t-12 until t-1. *Illiq* is the illiquidity measure of Amihud (2002), scaled by 10^5. *Rev* is a stock's return in month t-1. *Ltrev* is a stock's cumulative return from month t-36 until t-13. *Max* is the highest daily return in month t-1, while *Min* is the negative of the lowest daily return. *Skew* is the sample skewness of monthly returns over the three five years. *TK*, *Rev, Mom*, *Max* and *Min* are scaled by factor 100. The entire sample starts in January 1983 and ends in December 2017. *Panel A* shows the results of the Fama-MacBeth regressions for the months (N=355) not stamped as a bubble using a method by Phillips, Shi and Yu (2015), while *Panel B* shows the results for months (N=65) stamped as a bubble using the same method. The numbers in brackets show the t-statistics and bold type characters indicate significance at the 5% level.

Appendix E: Complete Table 8

Panel A.	Non-Bubble			
	(1) Arbrisk	(2) Tvol	(3) Size	(4) Illiq
TK	0.140***	-0.310***	-0.213***	-0.068**
	(8.17)	(-5.41)	(-5.77)	(-3.16)
TKarbrisk	**-0.022****			
	(-3.07)			
TKtvol		**0.017*****		
		(4.34)		
TKsize			**0.023*****	
			(5.06)	
TKilliq				**-0.115*****
				(-4.85)
Beta	0.499*	0.498*	0.515*	0.509*
	(2.13)	(2.29)	(2.15)	(2.12)
Size	-0.071**	-0.151***	0.031	-0.147***
	(-2.76)	(-3.42)	(0.86)	(-5.29)
Bm	0.192***	0.172***	0.176***	0.172***
	(4.60)	(4.10)	(4.17)	(4.05)
Mom	-0.006**	0.002	0.002	0.002
	(-3.03)	(1.11)	(1.28)	(1.02)
Illiq	0.265**	0.226**	0.250**	-1.018***
	(3.20)	(3.14)	(3.15)	(-3.69)
Rev	-0.064***	-0.056***	-0.055***	-0.056***
	(-15.17)	(-14.65)	(-14.16)	(-14.10)
Ltrev	-0.538***	-0.058	-0.047	-0.074
	(-6.49)	(-0.83)	(-0.63)	(-1.00)
Max	0.002	0.006	0.007	0.008*
	(0.54)	(1.70)	(1.81)	(2.00)
Min	-0.066***	-0.054***	-0.055***	-0.052***
	(-11.04)	(-8.96)	(-8.78)	(-8.25)
Arbrisk	0.404***			
	(3.51)			
Tvol		0.135***		

t statistics in parentheses * p<0.05, ** p<0.01, *** p<0.001 (continued)

Appendix E: Prospect theory value and proxies for limits to arbitrage (Based on Barberis, Mukherjee and Wang (2016), p. 3096).

Panel B.	Bubble			
	(1) Arbrisk	(2) Tvol	(3) Size	(4) Illiq
TK	0.084	-0.169	-0.195	-0.184**
	(1.87)	(-1.20)	(-1.86)	(-3.26)
TKarbrisk	-0.032			
	(-1.52)			
TKtvol		-0.001		
		(-0.11)		
TKsize			0.001	
			(0.08)	
TKilliq				-0.047
				(-1.58)
Beta	0.613	0.765*	0.768	0.754
	(1.70)	(2.19)	(1.92)	(1.89)
Size	-0.049	-0.123	-0.111	-0.135
	(-0.60)	(-0.97)	(-1.12)	(-1.45)
Bm	0.193	0.168	0.172	0.173
	(1.76)	(1.43)	(1.44)	(1.46)
Mom	-0.003	0.007	0.007	0.007
	(-0.65)	(1.62)	(1.68)	(1.68)
Illiq	0.176	0.289	0.286	-0.417
	(1.03)	(1.75)	(1.61)	(-0.96)
Rev	-0.076***	-0.068***	-0.066***	-0.065***
	(-8.37)	(-6.96)	(-6.71)	(-6.61)
Ltrev	-0.678**	-0.066	-0.040	-0.046
	(-3.02)	(-0.36)	(-0.18)	(-0.21)
Max	0.002	0.023	0.016	0.017
	(0.25)	(1.78)	(1.52)	(1.49)
Min	-0.055***	-0.048***	-0.046***	-0.044***
	(-5.26)	(-4.23)	(-3.97)	(-3.77)
Arbrisk	0.457			
	(1.12)			
Tvol		-0.008		

t statistics in parentheses * p<0.05, ** p<0.01, *** p<0.001

Appendix E: Prospect theory value and proxies for limits to arbitrage (Based on Barberis, Mukherjee and Wang (2016), p. 3096).

The table reports the result of Fama-MacBeth regressions with the independent variable percentage return. *Arbrisk* is the residual variance from a standard market model regression on market returns from month t-36 to t-1. *Tvol* is the average daily dollar trading value over year t-1. *Size* is the log equity market value at month t-1; the product of common shares outstanding in December of year t-1 and monthly stock price. *Illiq* is the illiquidity measure of Amihud (2002), scaled by 10^5. *TK* is scaled by 100. *Panel A* shows the results of the Fama-MacBeth regressions for the months (N=355) not stamped as a bubble using a method by Phillips, Shi and Yu (2015), while *Panel B* shows the results for months (N=65) stamped as a bubble using the same method. The numbers in brackets show the t-statistics.

Appendix F: Interactions of prospect theory components

	Non-Bubble		Bubble	
	LAPW	TK	LAPW	TK
TK	-0.239***	-0.256***	-0.537***	-0.612**
	(-3.86)	(-3.68)	(-3.45)	(-3.36)
Beta	0.160*	0.153*	0.240	0.225
	(2.20)	(2.15)	(1.94)	(1.90)
Size	-0.297***	-0.283***	-0.308	-0.270
	(-5.03)	(-4.81)	(-1.54)	(-1.37)
Bm	0.139***	0.139***	0.136	0.140
	(3.99)	(3.99)	(1.38)	(1.44)
Mom	0.060	0.063	0.260	0.276
	(0.98)	(1.04)	(1.60)	(1.68)
Illiq	0.207***	0.207***	0.170	0.170
	(4.10)	(4.09)	(1.58)	(1.57)
Rev	-0.689***	-0.689***	-0.811***	-0.809***
	(-14.09)	(-14.17)	(-6.59)	(-6.60)
Ltrev	-0.066	-0.057	-0.084	-0.037
	(-1.23)	(-1.01)	(-0.54)	(-0.23)
Max	0.459*	0.440	1.063	0.988
	(2.00)	(1.94)	(1.55)	(1.48)
Min	-0.266***	-0.268***	-0.224***	-0.231***
	(-8.13)	(-8.30)	(-3.59)	(-3.79)

t statistics in parentheses * $p<0.05$, ** $p<0.01$, *** $p<0.001$

Appendix F: Relative importance of prospect theory components (Based on Barberis, Mukherjee and Wang (2016), p.3101).

The table presents the results of Fama-MacBeth regressions on the monthly percentage return as dependent variable. The independent control variables are the same as in Table 7, but normalized to have mean zero and standard deviation one. Column (LAPW) is the prospect theory value computed with the parameters for probability weighting (PW) and loss aversion (LA) active. TK is the variable form Table 7, but standardized. The sample runs from January 1983 until December 2017, but is split into bubble (N=355) and non-bubble (N=65) periods. T-statistics are in parentheses.

Appendix G: Decile sorts on expected utility

Deciles	EU	Beta	Size	Bm	Rev	Illiq	Ltrev	Skew	Sd
1	-0.388	1.237	4.767	-0.905	0.027	0.262	1.120	0.736	0.205
2	-0.336	1.228	5.125	-0.681	0.018	0.223	0.732	0.636	0.171
3	-0.311	1.193	5.498	-0.584	0.015	0.188	0.585	0.510	0.150
4	-0.295	1.143	5.861	-0.546	0.013	0.137	0.483	0.369	0.131
5	-0.282	1.091	6.191	-0.540	0.011	0.103	0.405	0.206	0.116
6	-0.273	1.044	6.514	-0.534	0.008	0.078	0.330	0.034	0.102
7	-0.265	0.992	6.701	-0.518	0.006	0.062	0.246	-0.121	0.091
8	-0.259	0.935	6.716	-0.479	0.002	0.055	0.153	-0.247	0.080
9	-0.253	0.876	6.372	-0.399	-0.001	0.052	0.028	-0.371	0.068
10	-0.245	0.855	5.735	-0.227	-0.009	0.079	-0.184	-0.600	0.060
Total	-0.291	1.060	5.948	-0.541	0.010	0.124	0.390	0.115	0.117

Appendix G: Expected utility value portfolios (Based on Barberis, Mukherjee and Wang (2016), p.3100).

The Table reports the time series average of the monthly mean statistic of variables for deciles formed on EU. The description of variables can be found in Table 2. *Sd* is the standard deviation of monthly returns over the past five years. The sample starts in January 1983 and ends in December 2017.